education

Department:
Education
PROVINCE OF KWAZULU-NATAL

CURRICULUM GRADE 10 -12 DIRECTORATE

NCS (CAPS)

LEARNER SUPPORT

DOCUMENT GRADE 11

MATHEMATICS

STEP AHEAD PROGRAMME

2021

PREFACE

This support document serves to assist Mathematics learners on how to deal with curriculum gaps and learning losses as a result of the impact of COVID-19 in 2020. It also captures the challenging topics in the Grade 10 -12 work. Activities should serve as a guide on how various topics are assessed at different cognitive levels and also preparing learners for informal and formal tasks in Mathematics. It will cover the following topics:

TOPIC: Functions (Lesson 1)	Weighting	30 Marks	Grade	10

RESOURCES

Textbooks/ Handouts

NOTES:

What is a function?

A function is a mathematical relationship between two variables, where **every input** variable has **one** output variable.

Functions can be either:

a) One-to-one: where every single input variable has a unique output variable

Input (x)	-2	-1	0	1	2	3
Output (y)	8	2	2	-1	-4	-7

Notice that every **one** input value has **one** unique output value, hence **one-to-one.**

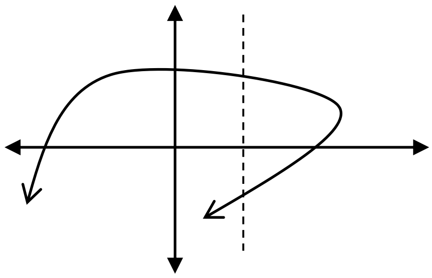
b) Many-to-one: where **two or more different input** values have **one** output value.

Input (x)	-4	-1	0	1	2	3
Output (y)	33	3	1	3	9	19

Notice that inputs -1 and 1 have the same output value, hence **many-to-one**.

But in both situations, there is no input value that has more than one output value, therefore both are functions.

Below is an example of a non-function:



Notice at the point marked by a dotted line, for **one input** value there is **more than one output**, thus **one-to-many**, therefore **not a function**.

Dependent and independent variables:

In functions, the x-variable is known as the input or independent variable, because its value can be chosen freely. The calculated y-variable is known as the output or dependent variable, because its value depends on the chosen input value.

Function Notation:

This is a very useful way to express a function. Another way of writing $y = 2x + 1$ is $f(x) = 2x + 1$. We say *f* of *x* is equal to 2*x* + 1. Any letter can be used, for example, *g(x), h(x), p(x)*, etc.

1. Determining the output value:

Find the value of the function for $x = -3$ can be written as: find *f(−3)*. Replace *x* with −3:

$f(-3) = 2(-3) + 1 = -5$

$\therefore f(-3) = -5$

This means that when $x = -3$, the value of the function is -5

2. Determining the output value:

Find the value of x that will give a y-value of 27 can be written as: find x if $f(x) = 27$.

We write the following equation and solve for x:

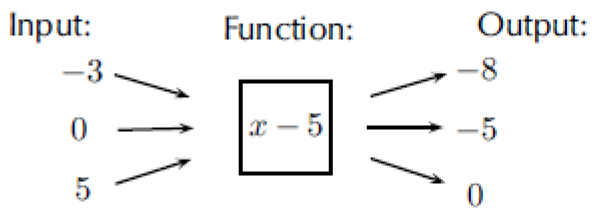
$2x + 1 = 27$

$\therefore x = 13$

This means that when $x = 13$ the value of the function is 27.

Functions can be expressed in many different ways for different purposes.

1. Words: "The relationship between two variables is such that one is always 5 less than the other."
2. Mapping diagram:

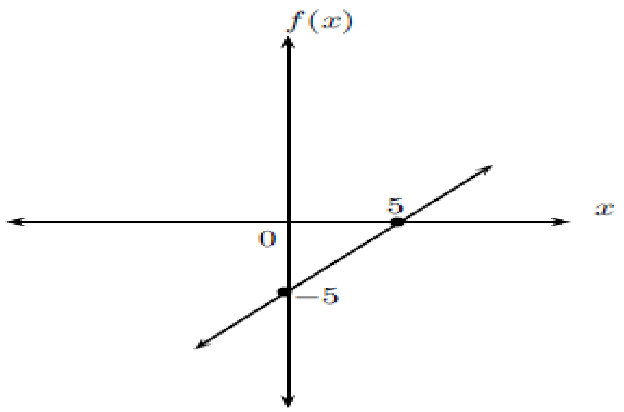


3. Table:

Input variable (x)	-3	0	5
Output variable (y)	-8	-5	0

4. Set of ordered number pairs: $(-3; -8)$, $(0; -5)$, $(5; 0)$
5. Algebraic formula: $f(x) = x - 5$

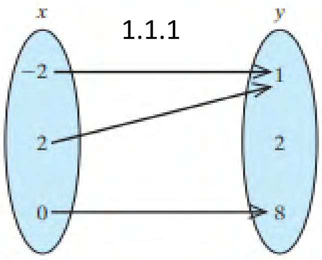
6. Graph:



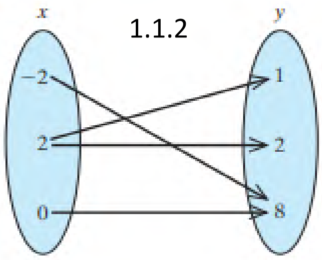
ACTIVITY 1.1

1.1 Using the notes provided, determine whether the following relations are functions or not. If it is a function, state whether it is a one-to-one or many-to-one function.

1.1.1



1.1.2



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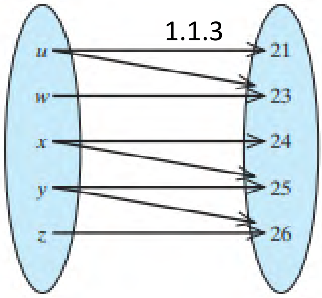
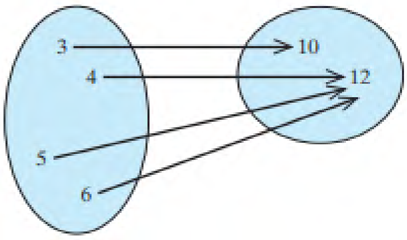


1.1.4



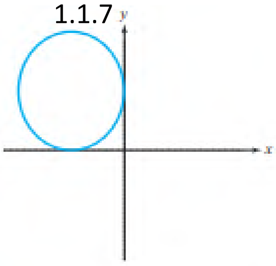
1.1.5

$\{(1,2),(3,4),(5,4),(-9,3)\}$

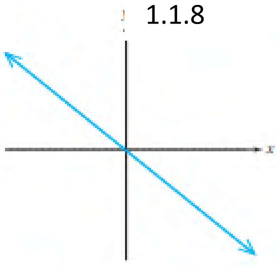
1.1.6

$\left\{(0,-1.1),\left(\frac{1}{2},8\right),(1.1,8),\left(4,\frac{1}{2}\right)\right\}$

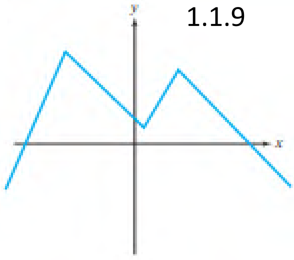
1.1.7



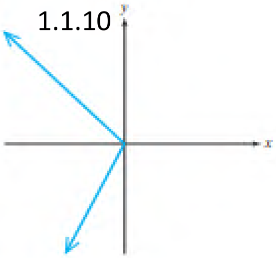
1.1.8



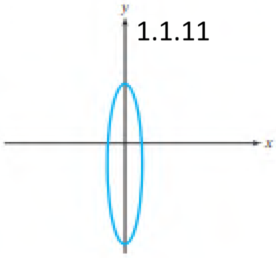
1.1.9



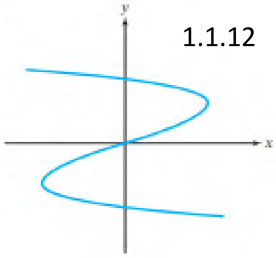
1.1.10



1.1.11



1.1.12



TOPIC: Functions: Lesson 2	Weighting	30	Grade	10

RESOURCES

Mind Action Series Gr. 10 page 103 – 136

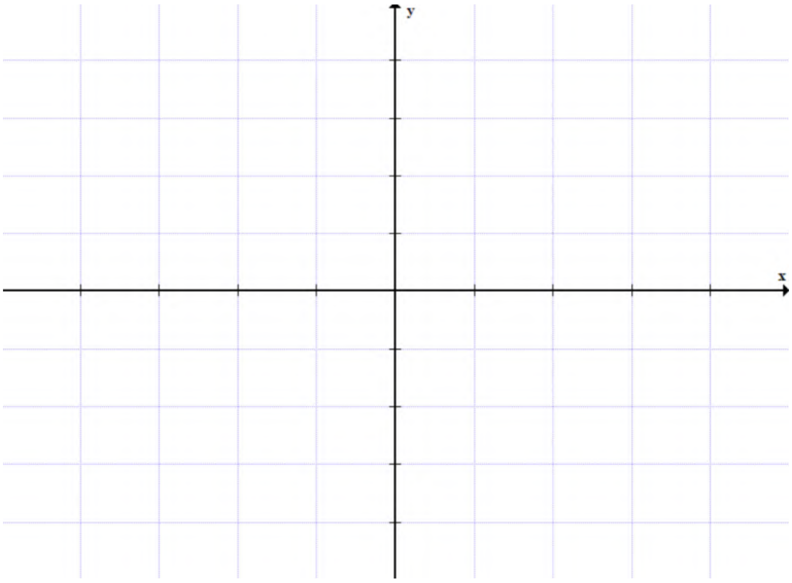
ACTIVITY 1.2 (INVESTIGATIVE APPROACH)

1.2.1. The effect of a on the linear function defined by $y = ax$

a) Complete the table below:

x	-3	-2	-1	0	1	2	3
$y = x$							
$y = \frac{1}{2}x$							
$y = 2x$							
$y = 3x$							

b) Now plot each of the functions on the same grid provided below. Use a different color for each function.



What if *a* is negative?

c) Complete the table below:

x	-3	-2	-1	0	1	2	3
$y=-x$							
$y=-\frac{1}{2}x$							
$y=-2x$							
$y=-3x$							

d) On the Cartesian plane provided in number b), plot these four more functions.

e) By looking at your graphs, describe the transformation from $y=x$ to $y=-x$, $y=\frac{1}{2}x$ to $y=-\frac{1}{2}x$, $y=2x$ to $y=-2x$ and $y=3x$ to $y=-3x$.

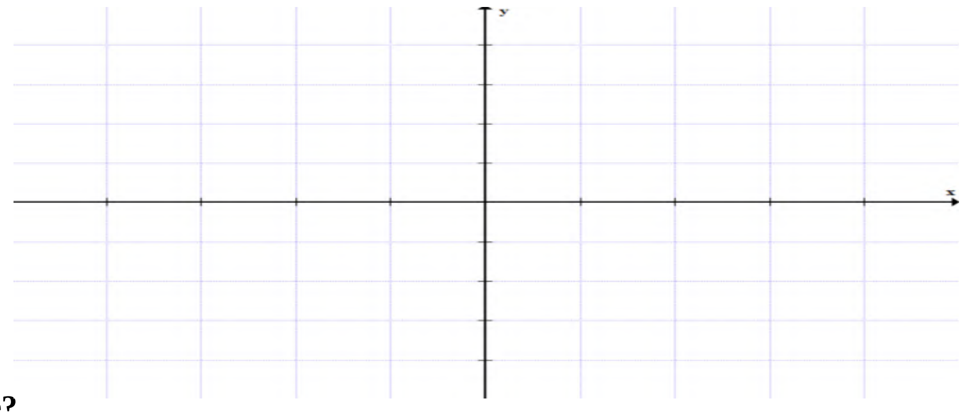
f) What conclusion can be made about the effect of a on $y=ax$?

1.2.2. The effect of q on the linear function defined by $y=ax+q$

a) Complete the table below:

x	-3	-2	-1	0	1	2	3
$y=x$							
$y=x+3$							
$y=x-2$							

b) Now plot each of the functions on the same grid provided below. Use a different color for each function.



What if *a* is negative?

c) Complete the table below and plot each of these functions on the same grid provided in number b)

x	-3	-2	-1	0	1	2	3
$y=-x+1$							
$y=-x-2$							

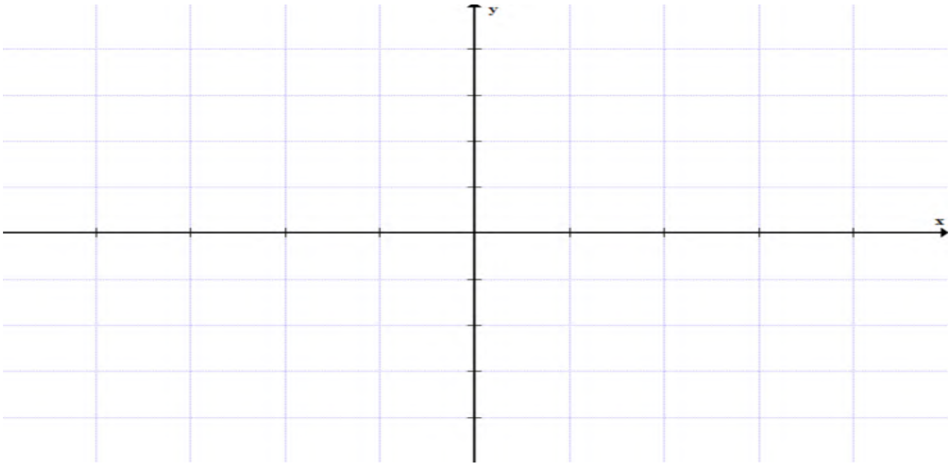
d) Using the data obtained, what can be concluded about the effect of q on the function defined by $y = ax + q$.

1.2.1. The effect of a on the quadratic function defined by $y = ax^2$

a) Complete the table below:

x	-3	-2	-1	0	1	2	3
$y = x^2$							
$y = \frac{1}{2}x^2$							
$y = 2x^2$							
$y = 3x^2$							

b) Now plot each of the functions on the same grid provided below. Use a different color for each function.



c) What conclusion can be made about the effect of a on $y = ax^2$?

What if *a* is negative?

d) Complete this table below and on the same grid provided in b) above, plot these functions.

x	-3	-2	-1	0	1	2	3
$y = -x^2$							
$y = -\frac{1}{2}x^2$							
$y = -2x^2$							
$y = -3x^2$							

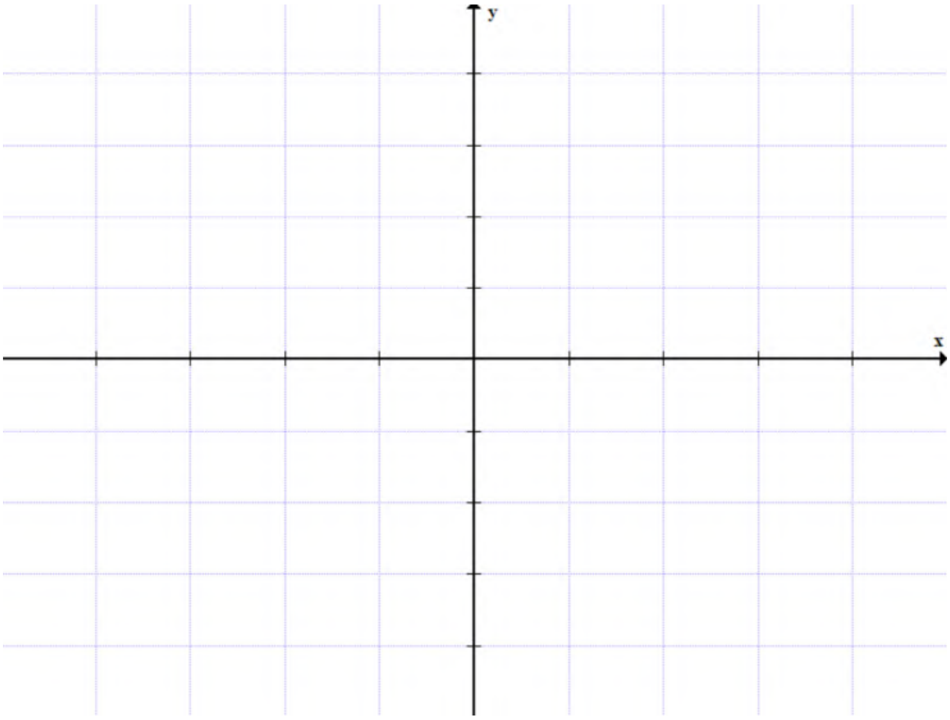
e) What relationship exists between the graphs of $y = x^2$ and $y = -x^2$, $y = \frac{1}{2}x^2$ and $y = \frac{1}{2}x^2$, $y = 2x^2$ and $y = -2x^2$ then $y = 3x^2$ and $y = -3x^2$.

1.2.2. The effect of q on the quadratic function defined by $y = ax^2 + q$

a) Complete the table below:

x	-3	-2	-1	0	1	2	3
$y = x^2$							
$y = x^2 - 1$							
$y = x^2 + 2$							

b) Plot each of the functions on the same grid provided below. Use a different color for each function.



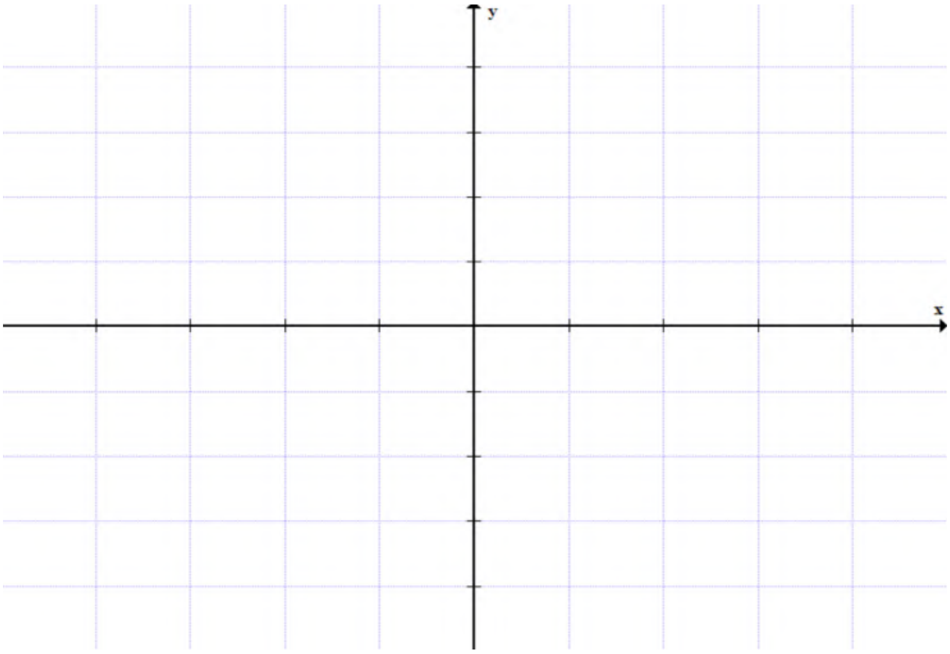
c) What conclusion can be made about the effect of q on the function $y = ax^2 + q$?

1.2.5. The effect of a on the hyperbolic function defined by $y = \frac{a}{x}$

a) Complete the table below:

x	-3	-2	-1	0	1	2	3
$y = \frac{1}{x}$							
$y = \frac{2}{x}$							
$y = \frac{4}{x}$							

b) Now plot each of the functions on the same grid provided below. Use a different color for each function.



NOTE: On the graphs drawn:

- An asymptote is a horizontal or vertical line that a graph approaches but never touches.
- The vertical line $x = 0$ which lies on the y-axis is called the vertical asymptote of the graph.
- The horizontal line $y = 0$ which lies on the x-axis is called the horizontal asymptote of the graph.

c) What conclusion can be made about the effect of *a* on $y = \frac{a}{x}$?

What if *a* is negative?

d) Complete this table and on the same set of axes provided in b), plot these functions.

x	-3	-2	-1	0	1	2	3
$y = \frac{-1}{x}$							
$y = \frac{-2}{x}$							
$y = \frac{-4}{x}$							

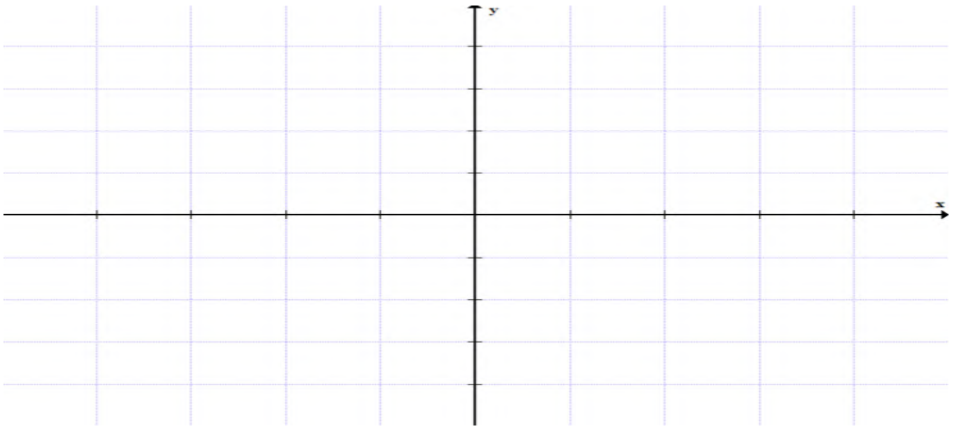
e) How are the functions drawn in b) get transformed when the sign of *a* is changed?

1.2.6. The effect of *q* on the hyperbolic function defined by $y = \frac{a}{x} + q$

a) Complete the table below:

x	-3	-2	-1	0	1	2	3
$y = \frac{1}{x}$							
$y = \frac{1}{x} - 3$							
$y = \frac{1}{x} + 2$							

b) On the new set of axes below, plot the three functions above. Use a different color for each.



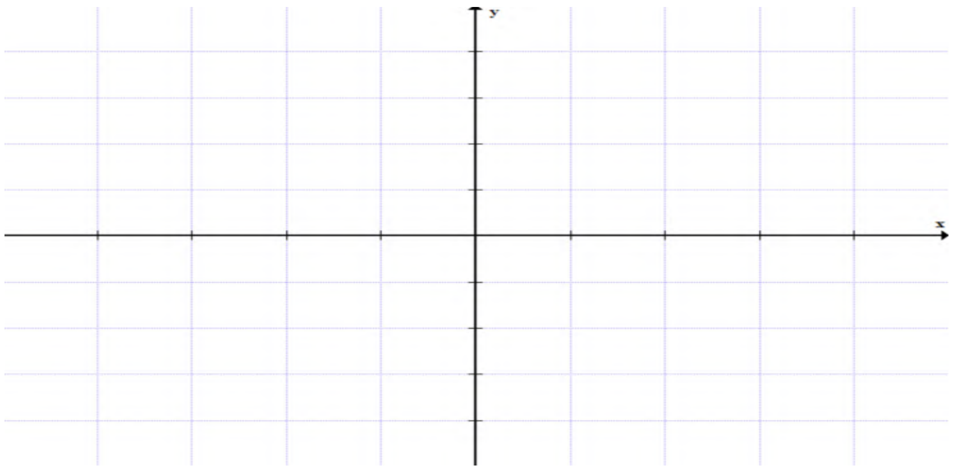
c) What effect does changing q have on the function $y = \frac{a}{x} + q$?

1.2.7. The effect of b on the exponential function defined by $y = b^x$ **where** $b > 1$

a) Complete the table below:

x	-3	-2	-1	0	1	2	3
$y = 2^x$							
$y = 3^x$							
$y = 4^x$							

b) Now plot each of the functions on the same grid provided below. Use a different color for each function.



When 0 < *b* < 1:

c) Complete this table and on the grid provided in b) plot these functions.

x	-3	-2	-1	0	1	2	3
$y = \left(\frac{1}{2}\right)^x$							
$y = \left(\frac{1}{3}\right)^x$							
$y = \left(\frac{1}{4}\right)^x$							

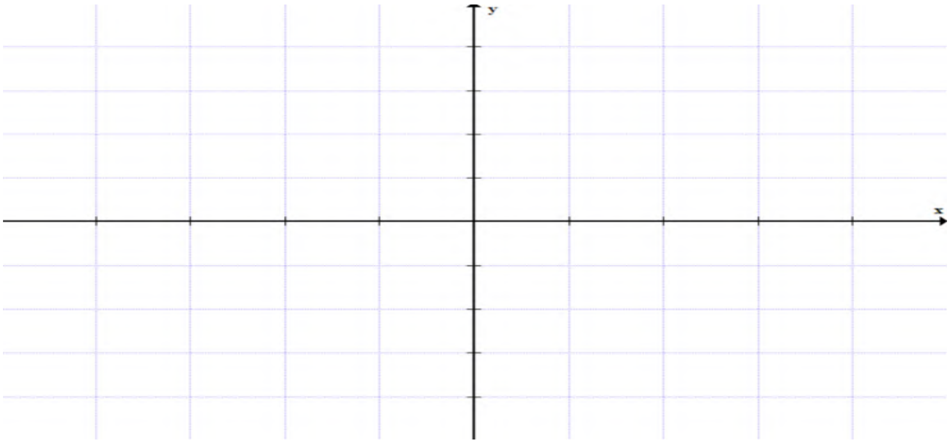
d) What conclusion can be made about the effect of b on $y = b^x$?

1.2.8. The effect of a on the exponential function defined by $y = a \times b^x$

a) Complete the following table:

x	-3	-2	-1	0	1	2	3
$y = 2^x$							
$y = 2 \times 2^x$							
$y = 3 \times 2^x$							
$y = 2 \times \left(\frac{1}{2}\right)^x$							

b) Now plot each of the functions on the same grid provided below. Use a different color for each function.



c) What conclusion can be made about the effect of a on $y = a \times b^x$?

What if *a* is negative?

d) Complete this table and plot these functions on the same grid provided in b) above.

x	-3	-2	-1	0	1	2	3
$y = -2^x$							
$y = -2 \times 2^x$							
$y = -3 \times 2^x$							
$y = -2 \times \left(\frac{1}{2}\right)^x$							

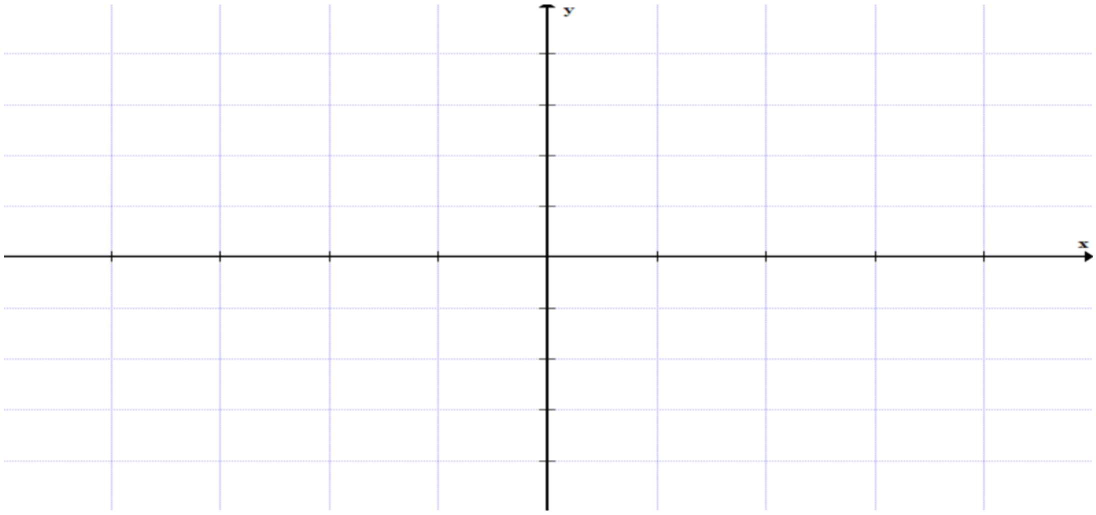
e) What effect does changing the sign of a have on the function $y = a \times b^x$?

1.2.9. Effect of q on $y = a \times b^x + q$

a) Complete the table below:

x	-3	-2	-1	0	1	2	3
$y = 2^x$							
$y = 2^x + 2$							
$y = 2^x - 3$							

b) Now plot each of the functions on the same grid provided below. Use a different color for each function.



c) What conclusion can be made about the effect of q on $y = a \times b^{x} + q$?

1.2.10 If the following is given, indicate the type of transformation that took place on each:

$y = 2^{x}$ transforms to $y = \dfrac{2^{-x}}{32}$

ADDITIONAL ACTIVITY

1. Did you notice that in the exponential function $y = a.b^{x} + q$ we only investigated the effect of b when $b > 1$ and when $0 < b < 1$?
   Investigate what happens when $b \leq 0$
   NOTE: When doing the above, please note that in $y = -2^{x}$, it is *a* that is less than zero not be. You are required to investigate what happens when *b* is less than zero. For example $y = (-2)^{x}$.
2. Write a summary of what each parameter does on different function.

   NOTE: Learners will read their summaries and together with the educator, come up with a comprehensive summary of this investigation.

TOPIC: Functions: (Lesson 3)	Weighting	30	Grade	10

RESOURCES

Mind Action Series Gr. 10 page 136 – 153

ACTIVITIES

1.3.1. Identify the type of graph (Linear, Parabola, Hyperbola and or Exponential) and using **dual intercept or table method**, draw a sketch graph of the following on different set of axes.

a) $y = 2x$

b) $y = \dfrac{2}{x}$

c) $y = 2x^{2}$

d) $g(x) = 2$

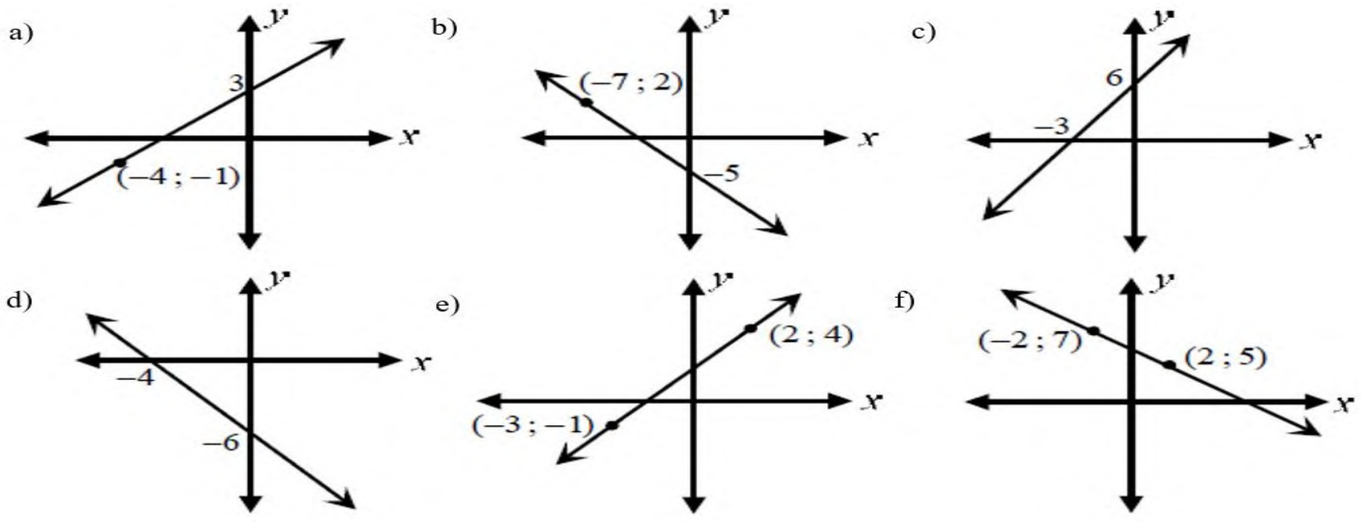
e) $f(x) = 2^{x}$

f) $h(x) = x^{2} + 2$

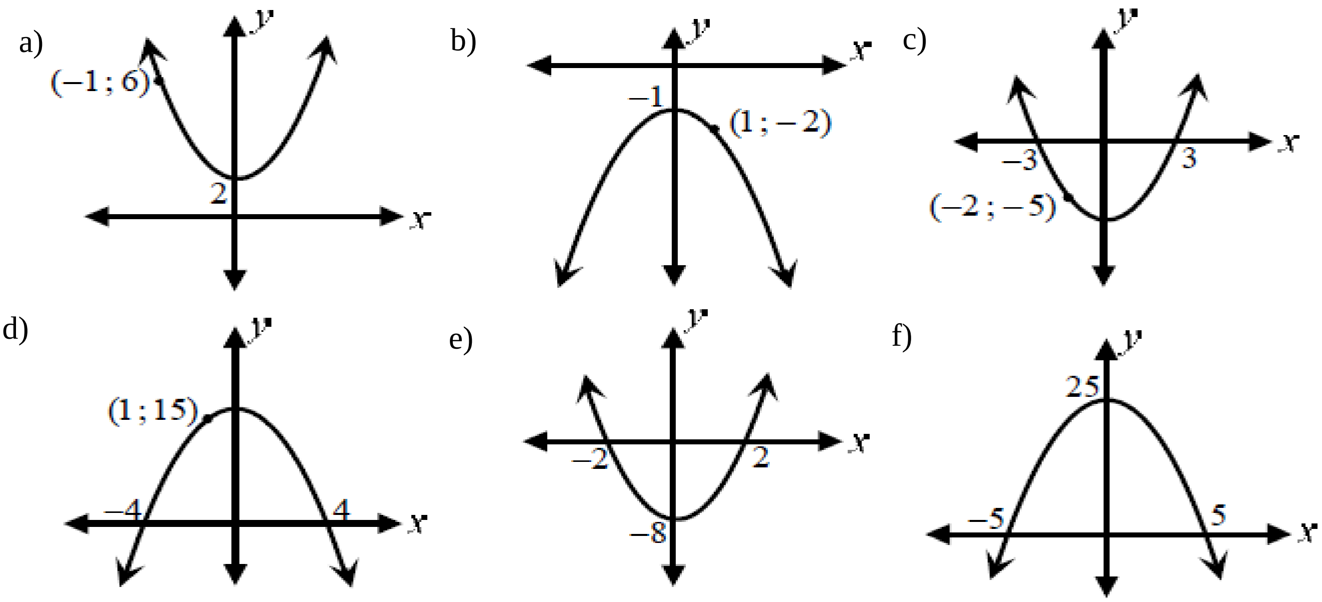
g) $p(x)=-2x^2+2$ h) $t(x)=-\frac{2}{x}+1$ i) $t(x)=-\left(\frac{1}{2}\right)^x+1$

j) $f(x)=-2x-2$ k) $x=2$ l) $k(x)=2.2^x-2$

1.3.2. Determine the equation of each of the following lines in the form $f(x)=ax+q$

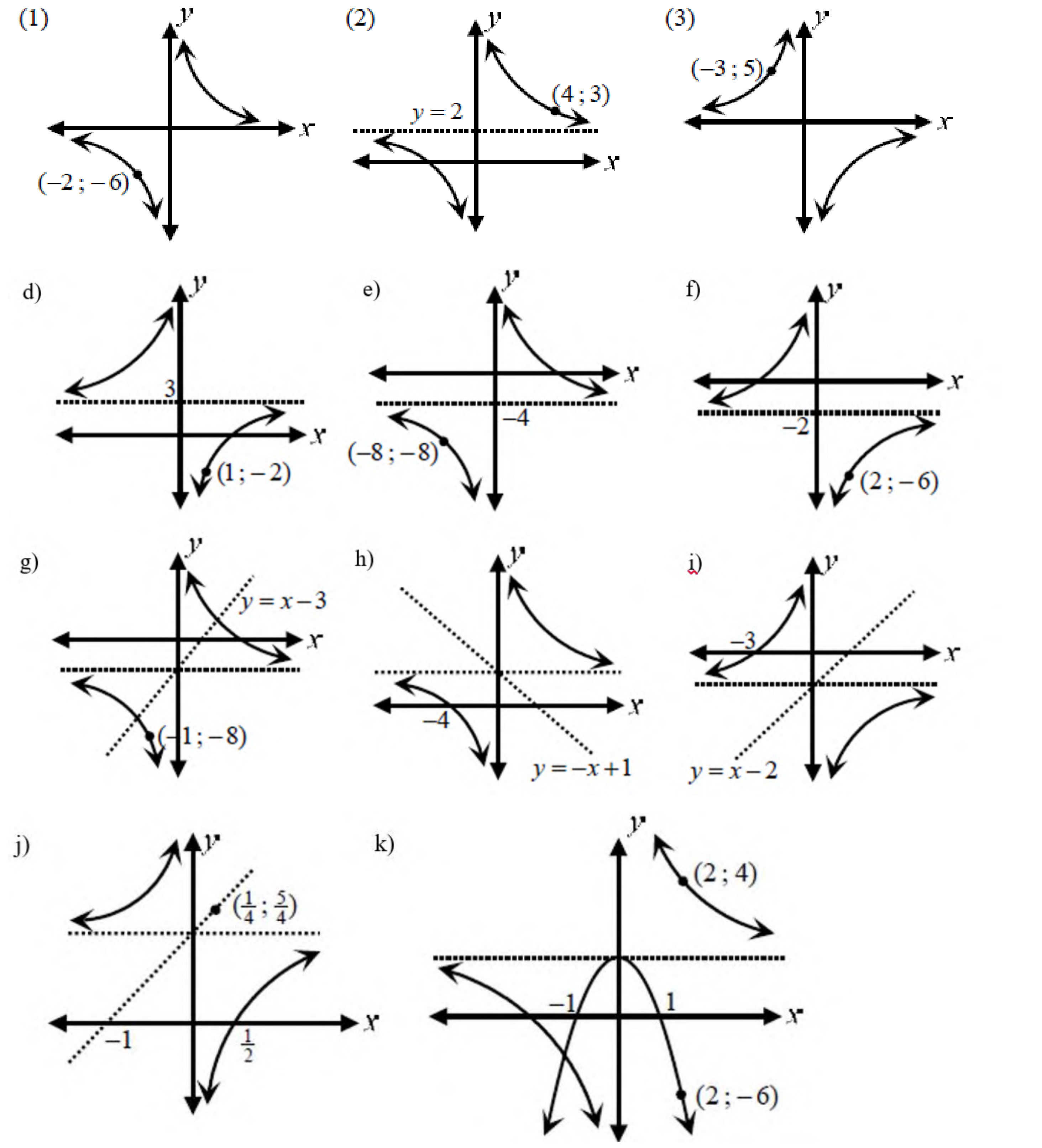


1.3.3 Determine the equation of each of the following parabolas in the form $f(x)=ax^2+q$



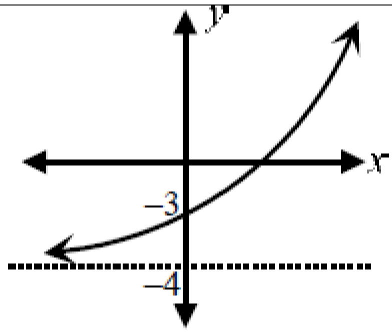
1.3.4. Determine the equation of each of the following hyperbolas in the form $f(x)=\frac{a}{x}+q$

a) b) c)

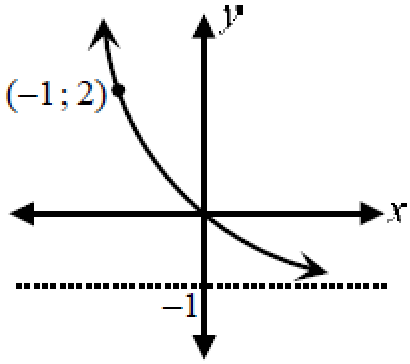


1.3.5 Do the following:

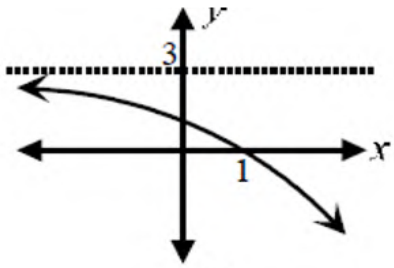
(a) Determine the equation of the given graph in the form $f(x) = a.2^x + q$



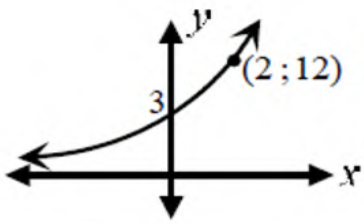
(b) Determine the equation of the given graph in the form $g(x) = b^x + q$



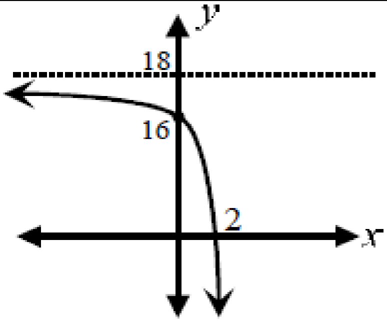
(c) Determine the equation of the given graph in the form $h(x) = -b^x + q$



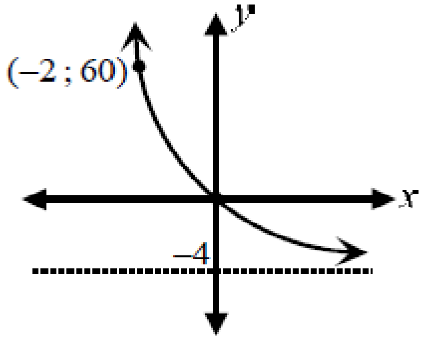
(d) Determine the equation of the given graph in the form $f(x) = a.b^x + q$



(e) Determine the equation of the given graph in the form $f(x)=a.b^x+q$



(f) Determine the equation of the given graph in the form $g(x)=a.b^x+q$



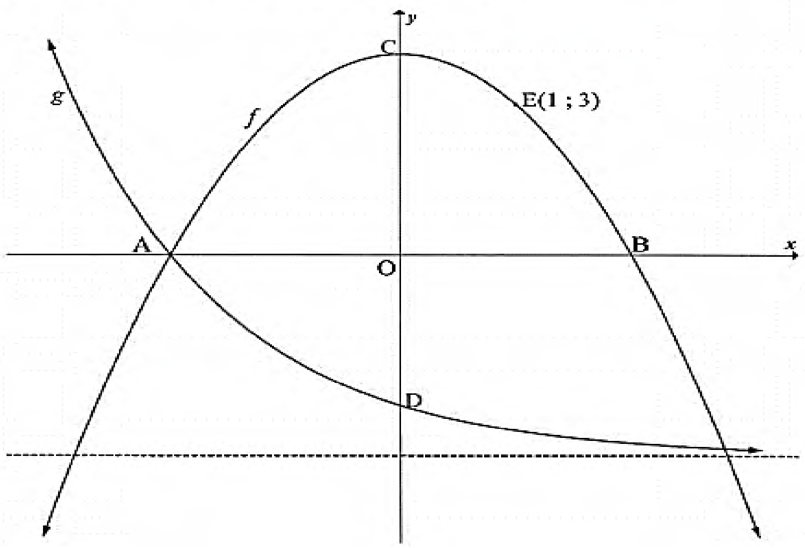
ADDITIONAL ACTIVITY: November 2018, DBE Gr.10

1.3.6

QUESTION 5

Sketched below are the graphs of $f(x)=ax^2+q$ and $g(x)=\left(\frac{1}{2}\right)^x-4$.

A and B are the x-intercepts of f. The graphs intersect at A and point E (1 ; 3) lies on f. C is the turning point of f and D is the y-intercept of g.



1.3.6.1 Write down the:

a. Coordinates of D (2)

b. Range of g (1)

1.3.6.2 Calculate the:

a. Coordinates of A (2)

b. Values of a and q (4)

1.3.6.3 Determine the:

a. Length of CD (2)

b. Equation of a straight line through A and D (3)

1.3.6.4 For which values of x is:

a. $f(x) > 0$? (2)

b. f decreasing? (1)
[17]

1.3.7

The equation of the function $g(x) = \frac{a}{x} + q$ passes through the point $(3;2)$ and has a range of $y \in (-\infty; 1) \cup (1; \infty)$.

1.3.7.1 Determine the:

a. Equation of g (3)

b. Equation of h, the axis of symmetry of g which has a positive gradient (2)

1.3.7.2 Sketch the graphs of g and h on the same system of axes. Clearly show ALL the asymptotes and intercepts with axes. (4)

1.3.7.3 Write the equations of the asymptotes of f if $f(x) = -g(x) + 5$. (3)
[12]

TOPIC: Functions: (Lesson 4)	Weighting	30	Grade	10
RESOURCES				
Mind Action Series Gr. 10 page 86 – 93				

For the following lesson, you need to know how basic trigonometric functions $y=\sin\theta$, $\cos\theta$ and $\tan\theta$ look like and know their properties namely: minimum and maximum value, range, and amplitude. So, let us start by discussing two of these three basic functions.

Consider:

a) $y=\sin\theta$

The table below represents the specific values of θ and the corresponding y values.

θ	$0°$	$30°$	$45°$	$60°$	$90°$	$120°$	$135°$	$150°$	$180°$
$y=\sin\theta$	0	0,5	0,7	0,9	1	0,9	0,7	0,5	0

θ	$210°$	$225°$	$240°$	$270°$	$300°$	$315°$	$330°$	$360°$
$y=\sin\theta$	−0,5	−0,7	−0,9	−1	−0,9	−0,7	−0,5	0

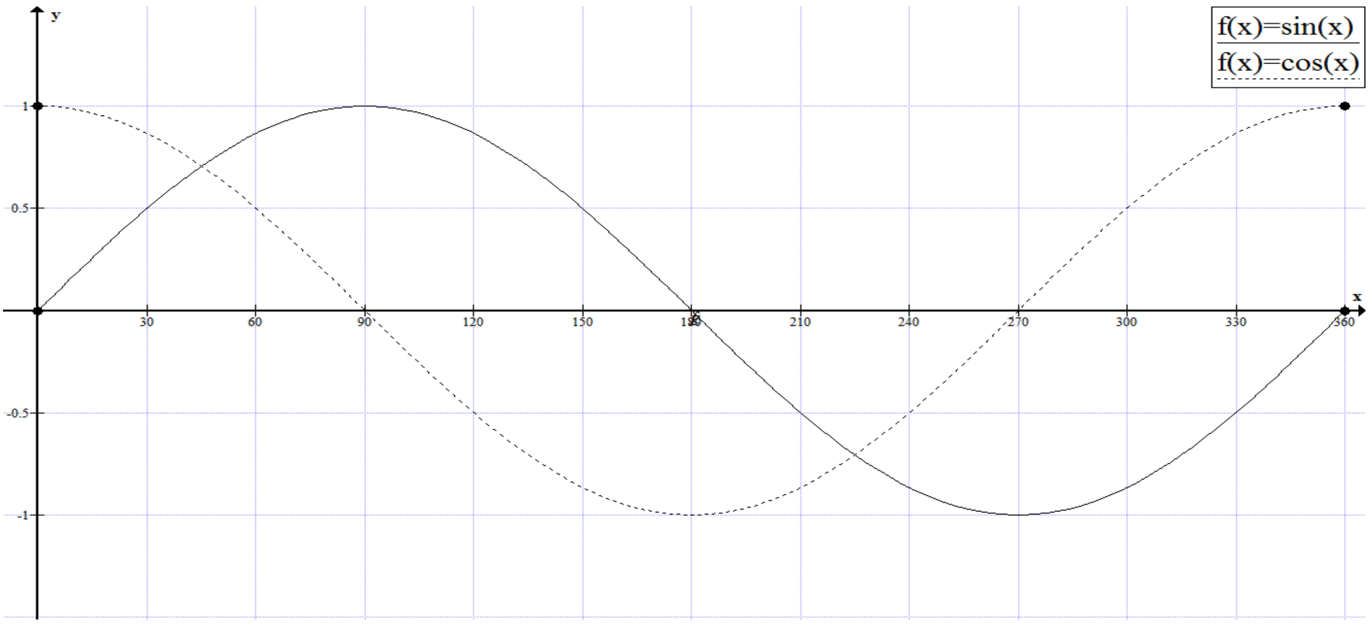
b) $y=\cos\theta$

The table below represents the specific values of θ and the corresponding y values.

θ	$0°$	$30°$	$45°$	$60°$	$90°$	$120°$	$135°$	$150°$	$180°$
$y=\cos\theta$	1	0,9	0,7	0,5	0	−0,5	−0,7	−0,9	−1

θ	$210°$	$225°$	$240°$	$270°$	$300°$	$315°$	$330°$	$360°$
$y=\cos\theta$	−0,9	−0,7	−0,5	0	0,5	0,7	0,9	1

The values of θ can be represented on the *x-axis* and the *y* values on the *y-axis* of a Cartesian plane and then these two function can be drawn as shown below:



The table below summarizes the characteristics of the two functions.

	Minimum value	Maximum value	Range	Amplitude	Period
$y=\sin\theta$	-1	1	$y\in[-1;1]$	$\frac{1}{2}[1-(-1)]=1$	$360°$
$y=\cos\theta$	-1	1	$y\in[-1;1]$	$\frac{1}{2}[1-(-1)]=1$	$360°$

Note: The amplitude of a graph is defined to be $\frac{1}{2}$ [distance between maximum and minimum]

A full basic graph of $y=\sin\theta$ and $y=\cos\theta$ is completed over a period of $360°$. Therefore the period of these two graphs is $360°$. This will be explored more in grade 11 trigonometric functions.

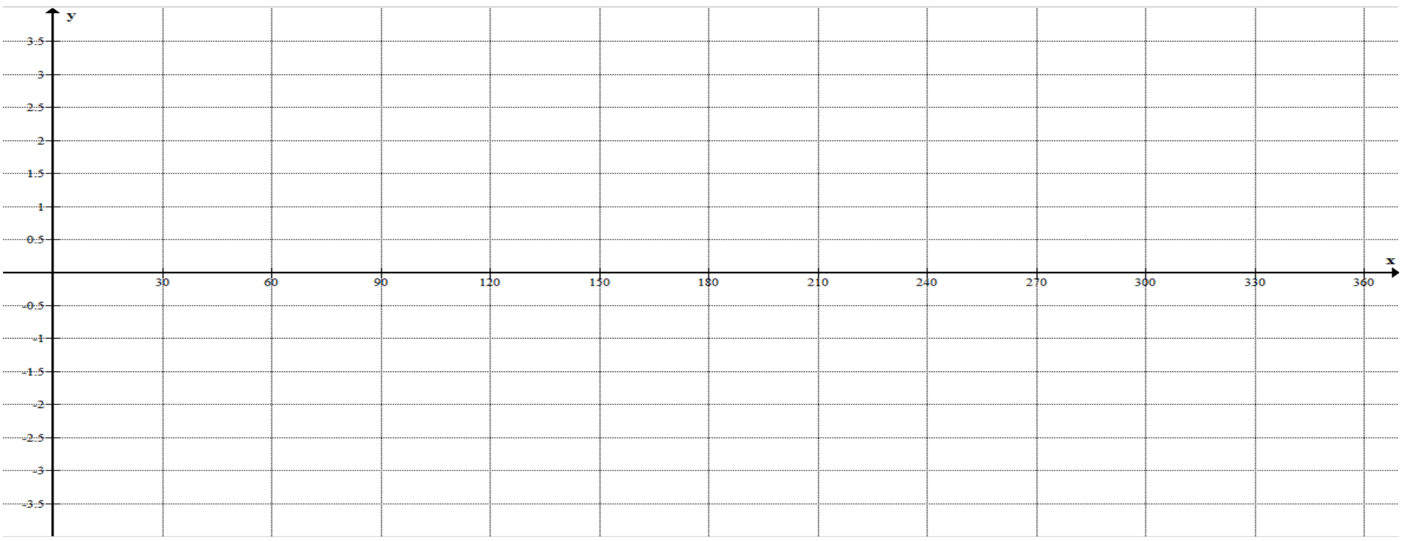
Now you will investigate the effect of *a* and *q* on the graphs of $y=a\sin\theta+q$ **and** $y=a\cos\theta+q$

1.4.1. The effect of a on $y=a\sin\theta$ and $y=a\cos\theta$

a) Complete the table below:

	30	60	90	120	150	180	210	240	270	300	330	360
$y=\sin\theta$												
$y=\cos\theta$												
$y=2\sin\theta$												
$y=2\cos\theta$												
$y=3\sin\theta$												
$y=3\cos\theta$												

b) Now plot these graphs on the same grid provided below.



c) Use your graphs to complete the table below:

	Minimum value	Maximum value	Range	Amplitude	Period
$y = \sin\theta$					
$y = \cos\theta$					
$y = 2\sin\theta$					
$y = 2\cos\theta$					
$y = 3\sin\theta$					
$y = 3\cos\theta$					

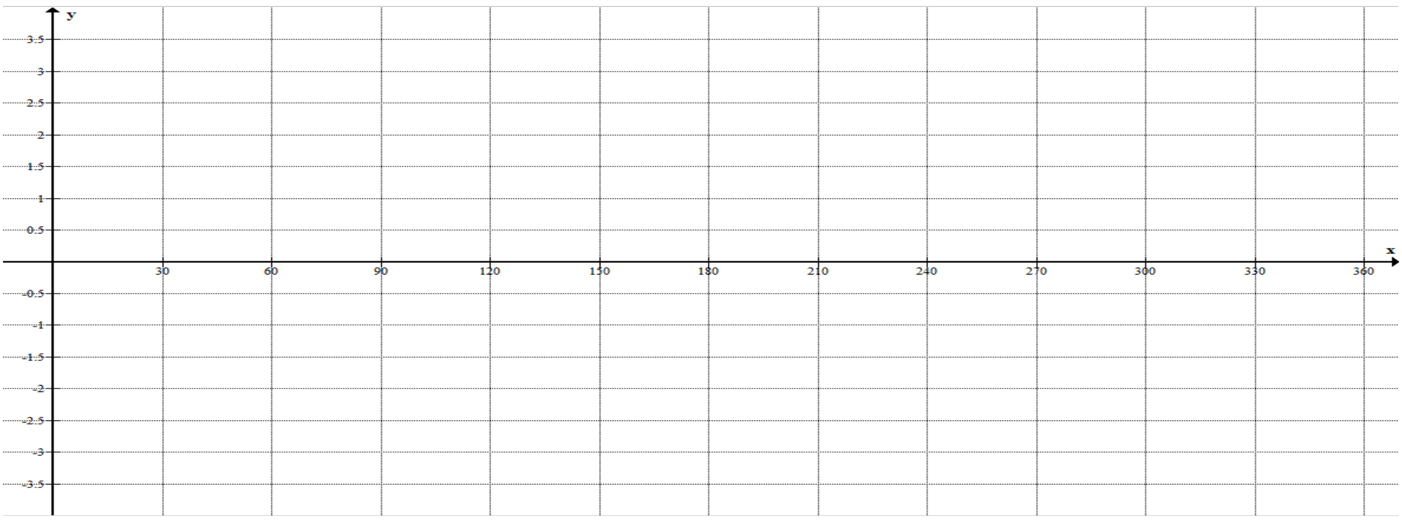
d) What can be concluded about the effect of *a* on functions defined by $y = a\sin\theta$ and $y = a\cos\theta$?

What if *a* is negative?

e) Complete the table below:

	30	60	90	120	150	180	210	240	270	300	330	360
$y = 2\sin\theta$												
$y = -2\sin\theta$												
$y = 3\cos\theta$												
$y = -3\cos\theta$												

f) Now plot these graphs on the same grid provided below:



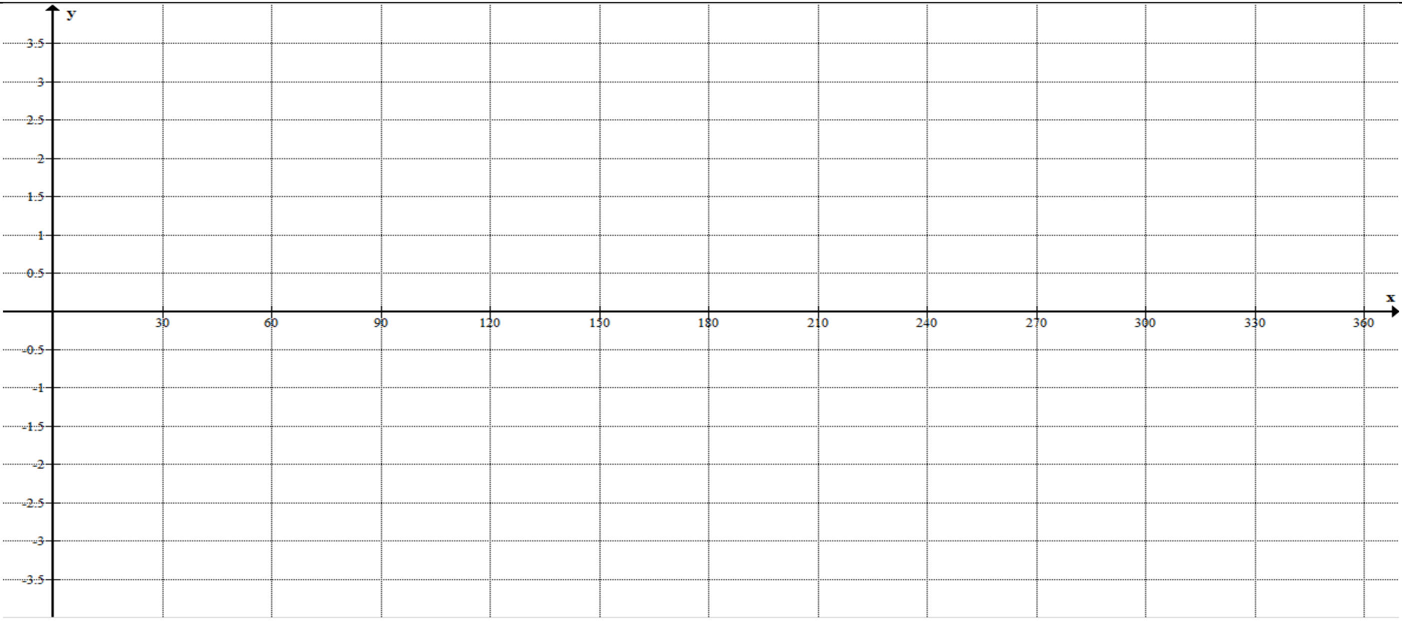
g) What effect does changing the sign of *a* have on these graphs?

1.4.2. The effect of *q* on $y = a\sin\theta + q$ and $y = a\cos\theta + q$

a) Complete the table below:

	30	60	90	120	150	180	210	240	270	300	330	360
$y = \sin\theta + 2$												
$y = \cos\theta + 2$												
$y = \sin\theta - 3$												
$y = \cos\theta - 3$												

b) Now plot these graphs on the same grid provided below.



c) Use your graphs to complete the table below:

	Minimum value	Maximum value	Range	Amplitude	Period
$y=\sin\theta+2$					
$y=\cos\theta+2$					
$y=\sin\theta-3$					
$y=\cos\theta-3$					

d) What conclusion can be made about the effect of q on $y=a\sin\theta+q$ and $y=a\cos\theta+q$?

Now let us discuss the last basic function $y=\tan\theta$ and its properties.

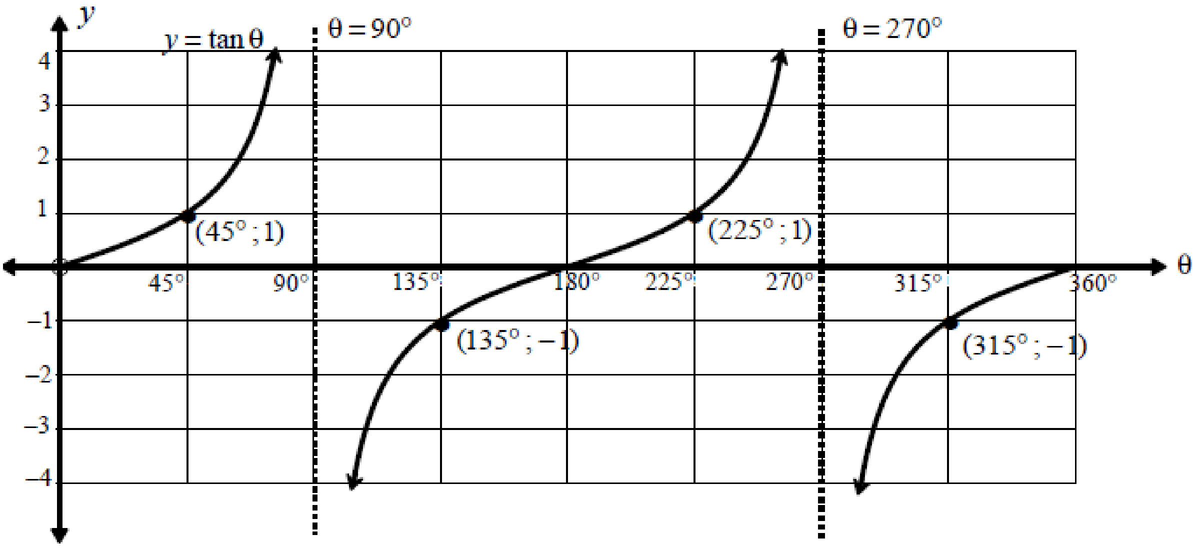
Consider: $y=\tan\theta$

The table below represents the specific values of θ and the corresponding y values.

θ	$0°$	$30°$	$45°$	$60°$	$89°$	$90°$	$91°$	$120°$	$135°$	$150°$	$180°$
$\tan\theta$	0	0,6	1	1,7	57,2	error	$-57,2$	$-1,7$	-1	$-0,6$	0

θ	$210°$	$225°$	$240°$	$269°$	$270°$	$271°$	$300°$	$315°$	$330°$	$360°$
$\tan\theta$	0,6	1	1,7	57,2	error	error	$-1,7$	-1	$-0,6$	0

The graph below represents this function.



The graph of $y = \tan\theta$ has the following characteristics:

a) The graph has no maximum and minimum value, therefore no amplitude.
b) The range is $y \in (-\infty;\infty)$
c) The graph is undefined at $\theta = 90°$ and $\theta = 270°$, therefore, as explained in lesson 2, these two lines are called **vertical asymptotes.**
d) A full basic graph of $y = \tan\theta$ is completed over a period of $180°$. Therefore the period of this graph is $180°$. This will be explored more in grade 11 trigonometric functions.

ADDITIONAL ACTIVITY

Write a summary of what the effect of each parameter is on each of the trigonometric functions.

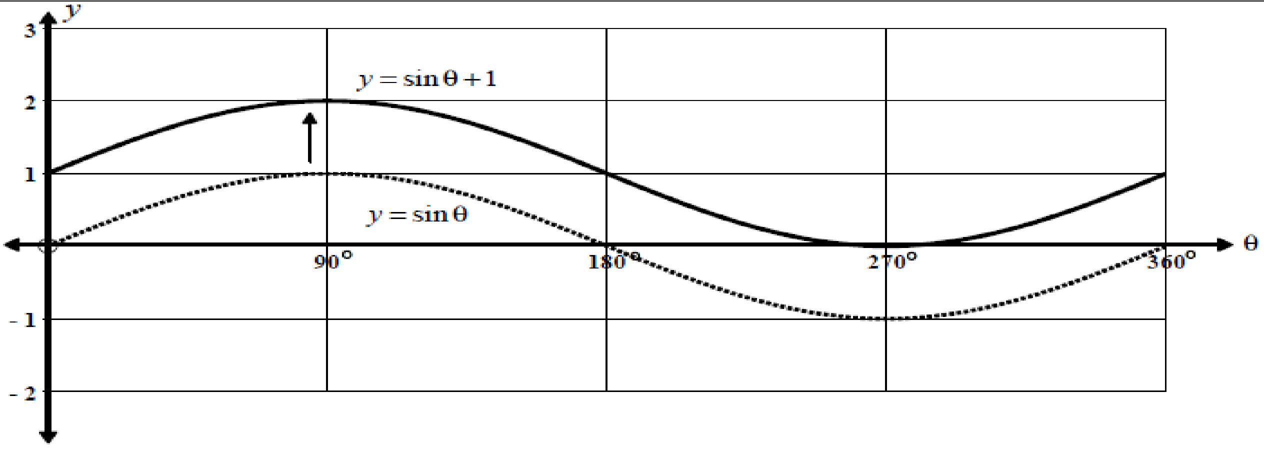
TOPIC: Functions: (Lesson 5)	Weighting	30	Grade	10
RESOURCES				
Mind Action Series Gr. 10 page 94, 97 and 98				

EXAMPLE: SKETCHING TRIGONOMETRIC GRAPHS

Sketch the graph of $y = \sin\theta + 1$ and $y = -\cos\theta - 1$ for $\theta \in [0°;360°]$

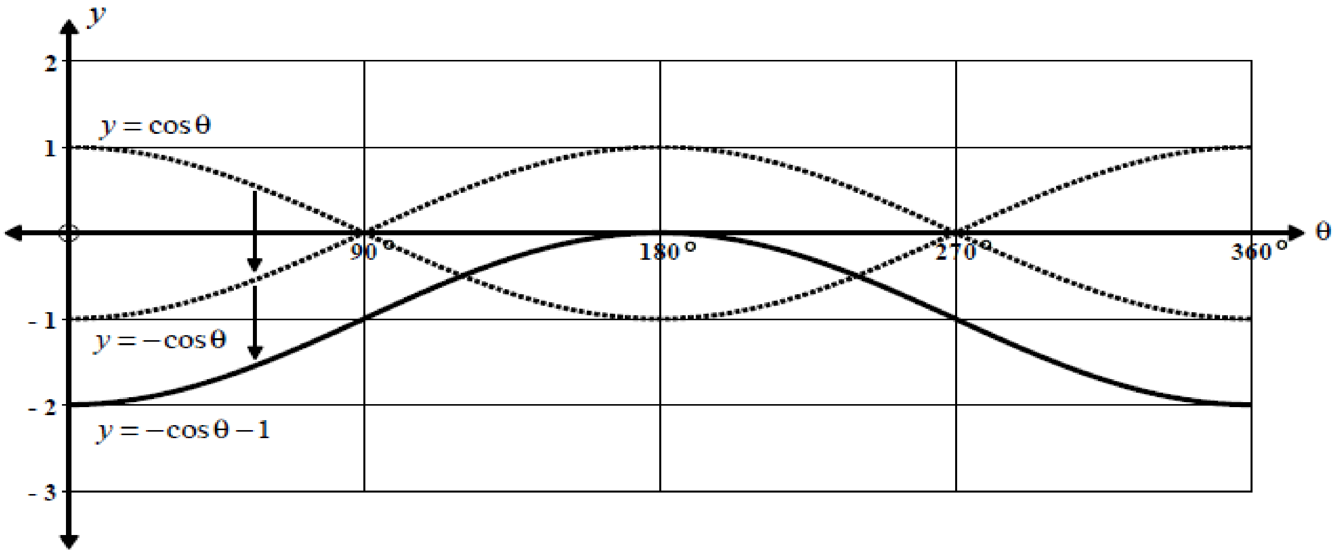
Solution

The graph of $y = \sin\theta + 1$ is the graph of $y = \sin\theta$ shifted 1 unit up. The graph is shown below:



The maximum value is 2 and the minimum value is 0. The range is $y \in [0;2]$. The amplitude is $\frac{1}{2}[2-0]=1$ and the period is $360°$.

The graph of $y = -\cos\theta - 1$ is the graph of $y = \cos\theta$ reflected in the *x*-axis and then shifted 1 unit down. The graph is shown below.



The maximum value is 0 and the minimum value is -2. The range is $y \in [-2;0]$. The amplitude is $\frac{1}{2}[0-(-2)]=1$ and the period is $360°$.

ACTIVITY 1.5

1.5.1. Given: $y = \sin\theta + 2$ and $y = \cos\theta - 1$

a) Sketch the graphs on the same set of axes for $\theta \in [0°;360°]$.

b) Write down the maximum and minimum values for each graph.

c) Write down the range, amplitude and period for each graph.

1.5.2. Given: $y = -\cos x + 3$ and $y = -\sin x - 2$

a) Sketch the graphs on the same set of axes for $x \in [0°;360°]$.

b) Write down the maximum and minimum values for each graph.

c) Write down the range, amplitude and period for each graph.

1.5.3. Given: $y = 2\sin\alpha + 4$ and $y = -3\cos\alpha - 1$

a) Sketch the graphs on the same set of axes for $\alpha \in [0°;270°]$.
b) Write down the maximum and minimum values for each graph.
c) Write down the range, amplitude and period for each graph.

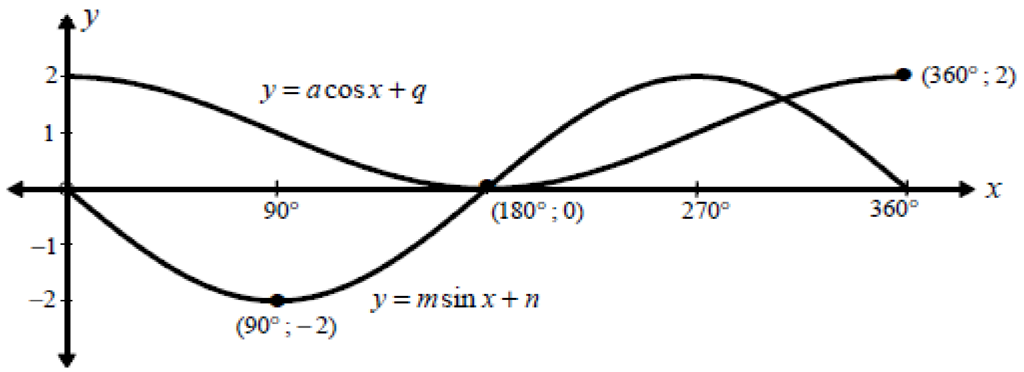
1.5.4. Sketch the graphs of the following in the indicated intervals:
a) $y = -\tan\theta$ for interval $\theta \in [0°;360°]$
b) $y = 3\tan\theta$ for interval $\theta \in [0°;360°]$
c) $y = -\frac{1}{2}\tan\theta$ for interval $\theta \in [0°;270°]$
d) $y = \tan\theta + 1$ for interval $\theta \in [0°;360°]$
e) $y = \tan\theta - 2$ for interval $\theta \in [90°;360°]$
f) $y = -2\tan\theta - 1$ for interval $\theta \in [0°;360°]$

1.5.5. Given: $y = 2\tan\theta$ and $y = -3\sin\theta - 3$
a) Sketch the graphs on the same set of axes for $\theta \in [0°;270°]$

Write down the period for each graph.

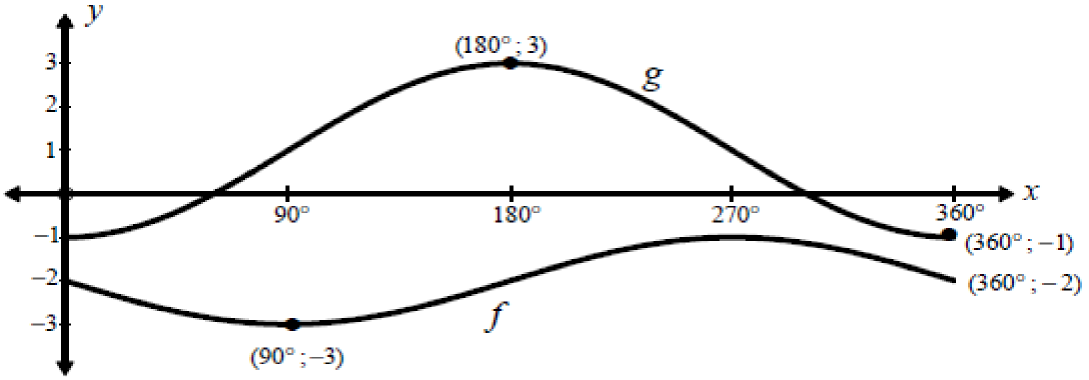
DETERMINING EQUATIONS OF GIVEN GRAPHS

1.5.6. In the diagram below, the graphs of $y = f(x) = a\cos x + q$ and $y = g(x) = m\sin x + n$ are shown for the domain $x \in [0°;360°]$.



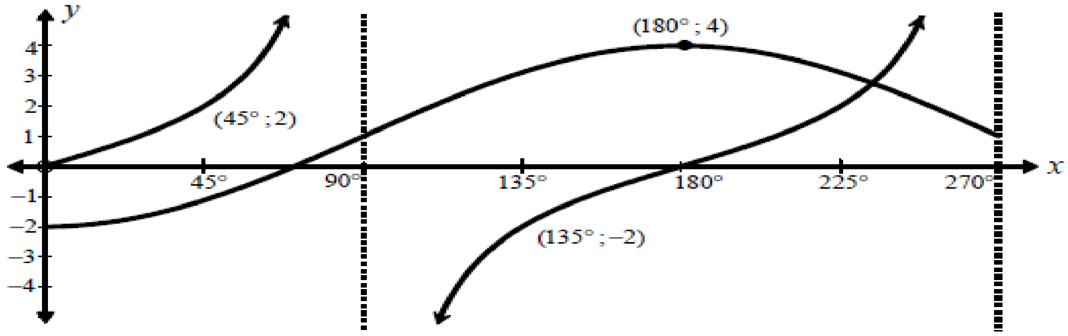
a) Write down the amplitude and range of *f*.
b) Write down the amplitude and range of *g*.
c) Determine the values of *a* and *q*.
d) Determine the values of *m* and *n*.

1.5.7. In the diagram below, the graphs of $y = f(x) = a\sin x + q$ and $y = g(x) = m\cos x + n$ are shown for the domain $x \in [0°;360°]$



a) Write down the amplitude and range and period of *f*.
b) Write down the amplitude and range and period of *g*.
c) Determine the values of *a* and *q*.
d) Determine the values of *m* and *n*.

1.5.8. In the diagram below, the graphs of two trigonometric functions are shown.



a) Determine the equations of the two graphs.
b) Write down the range, amplitude and period of each graph, where possible.

ADDITIONAL ACTIVITY

1.5.9 Consider the function $f(x) = -3\tan x$.

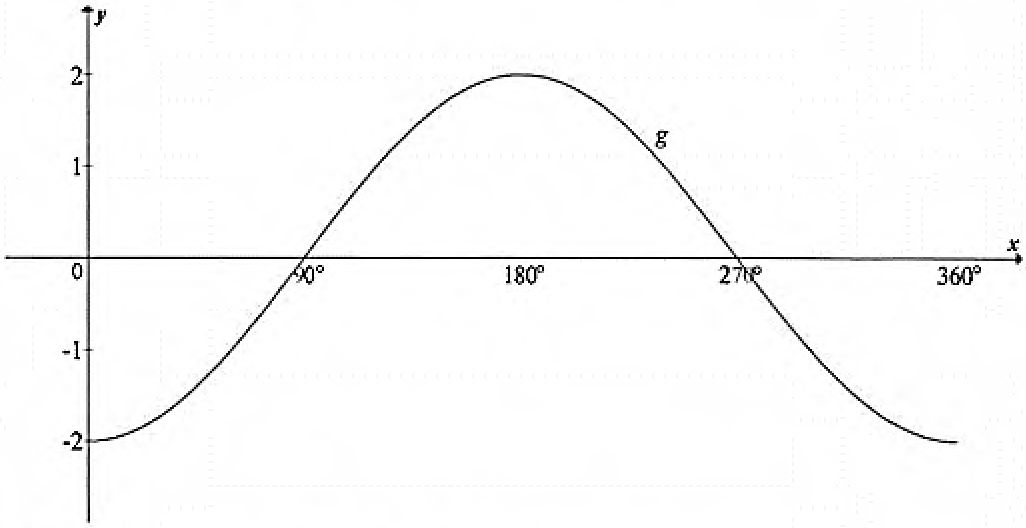
1.5.9.1 Sketch, on the grid provided in the ANSWER BOOK, the graph of f for $0° \leq x \leq 360°$. Clearly show ALL the intercepts and asympotes. (3)

1.5.9.2 Hence, or otherwise, write down the:

(a) Period of f (1)

(b) Equation of h if h is the reflection of f about the x-axis (1)

1.5.10 Sketched below is the graph of $g(x) = a.\cos b\theta$



a. Write down the values of a and b. (2)

b. Use the graph to determine the value(s) of x for which $g(x) > 0$. (1)

c. Determine the range of h if h is the image of g if g is shifted down TWO units. (2)

d. Determine, using the graph, the value of:

$$-2(\cos 0° + \cos 1° + \cos 2° + ... + \cos 358° + \cos 359° + \cos 360°)$$ (2)

[12]

HORIZONTAL SHIFTS EITHER TO THE LEFT OR TO THE RIGHT

Consider the graph $y = \sin x$ for $x \in [0^\circ;360^\circ]$ and $y = \sin(x + 30^\circ)$ for $x \in [0^\circ;360^\circ]$. Fill in the tables below and draw all 3 graphs on the same system of axes provided below.

1.5.11 $(y = \sin(x + p)$,

	x	-30°	0°	90°	180°	270°	360°
(a)	$y = \sin x$						
(b)	$y = \sin(x + 30^\circ)$,						
(c)	$y = \sin(x - 30^\circ)$,						

1.5.12 $y = \cos(x + p)$,

x	-30°	0°	90°	180°	270°	360°
$y = \cos x$						
$y = \cos(x + 30^\circ)$,						
$y = \cos(x - 30^\circ)$,						

1.5.13 $y = \tan(x + p)$

x	$-30°$	$0°$	$90°$	$180°$	$270°$	$360°$
$y = \tan x$						
$y = \tan(x+30°)$,						
$y = \tan(x-30°)$,						

1.5.14 If the following is given, indicate the type of transformation that took place on each:

(a) $f(x) = \sin x$ transforms to $h(x) = \sin(-x + 30°)$

(b) $t(x) = \cos x$ transforms to $h(x) = \sin(-x - 60°)$

(c) $f(x) = \tan x$ transforms to $g(x) = \tan(-45° - x)$

TOPIC: Functions: (Lesson 6)	Weighting	45	Grade	11
Grade 11 Textbook/ Handouts				

ACTIVITY 1.6.1:

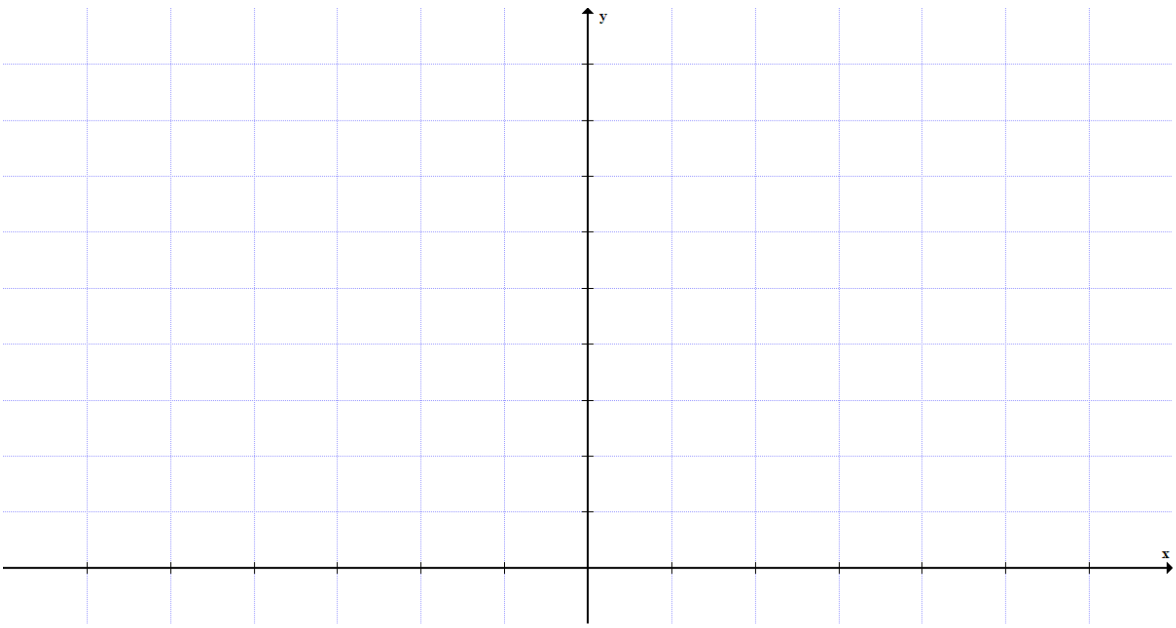
AIM: Investigate the effect of the parameter p on the quadratic, hyperbolic and exponential functions.

1.6.1.1. Effect of p on the quadratic function $y=a(x+p)^2+q$

(a) Complete the table below:

x	-3	-2	-1	0	1	2	3
$y=x^2$							
$y=(x-1)^2$							
$y=(x+1)^2$							
$y=(x-2)^2+1$							
$y=(x+2)^2+1$							

(b) Sketch the parabolas of the above functions on the same grid provided below: you may use coloured pens.



(c) Complete the table below to compare first graph with other graphs:

Graph	**Axis of symmetry**	**Turning Point**	**Shifted to the left or right**	**Shifted by how many units**
$y=x^2$	$x=0$	$(0;0)$	No shift	0
$y=(x-1)^2$				
$y=(x+1)^2$				
$y=(x-2)^2+1$				
$y=(x+2)^2+1$				

(d) Complete the sentence below to conclude about the effect of p on parabola:

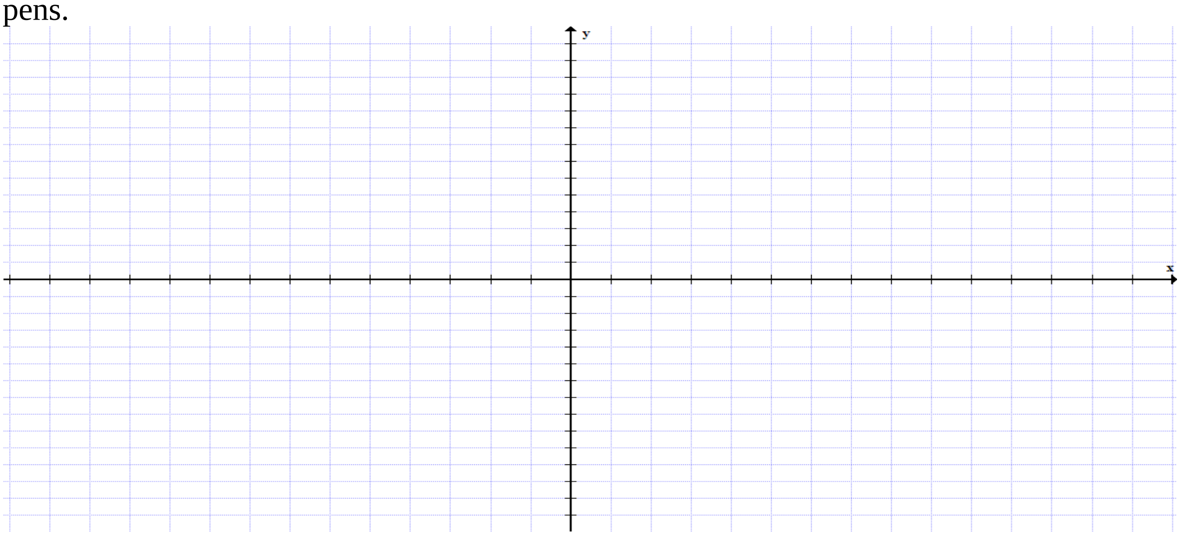
If p > 0, the graph will shift ……..units to the …………..direction and if p < 0, the graph will……

1.6.1.2. Effect of p on hyperbola $y=\dfrac{a}{x+p}+q$

(a) Complete the table below:

x	-3	-2	-1	0	1	2	3
$y=\dfrac{1}{x}$							
$y=\dfrac{1}{x-1}$							
$y=\dfrac{1}{x+1}$							
$y=\dfrac{1}{x-2}-1$							
$y=\dfrac{1}{x+2}-1$							

(b) Sketch the parabolas of the above functions on the same grid provided below: you may use colour pens.



(c) Complete the table below to compare first graph with other graphs:

Graph	**Equations of asymptotes**	**Shifted to the left or right**	**Shifted by how many units**	**Equations of the axis of symmetry**
$y=\dfrac{1}{x}$	$x=0$ $y=0$	No shift	0	$y=x$ $y=-x$
$y=\dfrac{1}{x-1}$				
$y=\dfrac{1}{x+1}$				
$y=\dfrac{1}{x-2}-1$				
$y=\dfrac{1}{x+2}-1$				

(d) Complete the sentence below to conclude about the effect of p on hyperbola:

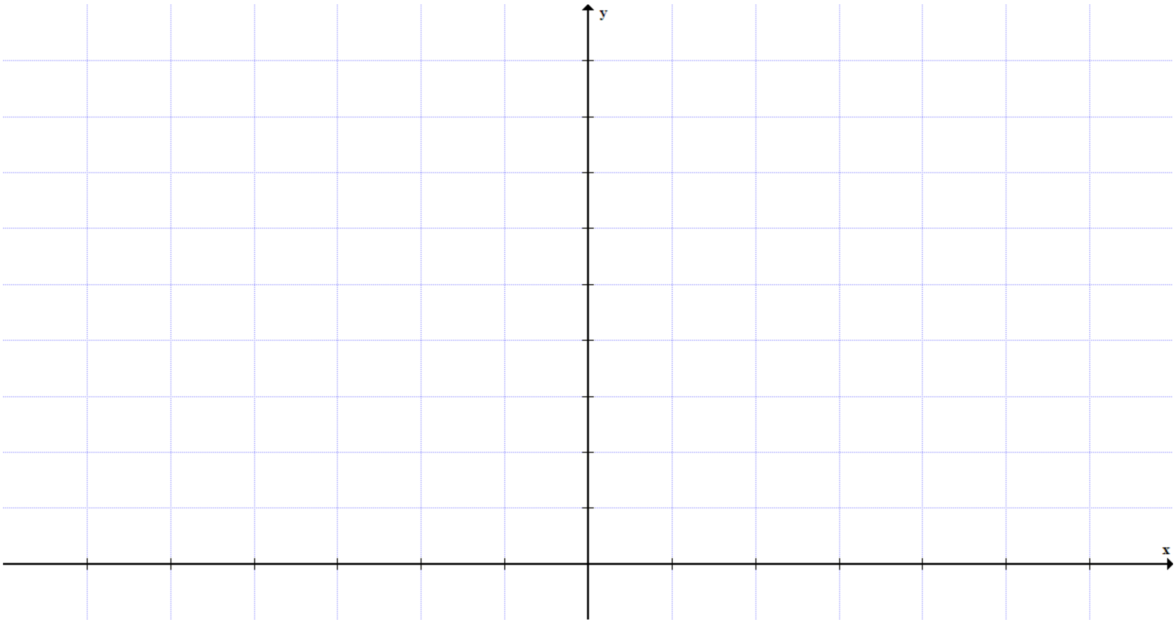
If p > 0, the graph will shift ……..units to the …………..direction and if p < 0, the graph will……

1.6.1.3 Effect of p on the exponential function $y = a.b^{x+p} + q$

(a) Complete the table below:

x	-3	-2	-1	0	1	2	3
$y = 2^x$							
$y = 2^{x+1}$							
$y = 2^{x-1}$							
$y = -2^{x+2} + 1$							
$y = -2^{x-2} + 1$							

(b) Sketch the parabolas of the above functions on the same grid provided below: you may use colour pens.



(c) Complete the table below to compare first graph with other graphs:

x	**Shifted to the left or right**	**Shifted by how many units**
$y = 2^x$	No shift	0
$y = 2^{x+1}$		
$y = 2^{x-1}$		
$y = -2^{x+2} + 1$		
$y = -2^{x-2} + 1$		

(d) Complete the sentence below to conclude about the effect of q on parabola:
If p > 0, the graph will shift ……..units to the …………..direction and if p < 0, the graph will……

1.6.1.4 Consider the following functions and describe the transformation performed on function (i) to get function (ii)

(a) (i) $y = x^2$

(ii) $y=(x+3)^2-4$

(b) (i) $y=-\dfrac{1}{x}$

(ii) $y=\dfrac{1}{2-x}+3$

ACTIVITY 1.6.2

1.6.2.1 Consider:

$f(x)=x^2+1$. Determine the equation of the new graph formed if the graph of *f* is:

(a) shifted 2 units to the left.

(b) shifted 3 units downwards.

(c) shifted 1 unit left and 4 units upwards.

1.6.2.2 Consider:

$f(x)=\dfrac{2}{x}$. Determine the equation of the new graph formed if the graph of *f* is:

(a) shifted 2 units upwards.

(b) shifted 3 units to the right.

(c) shifted 2 unit to the right and 2 units downwards.

1.6.2.3**.** If $f(x)=3^x+1$ and $g(x)=3^{x+1}-1$

(a) Sketch the graphs of *f* and g on the same system of axis.

(b) Explain how you would use the graph of *f* to sketch the graph of g.

TOPIC: Functions: (Lesson 7)	Weighting	45	Grade	11
RESOURCES				
Grade 11 Textbook/ Handouts/ Past QPs				
WORKED EXAMPLES				
For each of the following functions: (a) Calculate all the intercepts with the axis. (b) Determine the coordinates of the turning points. (c) Sketch the graph showing all the intercepts with the axis and the turning points. (d) State the domain and range. 1. $f(x)=2x^2+4x+2$ 2. $g(x)=-x^2+6x+7$ 3. $h(x)=2(x+3)^2-8$				
SOLUTIONS				
1. $f(x)=2x^2+4x+2$				

(a) for y-intercept let $x=0$

$$f(0)=2(0)^2+4(0)+2$$
$$=2$$
$$\therefore(0;2)$$

for x-intercept let $y=0$

$$2x^2+4x+2=0$$
$$x^2+2x+1=0$$
$$(x+1)(x+1)=0$$
$$x=-1\ /\ x=-1$$
$$\therefore(-1;0)$$

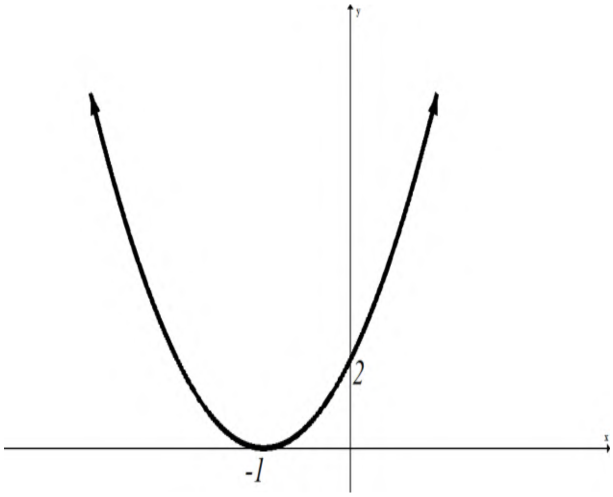
(b) first find axis of symmetry using: $x=-\frac{b}{2a}$

$$x=-\frac{4}{2(2)}=-1$$

Now find corresponding y value:

$$f(x)=2(-1)^2+4(-1)+2=0$$
$$\therefore TP(-1;0)$$

(c)



(d) domain: $x\in\mathbb{R}$

Range: $y\in\mathbb{R}; y\geq 0$ or $y\in[0;\infty)$ or $0\leq y<\infty$

2. $g(x)=-x^2+6x+7$

(a) for y-intercept let $x=0$

$$g(0)=-(0)^2+6(0)+7$$
$$=7$$
$$\therefore(0;7)$$

for x-intercept let $y=0$

$$-x^2+6x+7=0$$
$$x^2-6x-7=0$$
$$(x+1)(x-7)=0$$
$$x=-1\ /\ x=7$$
$$\therefore(-1;0)\qquad(7;0)$$

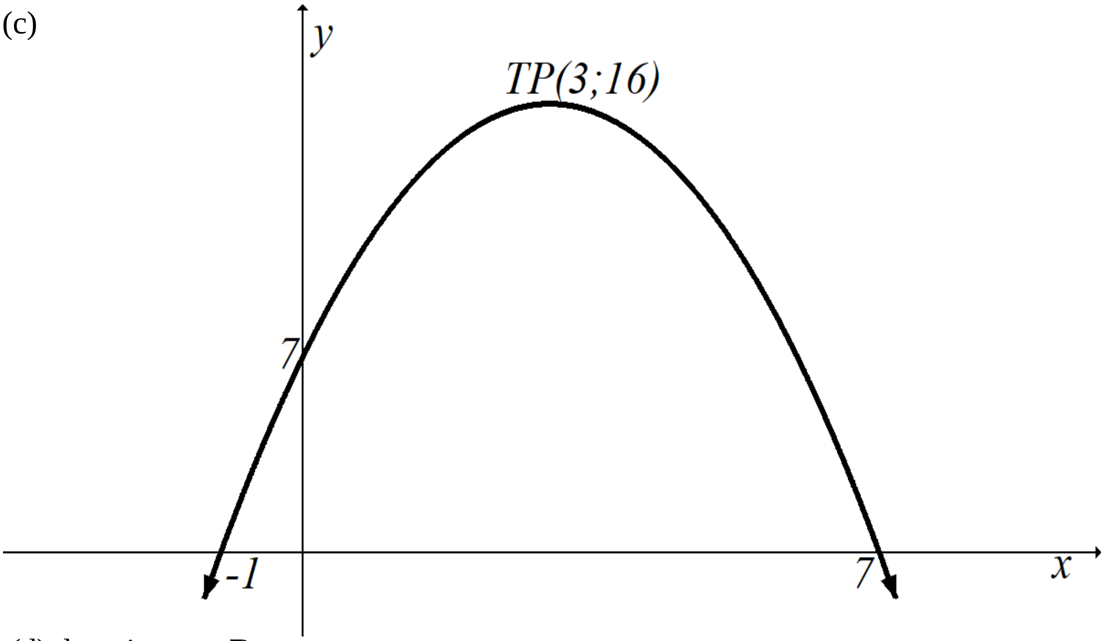
(b) first find axis of symmetry using: $x=-\frac{b}{2a}$

$$x=-\frac{6}{2(-1)}=3$$

Now find corresponding y value:

$$g(3)=-(3)^2+6(3)+7=16$$
$$\therefore TP(3;16)$$

(c)



(d) domain: $x \in \mathbb{R}$

Range: $y \in \mathbb{R}; y \leq 16$ or $y \in (-\infty; 16]$ or $-\infty < y \leq 16$

3. $h(x) = 2(x+3)^2 - 8$

(a) for y-intercept let $x = 0$

$$h(0) = 2(0+3)^2 - 8$$
$$= 10$$
$$\therefore (0;10)$$

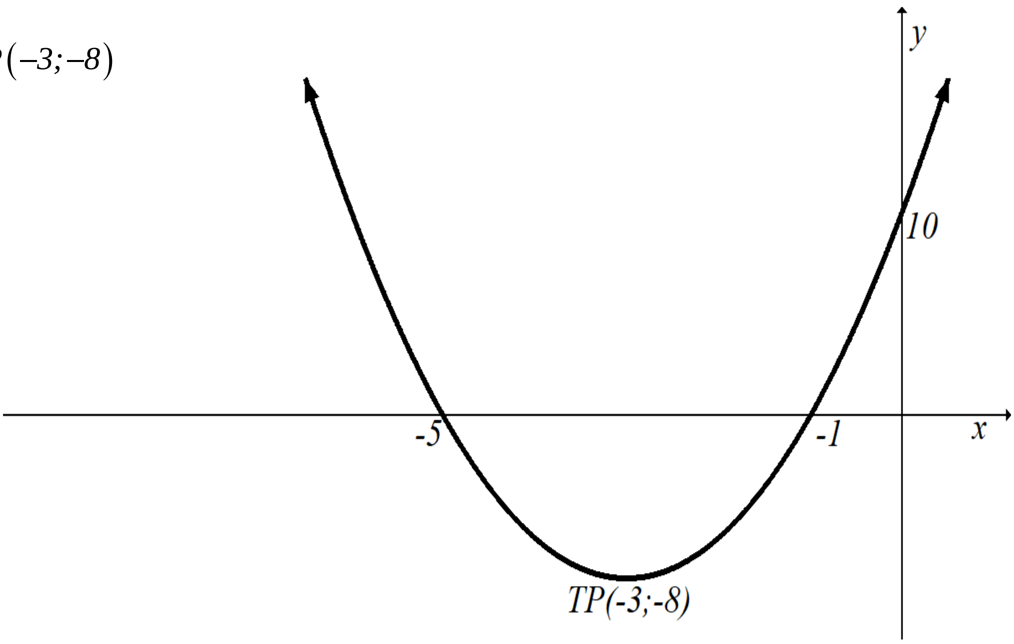
for x-intercept let $y = 0$

$$2(x+3)^2 - 8 = 0$$
$$2(x+3)^2 = 8$$
$$(x+3)^2 = 4$$
$$x+3 = \pm 2$$
$$x = -5 \,/\, x = -1$$
$$\therefore (-5;0) \qquad (-1;0)$$

(b) Read the turning point from the equation:

$\therefore TP(-3;-8)$

(c)



(d) domain: $x \in \mathbb{R}$

Range: $y \in \mathbb{R}; y \geq -8$ or $y \in [-8; \infty)$ or $-8 \leq y < \infty$

ACTIVITY 1.7.1

For each of the following functions:
(a) Calculate all the intercepts with the axis.
(b) Determine the coordinates of the turning points.
(c) Sketch the graph showing all the intercepts with the axis and the turning points.
(d) State the domain and range.

1.7.1.1 $f(x) = x^2 + 4x - 5$

1.7.1.2 $g(x) = -x^2 - 3x - 2$

1.7.1.3 $h(x) = (x-1)^2 - 9$

1.7.1.4 $k(x) = -2(x+2)^2 + 18$

ACTIVITY 1.7.2

Consider the functions alongside: 1.7.2.1 $f(x) = -x^2 + 4x + 5$

1.7.2.2 $g(x) = 3(x-2)^2 + 1$

For each of the functions above:
(a) Determine:
(i) all the intercepts with the axes.
(ii) coordinates of the turning point.
(iii) domain and range,
(b) sketch the graph.

1.7.2.3 Use the given information to sketch the graphs of:
(a) $y = ax^2 + bx + c$ if $a > 0, b > 0, c > 0$
(b) $y = ax^2 + bx + c$ if $a > 0, b = 0, c > 0$
(c) $y = a(x+p)^2 + q$ if $a < 0, p < 0, q > 0$ and one root is zero

1.7.2.4. Determine the new equation (in the form $y = ax^2 + bx + c$) if:
(a) $y = 2x^2 + 4x + 1$ is reflected about y axis.
(b) $y = 2x^2 + 4x + 1$ is reflected about x axis.
(c) $y = 2x^2 + 4x + 1$ is shifted 3 units down.
(d) $y = 2x^2 + 4x + 1$ is shifted 1 unit up and 2 units to the left.

1.7.2.5

Given: $f(x) = -2x^2 + x + 6$

5.1 Calculate the coordinates of the turning point of f. (4)

5.2 Determine the y-intercept of f. (1)

5.3 Determine the x-intercepts of f. (4)

5.4 Sketch the graph of f showing clearly all intercepts with the axes and turning point. (3)

5.5 Determine the values of k such that $f(x) = k$ has equal roots. (2)

5.6 If the graph of f is shifted two units to the right and one unit upwards to form h, determine the equation h in the form $y = a(x+p)^2 + q$. (3)

[17]

TOPIC: Functions: (Lesson 8)	Weighting	45	Grade	11

RESOURCES

EXAMPLES

Given $f(x)=\dfrac{3}{x+1}-2$

(a) Calculate intercepts with the axis.
(b) Determine the equations of asymptotes.
(c) Sketch the graph showing all the intercepts with the axis and asymptotes.
(d) State the domain and range.
(e) Determine the equations of the axis of symmetry.

SOLUTIONS

(a) for y-intercept let $x=0$

$$f(0)=\frac{3}{0+1}-2$$
$$=1$$
$$\therefore(0;1)$$

for x-intercept let $y=0$

$$\frac{3}{x+1}-2=0$$
$$\frac{3}{x+1}=2$$
$$2x+2=3$$
$$2x=1$$
$$x=\frac{1}{2}$$
$$\therefore\left(\frac{1}{2};0\right)$$

(b) Vertical asymptote:

Denominator is equal to zero $x+p=0$ and solve for x.

$$x+1=0$$
$$\therefore x=-1$$

Horizontal asymptote:

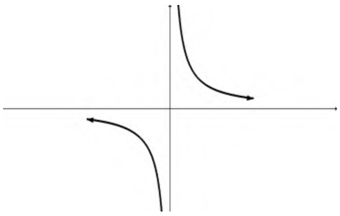
$y=q$

$y=-2$

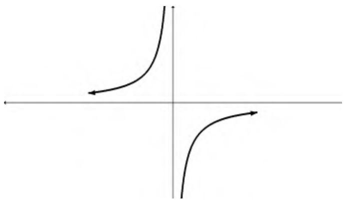
(c) To sketch the graph:

- Draw vertical and horizontal asymptotes.
- Indicate the intercepts with the axis if they exist.
- Draw the two curves passing through the relevant point but they must not touch the asymptote, they only approach the asymptotes.
- Keep in mind the shape of the graph

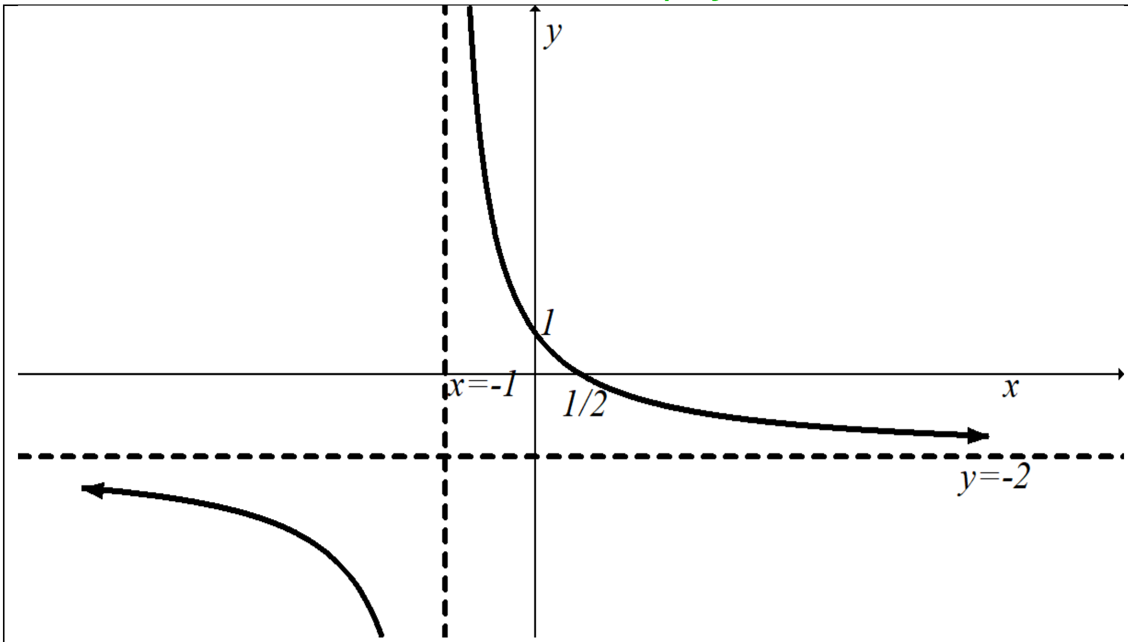
$a>0$



$a<0$



In this case $a>0$:



(d) domain: $x \in \mathbb{R}$ but $x \neq -1$
Range: $y \in \mathbb{R}$ but $y \neq -2$

(e) **Remember:**

- Axis of symmetry is a line divides the graph symmetrically.
- It passes through the point of intersection of the asymptotes.
- There are two axis of symmetry on the hyperbola.
  - ❖ $y = x + c$ (have positive gradient)
  - ❖ $y = -x + c$ (have negative gradient)
  - ❖ To calculate c, use point of intersection of the asymptotes.

In our example the point of intersection of the asymptotes is:
$(-1;-2)$

$\therefore\ y = x + c$ $\qquad\qquad$ $y = -x + c$

Substitute the point:

$-2 = -1 + c$ $\qquad\qquad$ $-2 = -(-1) + c$

$-1 = c$ $\qquad\qquad$ $-3 = c$

The equations are:

$y = x - 1$ $\qquad\qquad$ $y = -x - 3$

NB: Teacher must draw the axis of symmetry on the graph above.

ACTIVITY 1.8.1

For each of the following functions:
(a) Calculate intercepts with the axis.
(b) Determine the equations of asymptotes.
(c) Sketch the graph showing all the intercepts with the axis and asymptotes.
(d) State the domain and range.
(e) Determine the equations of the axis of symmetry.

1.8.1.1 $f(x) = \dfrac{2}{x-1}$ $\qquad\qquad$ 1.8.1.2 $g(x) = -\dfrac{1}{x+2} + 2$

ACTIVITY 1.8.2

1.8.2.1. Given:

$$f(x)=\frac{1}{x-3}+4$$

(a) Determine the intercepts with the axis.
(b) Determine the equations of asymptotes.
(c) Sketch the graph showing all the intercepts with the axis and asymptotes.
(d) State the domain and range.
(e) Determine the equations of the axis of symmetry with negative gradient.
(f) Write down the equation of h if $h(x)=-f(x-3)$.

1.8.2.2 Given: $g(x)=\dfrac{a}{x+p}+q$

- Domain of g: $x\in\mathbb{R};\ x\neq -1$
- Horizontal asymptote of g: $y=2$
- g passes through the points $(0;4)$
- $a>0$

Use the information above to sketch the graph of g.

1.8.2.3. Given: $f(x)=\dfrac{x-2}{x+3}$

(a) Write down the equation of *f* in the form of $y=\dfrac{a}{x+p}+q$

(b) Hence write down the range of h if the graph of h is obtained by reflecting the graph of *f* about line $y=0$ and then shifted 2 units upwards.

1.8.2.4

Given: $f(x)=\dfrac{8}{x-2}+3$

a.	Write down the equations of the asymptotes of *f*.	(2)
b.	Calculate the *x*- and *y*-intercepts of *f*.	(3)
c.	Sketch the graph of *f*. Show clearly the intercepts with the axes and the asymptotes.	(3)
d.	If $y=x+k$ is an equation of the line of symmetry of *f*, calculate the value of *k*.	(2)
		[10]

TOPIC: Functions: (Lesson 9)	Weighting	45	Grade	11
RESOURCES				
Grade 11 Textbook/ Handouts/ Past papers				
ACTIVITY 1.9.1.				

For each of the following functions:
(a) Calculate intercepts with the axis.
(b) Determine the equation of asymptotes.
(c) Sketch the graph showing all the intercepts with the axis and asymptotes.
(d) State the domain and range.

1.9.1.1 $f(x)=2^{x+1}$

1.9.1.2 $g(x)=2.2^{x}-4$

1.9.1.3 $y = 3^{-x} - 9$

1.9.1.4 $h(x) = \left(\frac{1}{2}\right)^{x+2} - 1$

1.9.1.5 $y = -4^{x+2} - 2$

ACTIVITY 1.9.2

Given: $f(x) = -\left(\frac{1}{4}\right)^x + 4$

a. Write down an equation of the asymptote of f. (1)

b. Determine the coordinates of the y-intercept of f. (2)

c. Determine the coordinates of the x-intercept of f. (3)

d. Sketch a graph of $y = f(x)$, clearly indicating the asymptote and the coordinates of all intercepts with the x- and y-axes. (4)

e. If the graph of f is now reflected in the line $y = 4$ to create the graph of k, write down a formula for k in the form $y = \ldots$ (2)

[12]

TOPIC: Functions: (Lesson 10)	Weighting	45	Grade	11

<u>TYPE 1</u>

Here we are given the x-intercepts and one point on the graph. We will use the structure $y = a(x - x_1)(x - x_2)$ where x_1 and x_2 are the x-intercepts.

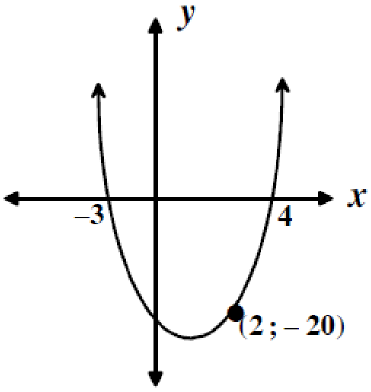
<u>EXAMPLE 15</u>

Determine the equation of the parabola in the form $f(x) = ax^2 + bx + c$.

$y = a(x - x_1)(x - x_2)$

$\therefore y = a(x - (-3))(x - 4)$

$\therefore y = a(x + 3)(x - 4)$



Substitute $(2 ; -20)$ to find the value of a.

$-20 = a(2 + 3)(2 - 4)$

$\therefore -20 = a(5)(-2)$

$\therefore -20 = -10a$

$\therefore a = 2$

$\therefore y = 2(x + 3)(x - 4)$

$\therefore y = 2(x^2 - x - 12)$

$\therefore f(x) = 2x^2 - 2x - 24$

<u>TYPE 2</u>

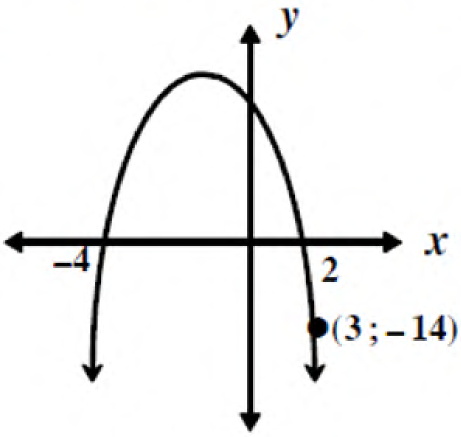
Here we are given the turning point and one other point on the graph. We make use of the structure $y = a(x + p)^2 + q$

<u>EXAMPLE 16</u>

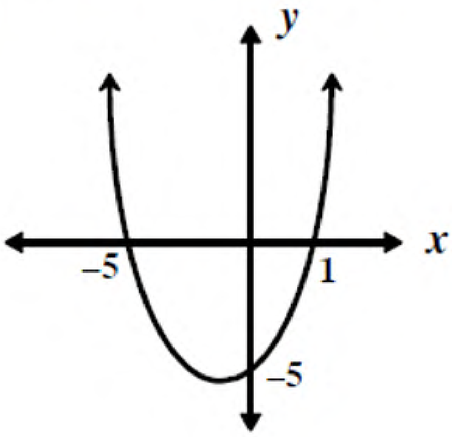
Activity 1.10.1

1.10.1 Determine the equations of the following in the form $y = ax^2 + bx + c$:

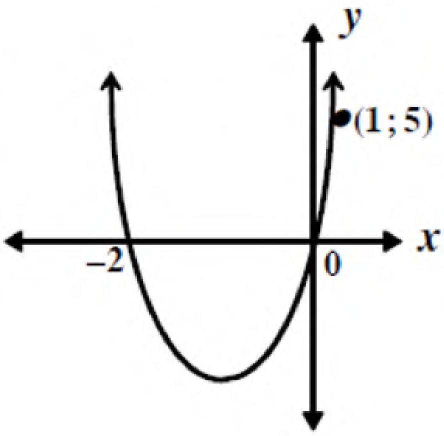
(a)



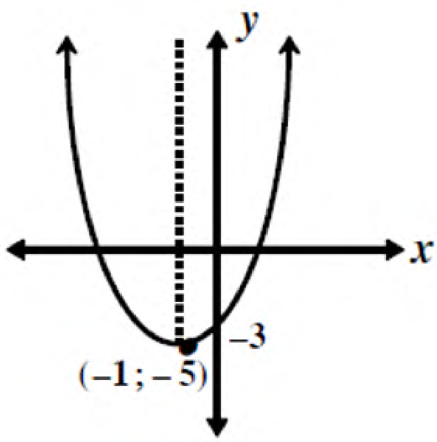
(b)



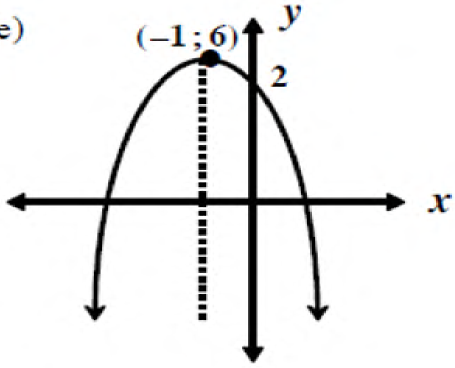
(c)



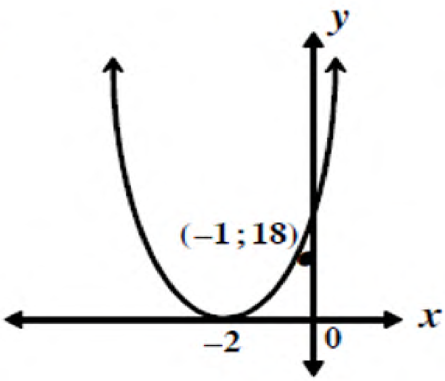
(d)



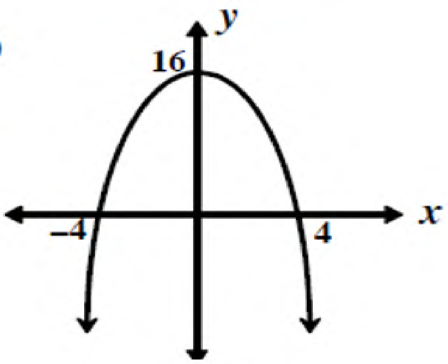
(e)



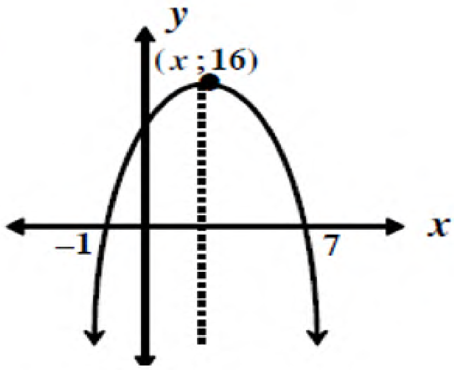
(f)



(g)



(h)



TOPIC: Functions: (Lesson 11)

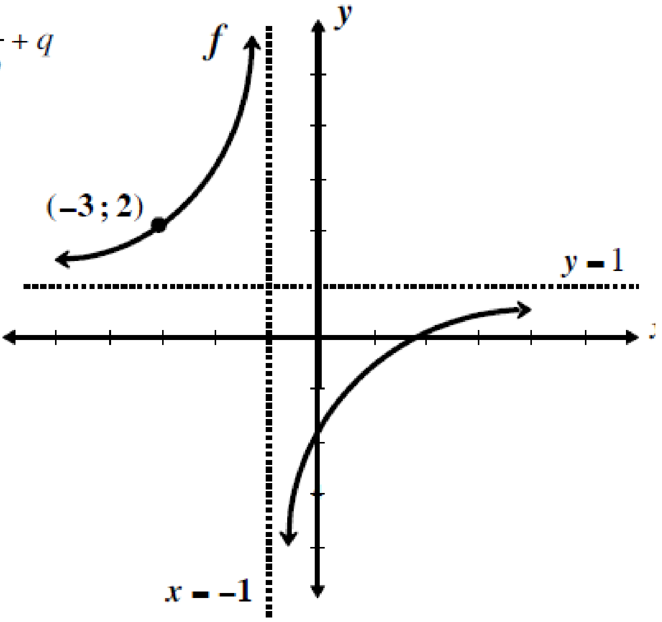
All Grade 10 Functions concepts and solving equations involving fractions and exponents.

WORKED EXAMPLES

DETERMINING THE EQUATION OF A HYPERBOLA

1.11.a

Determine the equation of $f(x) = \dfrac{a}{x+p} + q$



Vertical asymptote: $x = -1$

$\therefore x + 1 = 0$

Therefore the expression $x+1$ is in the denominator of the equation.

Horizontal asymptote: $y = 1$

Therefore the value of q is 1.

Therefore the equation becomes:

$$y = \frac{a}{x+1} + 1$$

All we need to now do is substitute the point $(-3\,;2)$ into the equation to get the value of a.

$$\therefore 2 = \frac{a}{-3+1} + 1$$

$$\therefore 2 = \frac{a}{-2} + 1$$

$$\therefore 2 - 1 = \frac{a}{-2}$$

$$\therefore 1 = \frac{a}{-2}$$

$$\therefore a = -2$$

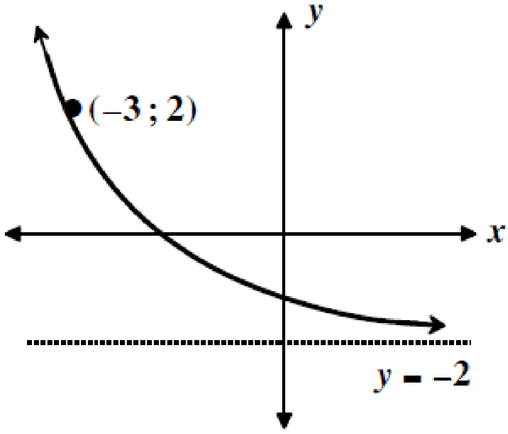
Therefore, substitute this value of a into the equation to get the final equation:

$$y = \frac{-2}{x+1} + 1$$

DETERMINING THE EQUATION OF AN EXPONENTIAL GRAPH

1.11.b

Determine the equation of $g(x) = b^{x+1} + q$



Horizontal asymptote: $y = -2$

Therefore the value of q is -2

$\therefore y = b^{x+1} - 2$

Now substitute the point $(-3\,;2)$ into the equation to get b.

$2 = b^{-3+1} - 2$

$\therefore 4 = b^{-2}$

$\therefore 4 = \dfrac{1}{b^2}$

$\therefore 4b^2 = 1$

$\therefore 4b^2 - 1 = 0$

$\therefore (2b+1)(2b-1) = 0$

$\therefore b = -\dfrac{1}{2}$ or $b = \dfrac{1}{2}$

But $b \neq -\dfrac{1}{2}$

$\therefore b = \dfrac{1}{2}$

Therefore the equation is: $g(x) = \left(\dfrac{1}{2}\right)^{x+1} - 2$

Note:
Since $b > 0$, you could have solved the equation $4b^2 = 1$ as follows:

$4b^2 = 1$

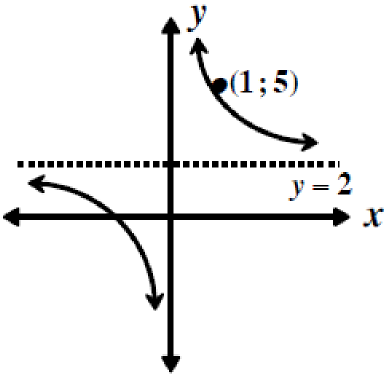
$\therefore b^2 = \dfrac{1}{4}$

$\therefore b = \dfrac{1}{2}$

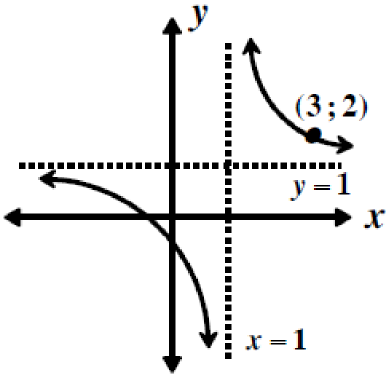
ACTIVITY 1.11.1

Determine the equations of the following graphs:

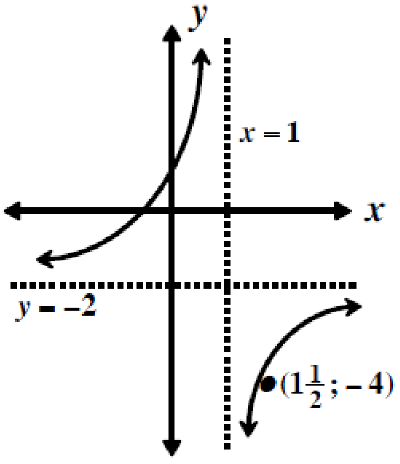
(a) $f(x)=\dfrac{a}{x}+q$



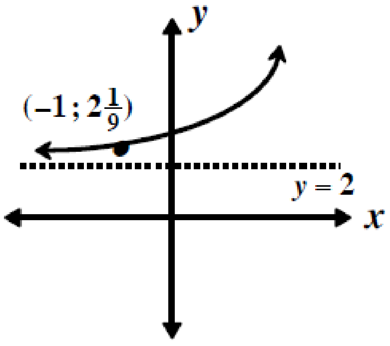
(b) $g(x)=\dfrac{a}{x+p}+q$



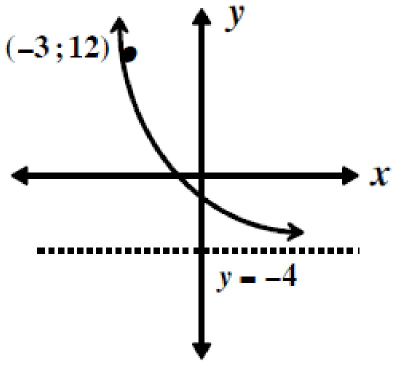
(c) $f(x)=\dfrac{a}{x+p}+q$



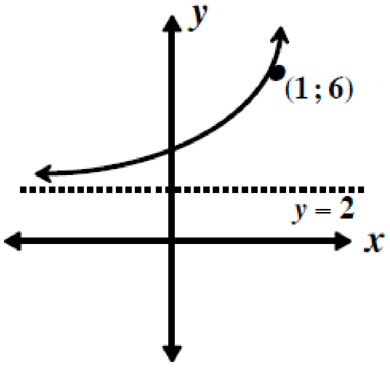
(d) $y=b^{x-1}+q$



(e) $y=b^{x+1}+q$



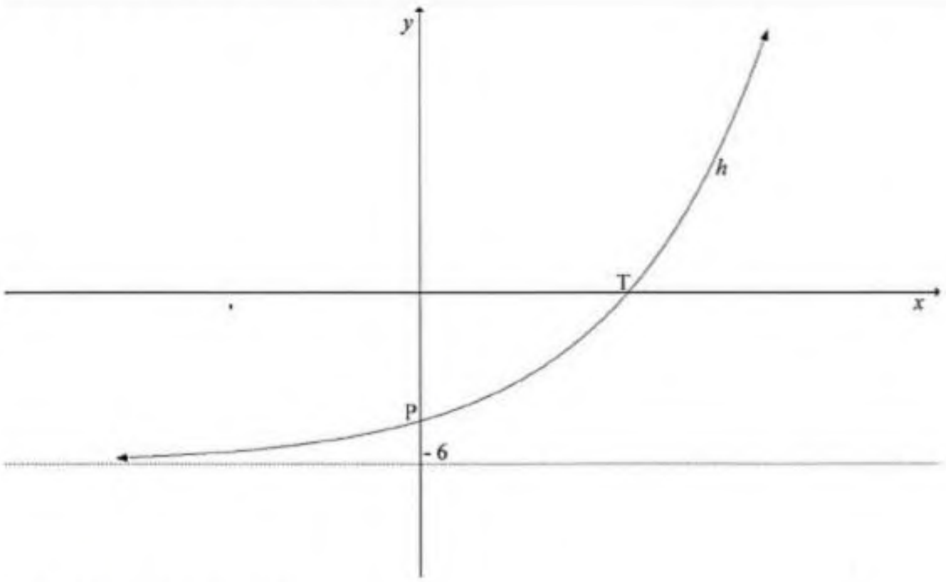
(f) $y=2^{x+p}+q$



ACTIVITY 1.11.2

1.11.2.1

Given: $h(x) = a.2^{x-1} + q$. The line $y = -6$ is an asymptote to the graph of h. P is the y-intercept of h and T is the x-intercept of h.



a. Write down the value of q. (1)

b. If the graph of h passes through the point $\left(-1;-5\frac{1}{4}\right)$, calculate the value of a. (4)

c. Calculate the average gradient between the x-intercept and the y-intercept of h. (5)

d. Determine the equation of p if $p(x) = h(x-2)$ in the form $p(x) = a.2^{x-1} + q$. (2)

[12]

1.11.2.2

Given: $f(x) = ax^2 + bx + c$

$(m-5)$ and $(m+3)$ are roots of f.
The maximum value of f occurs when $x = 2$.

a. Calculate the value of m. (3)

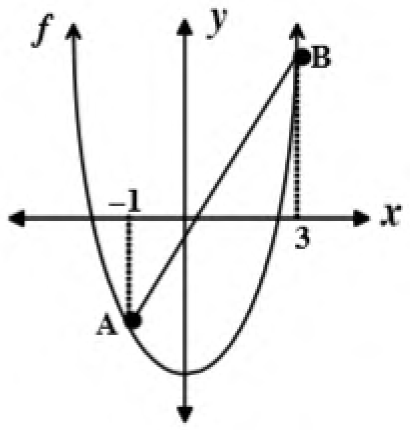
b. Determine the equation of f, in the form $y = ax^2 + bx + c$, if it is also given that $f(1) = 15$. (5)

[8]

TOPIC: Functions: (Lesson 12)	Weighting	45	Grade	11
RESOURCES				
Mind Action Series Gr. 11				

Examples

1.1 Determine the average gradient of the graph of $f(x) = x^2 - 4$ between $x = -1$ and $x = 3$



1.2 Given: $g(x) = -3x^2 + 2$

Calculate the average gradient of g between $x = -4$ and $x = 5$.

1.3 Consider the graph of $f(x) = x^2 - 2x - 3 = (x-1)^2 - 4$

Determine the value(s) of k for which:

1.3.1 $x^2 - 2x + k = 0$ has equal solutions(roots)

1.3.2 $x^2 - 2x + k = 0$ has non-real solutions

1.3.3 $x^2 - 2x + k = 0$ has two real, unequal roots

1.3.4 $x^2 - 2x - 3 = k$ has equal roots

1.3.5 $x^2 - 2x - 3 = k$ has non-real roots

1.3.6 $x^2 - 2x - 3 = k$ has two real, unequal roots

SOLUTIONS

1.1

$y = (-1)^2 - 4 = -3$

$A(-1;-3)$

$y = (3)^2 - 4 = 5$

$B(3;5)$

$$m_{AB} = \frac{y_B - y_A}{x_B - x_A} = \frac{5-(-3)}{3-(-1)} = 2$$

1.2 $g(-4) = -3(-4)^2 + 2 = -46$

$g(5) = -3(5)^2 + 2 = -73$

$$\text{AG} = \frac{y_2 - y_1}{x_2 - x_2} = \frac{-46-(-73)}{-4-5} = -3$$

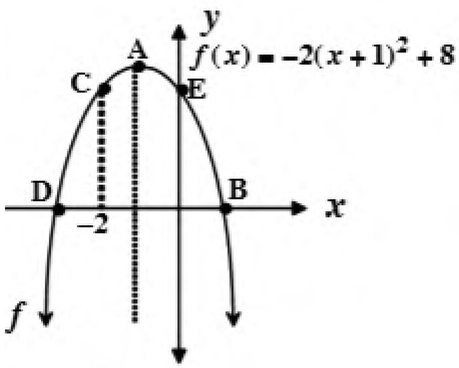
1.3

1.3.1 $k = 1$
1.3.2 $k > 1$
1.3.3 $k < 1$
1.3.4 $k = -4$
1.3.5 $k < -4$
1.3.6 $k > -4$

ACTIVITY 1.12.1

1.12.1.1 Determine the average gradient of f between:

a) A and B
b) B and C
c) D and E



1.12.1.2. **Consider** $f(x) = -x^2 - 2x + 8$

Determine the value(s) of k **for which:**

a) $-x^2 - 2x + k = 0$ **has real and equal solutions**

b) $-x^2 - 2x + k = 0$ **has non-real solutions**

c) $-x^2 - 2x + k = 0$ **has real and unequal solutions**

d) $-x^2 - 2x + k = 0$ **has two unequal, negative solutions**

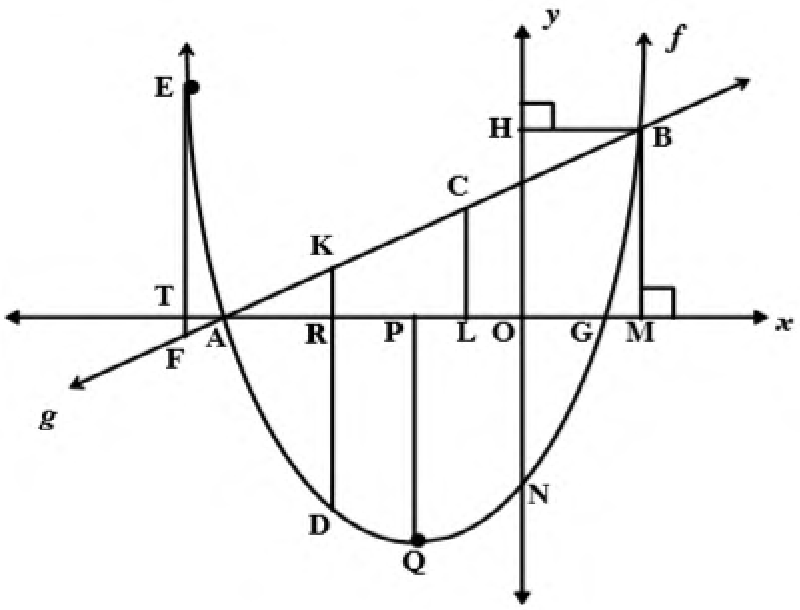
e) $-x^2 - 2x + 8 = k$ **has non-real solutions**
f) $-x^2 - 2x + 8 = k$ **has equal solutions**

g) $-x^2 - 2x + 8 = k$ **has two distinct real solutions**
h) $-x^2 - 2x + 8 = k$ **has two unequal solutions which differ in sign**

TOPIC: Functions: (Lesson 13)	Weighting	45	Grade	11
RESOURCES				
Mind Action Series Gr. 11				

EXAMPLE

1. In the diagram below are the graphs of $f(x)=x^2+3x-4$ **and** $g(x)=x+4$



Determine:

(a) the length of AG and ON.
(b) the length of CL if $\text{OL}=1$ unit
(c) the length of OT if $\text{ET}=6$ units
(d) the length of KD if $\text{OR}=2$ units
(e) the length of OT if $\text{EF}=7$ units
(f) the length of BM and HB
(g) Determine the coordinates of Q
(h) Determine the length of PQ
(i) the maximum length of KD

a. A and G are the x-intercepts of the parabola.

$\therefore 0=x^2+3x-4$

$\therefore 0=(x+4)(x-1)$

$x=-4$ or $x=1$

AG=5 units

N is the y-intercept of the parabola

$\therefore$ ON=4 units

b. Since OL= 1 unit, therefore $x_L=-1$.

$\therefore y=-1+4=3$

$\therefore$ CL= 3 units

$$6 = x^2 + 3x - 4$$
$$0 = x^2 + 3x - 10$$

c.
$$0 = (x+5)(x-2)$$
$x = -5$ or $x = 2$

$\therefore$ OT= 5 units

d. KD= $y_K - y_D$

$$KD = (x+4) - (x^2 + 3x - 4)$$
$$KD = x + 4 - x^2 - 3x + 4$$
$$KD = -x^2 - 2x + 8$$
$$KD = -(-2)^2 - 2(-2) + 8 = 8$$

e. EF= $y_E - y_F$

$$EF = (x^2 + 3x - 4) - (x + 4)$$
$$7 = x^2 + 3x - 4 - x - 4$$
$$0 = x^2 + 2x - 15$$
$$0 = (x+5)(x-3)$$
$$x = -5$$

f. Determine the maximum length of KD.
To find the maximum length of the line segment KD, we use completing the square.

$$KD = (x+4) - (x^2 + 3x - 4)$$
$$\therefore KD = x + 4 - x^2 - 3x + 4$$
$$\therefore KD = -x^2 - 2x + 8$$
$$\therefore KD = -(x^2 + 2x) + 8$$

or $x = 3$

$\therefore$ OT= 5 units

$$x^2 + 3x - 4 = x + 4$$
$$x^2 + 2x - 8 = 0$$

g.
$$(x+4)(x-2) = 0$$
$x = -4$ or $x = 2$

$$y = 2 + 4 = 6$$

$\therefore$ BM = 6 units and HB = 2 units

$x_Q = -\frac{3}{2(1)} = -\frac{3}{2}$

h. $y = \left(-\frac{3}{2}\right)^2 + 3\left(-\frac{3}{2}\right) - 4 = -\frac{25}{4}$

$\therefore Q\left(-\frac{3}{2};-\frac{25}{4}\right)$

i. $PQ = \frac{25}{4}$ units

$KD = (x+4) - (x^2 + 3x - 4)$

$KD = -x^2 - 2x + 8$

$KD = -(x^2 + 2x) + 8$

j. $\therefore KD = -\left[x^2 + 2x + \left(\frac{1}{2}(2)\right)^2 - \left(\frac{1}{2}(2)\right)^2\right] + 8$

$KD = -[x^2 + 2x + 1 - 1] + 8$

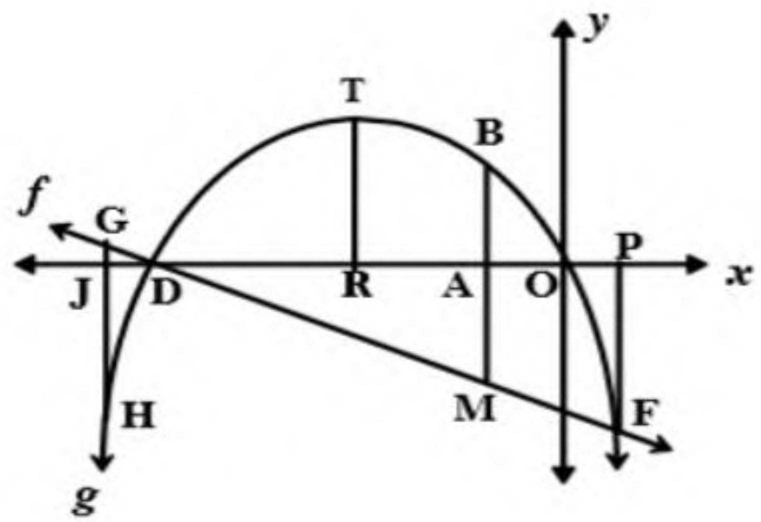
$KD = -[(x+1)^2 - 1] + 8$

$KD = -(x+1)^2 + 9$

KD = 9 units

ACTIVITY1.13.1

The graphs of $f(x) = -2x - 8$ and $g(x) = -2x^2 - 8x$ are represented in the diagram below.



T is the turning point of g.Determine:

a) The length of OD.

b) The length of TR.

c) The equation of TR

d) BM if OA = 1

e) OJ if GH = 28

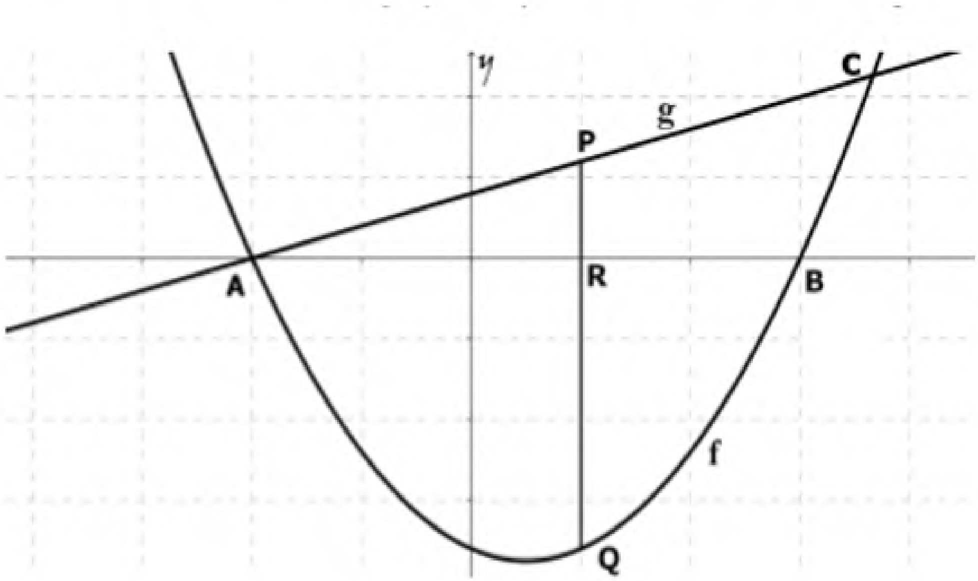
f) The length of FP

g) The maximum length of BM.

TOPIC: Functions: (Lesson 14)	Weighting	45	Grade	11
RESOURCES				
Mind Action Series Gr. 11				

EXAMPLE

1. The figure sketched below shows graphs of $f(x) = 3x^2 - 3x - 18$ **and** $g(x) = 2x + 4$.



Determine:

1.1.1 **the coordinates A and B**
1.1.2 **the coordinates of C**

1.2 For which value of x **will :**

1.2.1 $f(x) > 0$

1.2.2 $f(x) \leq 0$

1.2.3 $f(x) \geq g(x)$

1.2.4 $f(x) \leq g(x)$

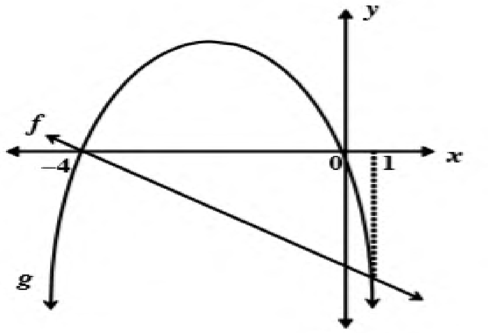
1.2.5 $g(x).f(x) > 0$

1.2.6 $x.f(x) > 0$

ACTIVITY 1.14.1

The graphs of $f(x) = -2x - 8$ and $g(x) = -2x^2 - 8x$ are represented in the diagram below.

Determine the values of x for which:



a) $f(x) > 0$

b) $f(x) \leq 0$

c) $f(x) = g(x)$

d) $f(x) \geq g(x)$

e) $f(x) < g(x)$

f) $g(x) - f(x) = 8$

g) $g(x) - f(x) = 12$

h) $f(x).g(x) \geq 0$

i) $f(x).g(x) > 0$

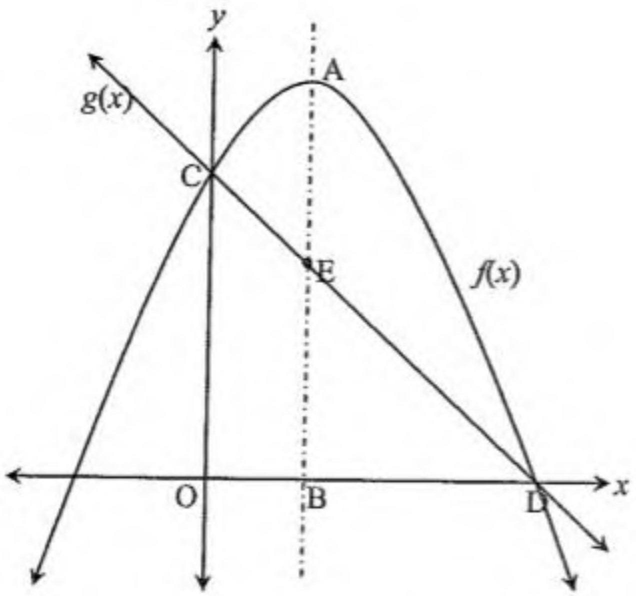
j) $f(x).g(x) \leq 0$

k) $f(x).g(x) < 0$

l) $x.f(x) > 0$

TOPIC: Functions: (Lesson 15)	Weighting	45	Grade	11
RESOURCES				
Past exam papers				

1.15.1 In the diagram below, $f(x) = -x^2 + x + 12$ and $g(x) = mx + c$



a. Determine the coordinates of C and D. (3)

b. Determine the values of m and c, hence determine the equation of $g(x)$ (2)

c. If $\text{OB} = \frac{1}{2}$, find the length of AE. (3)

d. For which values of x is $f(x)$ decreasing? (1)

e. Write down the range of $f(x)$

1.15.2 Given $f(x) = \frac{1}{x-3} - \frac{2x+6}{x+3}$

a. Show that $f(x)$ can be written a $f(x) = \frac{1}{x-3} - 2$ (2)

b. Write down the equations of the asymptotes of f. (2)

c. Determine the x-intercept of f. (3)

d. Determine the y-intercept of f. (2)

e. Sketch the graph of f. Show clearly ALL the intercepts with the axes and the asymptotes. (3)

f. Determine the equation of the axis of symmetry of f having positive gradient. (3)

g. The graph of f is transformed to obtain the graph of $h(x) = \frac{1}{x}$. Describe the transformation from f to h (2)

h. Write down the domain of h. (2)

1.15.3 Given: $f(x) = -x^2 + 2x + 3$ and $g(x) = 1 - 2^x$

a. Sketch the graphs of f and g on the same set of axes.

b. Determine the average gradient of f between $x=-3$ and $x=0$.

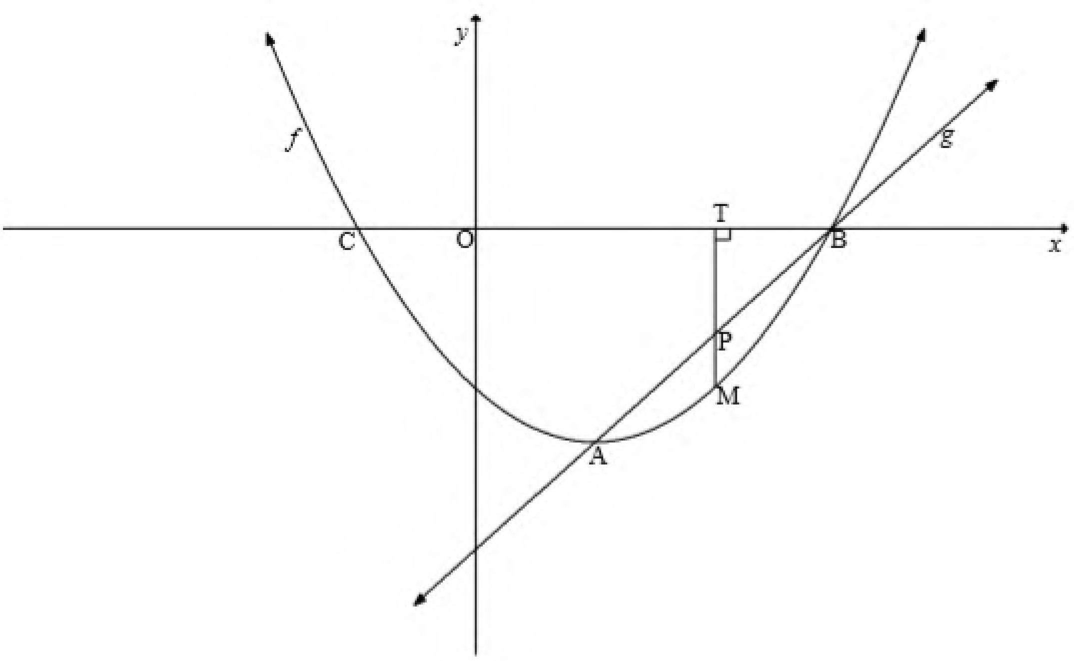
c. For which value(s) of x is $f(x).g(x)\geq 0$?

d. Determine the value of c such that the x-axis will be a tangent to the graph of h, where $h(x)=f(x)+c$

e. Determine the y-intercept of t if $t(x)=-g(x)+1$

f. The graph of k is a reflection of g about the y-axis. Write down the equation of k.

1.15.4 The graph of $f(x)=x^2+bx+c$ and the straight line g are sketched below. A and B are the points of intersection of f and g. A is also the turning point of f. The graph of f intercects the x-axis at B (3;0) and C. The axis of symmetry of f is $x=1$.



a. Write down the coordinates of C. (1)

b. Determine the equation of f in the form $y=ax^2+bx+c$ (3)

c. Determine the range of f. (2)

d. Calculate the equation of g in the form $y=mx+c$ (3)

e. For which values of x will :

(i) $f(x)\geq 0$

(ii) $\dfrac{f(x)}{g(x)}>0$ (2)

(iii) $x.f(x)>0$ (2)

f. For what values of p will $x^2 - 2x = p$ have non-real roots? (2)

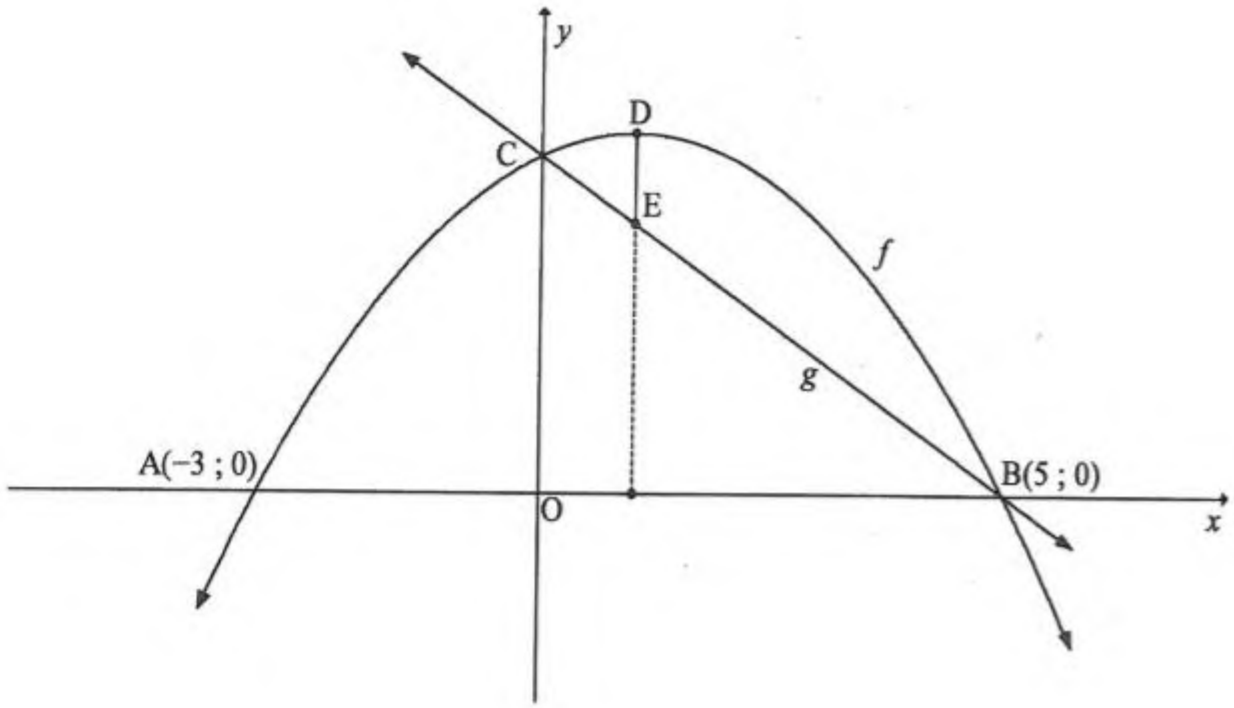
g. T is a point on the x-axis and M is a point on f such that TM $\perp$ x-axis.

TM intersects g at P. Calculate the maximum length of PM. (4)

1.15.5 Write $y = \dfrac{7-x}{x-1}$ in the form $y = \dfrac{k}{x-p} + q$ (4)

1.15.6 The diagram below shows the graphs of $f(x) = -x^2 + 2x + 15$ and $g(x) - 3x + k$.

Graph f cuts the x-axis at $A(-3;0)$ and $B(5;0)$, the y-axis at C and has a turning point at D. Graph g cuts the x-axis at B and the y-axis at C. E is a point on g such DE is parallel to the y-axis.



a. Show that $k = 15$. (1)

b. Determine the coordinates of D , the turning point of f . (3)

c. Determine the value(s) of x for which f is increasing . (1)

d. Calculate the average gradient between points A and B . (3)

e. Calculate the length DE. (2)

f. If $h(x) = f(x-1) - 2$, determine the equation of h in the form $h(x) = a(x+p)^2 + q$. (4)

g. Determine the maximum value of $p(x) = 3^{f(x)-12}$ (3)

h. Determine the value(s) of x for which $f(x)+k=0$ will have two distinct real roots. (2)

TOPIC: FINANCE: GROWTH AND DECAY (Lesson 1)	Weighting	15	Grade	10

RESOURCES

- Grade10 and 11 text books
- KZN DOE document (JIT term 4)

Example 1

Thando has R 100 in his savings account. The bank pays him a simple interest rate of 10% p.a. a) Calculate the amount that Thando receives if she decides to withdraw the money after 1year, 2 years,3years respectively(calculate the simple interest.

Solution. $after\ the\ 1^{st}\ year: SI = P.i.n = 100 \times 0{,}1 \times 1 = R10$

$after\ the\ 2^{nd}\ year: SI = P.i.n = 100 \times 0{,}1 \times 2 = R20$

$after\ the\ 3^{rd}\ year: SI = P.i.n = 100 \times 0{,}1 \times 3 = R30$

Therefore the Total amount Thando is R100 + 30 = R 130

b) Calculate the amount that Thando receives if he decides to withdraw the money after 3years using

$A = P(1 + i.n)$

Solution.

$(b) A = P(1 + i.n) = 100\left(1 + \frac{10}{100}.3\right) = R130{,}00$ Thando would have $R130{,}00$

Example 2

What amount of money should Godfrey invest for 5 years at an interest rate of 8 per annum on a simple investment to have an amount of R500 saved?

Solution

$A = P(1 + i.n) = 500\left(1 + \frac{8}{100}.5\right) = R700{,}00$ Godfrey would have $R700{,}00$

ACTIVITIES / ASSESSMENT

2.1.1. Mary invested R20 000 at a bank that offered her a simple interest of 18% p.a. Calculate the total amount that she will have after 6 years.

2.1.2. Steven wants to save money to buy a computer worth R6 000 in 3 years' time. The interest rate offered is 5% p.a. simple interest. How much money does he need to invest now?

TOPIC: Growth and Decay (Lesson 2)	Weighting	15	Grade	10

RESOURCES

- Grade10 and 11 text books
- KZN DOE document (JIT term 4)

Example 1
Than do has R 500 in his savings account. The bank pays him a compound interest rate of 10% p.a. Calculate the amount that Thando receives if he decides to withdraw the money after 3years.
Solution.
$A = P(1+i)^n \ = 500\left(1+\frac{10}{100}\right)^3 = R665{,}50$ Thando would have $R665{,}50$

Example 2
What amount of money should Godfrey invest for 5 years at an interest rate of 8, 8% per annum to have an amount of R15 000 saved?
Solution

$$A = P(1+i)^n$$
$$15000 = P\left(1+\frac{8{,}8}{100}\right)^5$$
$$P = \frac{15000}{\left(1+\frac{8{,}8}{100}\right)^5} = R9838{,}91$$

He must invest $R9838{,}91$

ACTIVITIES / ASSESSMENT

2.2.1. Katleho starts a business and borrows R28 000 for 3 years to get the business going. Katleho agrees to pay it back at 12% p.a. compounded annually. Calculate the total amount that Katleho will have to pay back.

2.2.2. Steven wants to save money to buy a computer worth R6 000 in 3 years' time. The interest rate offered is 5% p.a. compounded. How much money does he need to invest now?

2.2.3. Your uncle borrows R8 000 and pays it back over a 9-year period. In total, he must pay back R18 000. What compound interest rate was he charged? Round your answer to 2 decimal places.

TOPIC: Growth and Decay (Lesson 3)	Weighting	15	Grade	10

RESOURCES

- Grade10 and 11 text books
- KZN DOE document (JIT term 4)

ERRORS/MISCONCEPTIONS/PROBLEM AREAS

- Language barrier
- Conversion of interest rate from % to **decimal**.
- Incorrect substitution into a formula, e.g. A in P and vice versa

Example 1. The following advertisement appeared with regards to buying a bicycle on a hire purchase agreement loan:

- Purchase price: R5999
- Required deposit: R600
- Loan term: ONLY 18 months at 8% p.a. simple interest

a) Calculate the monthly amount that a person must budget for, in order to pay for the bicycle.

Solution (a) Loan amount required:

$$R5999 - R600 = R5399$$
$$Total\ owing: A = P(1 + n.i)$$
$$A = 5399(1 + (1{,}5)(0{,}08))$$
$$A = 6046{,}88$$

Monthly payment: 6046,88÷18 = 335,94

R335,94 will need to be budgeted

b) How much interest does one have to pay over the full term of the loan?

Solution (b) R6046,88 - R5399 = R647,88

Example 2. The following information is given:

- 1 ounce = 28,35g
- \$1 = R8,79

Calculate the rand value of a 1kg gold bar, if 1 ounce of gold is worth \$978, 34.

Solution:

$$1kg = 1000g$$
$$\frac{1000}{28{,}35} = 35{,}2733$$
$$cost\ in\ dollars = 35{,}2733 \times 978{,}34 = 34509{,}347$$
$$cost\ in\ rands = 34509{,}347 \times 8{,}79 = R303337{,}16$$

ACTIVITIES / ASSESSMENT

2.3.1 Mary wants to buy a fridge that costs R15 550. She must pay a deposit of 15% of the cost, and the balance by means of a hire-purchase agreement. The rate of interest on the loan 16, 25% p.a. simple interest. The repayment period of the loan is 54 months. In addition to the hire-purchase agreement, an annual insurance premium of 1, 5% of the total cost of the fridge is added. The annual insurance premium should be paid in monthly instalments.

a) Calculate the value of the loan that Mary will take.

b) Calculate the total amount that must be repaid on the hire-purchase agreement.

c) Calculate the monthly repayment which includes the monthly insurance premium.

2.3.2. Table below shows the Rand equivalent of one British pound and one US dollar

Country	Currency	Rate of exchange of rands
Britain (United kingdom)	Pound £	21,41
United State of America	Dollar \$	13,35

A South African nurse works in the United States of America.

a) The nurse saves the equivalent of R4 800 per month. Calculate the amount in US dollars that she saves per month.

b) The nurse ordered a book from the United Kingdom and paid \$85 for it. Calculate the price of the book in pounds.

TOPIC: Growth and Decay (Lesson 4)	Weighting	15	Grade	11

RESOURCES

- Grade10 and 11 text books
- KZN DOE document (JIT term 4)

Example 1

A fridge costs R9999. Calculate what it will be worth in 5 years' time if it depreciates:
a) On a reducing balance at 8% p.a.
b) On a straight-line basis at 10% p.a.

Solution: $A = P(1-i)^n = 9999\left(1-\frac{8}{100}\right)^5 = R6590{,}16$

The fridge would be $R6590{,}16$

Solution

b) $A = P(1-n, i) = 9999\big(1-(5\times 0{,}1)\big) = R4999{,}50$

Example 2

The price of a new school bus is R540 000. The value of the bus decreases at 11% per annum according to the diminishing balance method. Calculate the value of the bus after 8 years.

Solution: $A = P(1-i)^n = 540000\left(1-\frac{11}{100}\right)^8 = R212575{,}80$

The value of the bus would be $R212575{,}80$

ACTIVITIES / ASSESSMENT

2.4.1 A school buys laptop at a total cost of R10 000. If the average rate of inflation is 6, 1% p.a. on a reduced balance method over the next 4 years, determine the value of the laptop after 4 years.

2.4.2 A machine costs R25 000 in 2016. Calculate the book value of the machine after 6 years if it depreciates at 9%p.a. according to the reducing balance method.

TOPIC: Growth and Decay (Lesson 5)	Weighting	15	Grade	11

RESOURCES

- All text books
- Grade10 and 11 text books
- KZN DOE document (JIT term 4)

Example 1

A cell phone is currently worth R3037, 50. It depreciated for 3 years at a rate of 25% per annum on a reducing balance method. What was the original price of a cell phone?
Solution

$$A = P(1-i)^n$$

$$3037{,}50 = P\left(1-\frac{25}{100}\right)^3$$

$$P = \frac{3037{,}50}{\left(1-\frac{25}{100}\right)^3} = R7200{,}00$$

The cell phone was R7 200, 3 years ago

Example 2
A tractor bought for R120 000 depreciates to R11 090, 41 after 12 years by using the reducing balance method. Calculate the rate of depreciation per annum. (The rate was fixed over the 12 years).
Solution:

$$A = P(1-i)^n$$
$$11090{,}41 = 120000(1-i)^{12}$$
$$\frac{11090{,}41}{120000} = (1-i)^{12}$$
$$\sqrt[12]{\frac{11090{,}41}{120000}} = (1-i)$$
$$\sqrt[12]{\frac{11090{,}41}{120000}} - 1 = -i$$
$$-0{,}1799999 = -i$$
$$i = 0{,}179999$$

The rate of depreciation is 18%

ACTIVITIES / ASSESSMENT

2.5.1. A car costing R201000 depreciate at 12, 21% pa compounded quarterly. Calculate the price of the car after 5 years.

2.5.2. A small bus company buys a bus for R1, 2 million. The depreciation rate on the bus is 20% p.a compounded monthly on a reduced balance method. Calculate the value of the bus after 6 years.

2.5.3. Machinery used in a factory workshop cost R400000. After four years the machinery is worth R200000. Calculate the rate of depreciation p.a if the depreciation is calculated on a reducing balance method.

2.5.4. If the value of a car purchased for R318 660 is R193 450, 76 after 5 years, calculate the rate of depreciation p.a based on a reduced balance method.

TOPIC: Growth and Decay (Lesson 6)	Weighting	15	Grade	11

RESOURCES

- All text books
- Grade10 and 11 text books
- KZN DOE document (JIT term 4)

Example 1
What amount of money should Godfrey invest for 6 years at an interest rate of 8, 8% per annum compounded semi-annually to have an amount of R15 000 saved?
Solution:

$$A = P(1+i)^n$$
$$15000 = P\left(1+\frac{8{,}8}{200}\right)^{12}$$
$$P = \frac{15000}{\left(1+\frac{8{,}8}{200}\right)^{12}} = R8947{,}16$$

He must invest $R8947{,}16$

Example 2

Cyril has R5000 and wants to invest it so that it grows to R8000. He has 4 years to do so. What interest rate compounded monthly, needs to be offered for this to be possible?

Solution:

$$x = P(1+i)^n$$

$$8000 = 5000\left(1+\frac{i}{1200}\right)^{48}$$

$$\sqrt[48]{\frac{8000}{5000}} = \left(1+\frac{i}{1200}\right)$$

$$\sqrt[48]{\frac{8}{5}} - 1 = \frac{i}{1200}$$

$$0{,}00983983824 = \frac{i}{1200}$$

$$i = 11{,}80780588471$$

The interest rate is 11,81%

ACTIVITIES / ASSESSMENT

2.6.1. Steven plan to investment R5 000 for 4 years, he went to the following 4 different banks which offered him the following: if:

- **Bank A**, the interest rate of 8, 5% p.a compounded annually.
- **Bank B**, the interest rate of 7, 55% p.a compounded semi-annually.
- **Bank C**, the interest rate of 7, 8% p.a compounded quarterly.
- **Bank D**, the interest rate of 7, 5% p.a compounded monthly.

Which bank should Steven invest his money in order to have greater investment after 4years?

2.6.2. What amount of money should Godfrey invest for 6 years at an interest rate of 8, 8% per annum compounded semi-annually to have an amount of R15 000 saved?

2.6.3. Cyril has R5000 and wants to invest it so that it grows to R8000. He has 4 years to do so. What interest rate compounded monthly, needs to be offered for this to be possible?

TOPIC: Growth and Decay (Lesson 7)	Weighting	15	Grade	11

RESOURCES

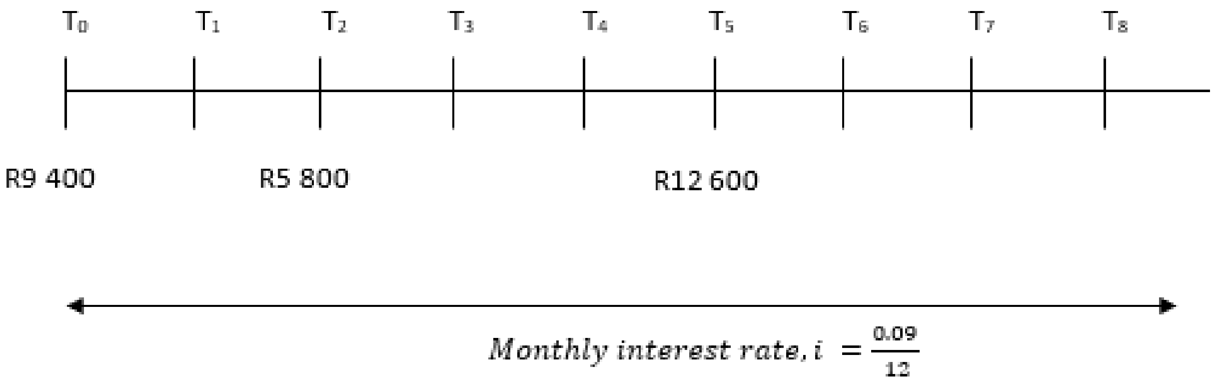
KZN DOE document grade 11

All textbooks

Previous question papers

Example 1

A savings account is opened with a deposit of R9 400 and two years later a further R5 800 is added to the savings account. Five years after the saving account was opened, another R12 600 is deposited into the account. The interest paid on the savings is 9% p.a. compounded monthly. Calculate the total amount saved at the end of the eight years.



Solution 1

At the end of the second year the total amount in the savings account is made up of:

- the initial deposit of R9 400 plus two years' interest on R9 400
- the second deposit of R5 800.

$$A_1 = 9\ 400\left(1 + \frac{0{,}09}{12}\right)^{2 \times 12} + 5\ 800$$
$$= \text{R}17\ 046{,}29$$

At the end of the fifth year the total amount accumulated in the savings account will be made up of:

- R17 046,29 plus the interest paid on R17 046,29 for three years
- the third deposit of R12 600.

$$A_2 = 17\ 046{,}29\left(1 + \frac{0{,}09}{12}\right)^{3 \times 12} + 12\ 600$$
$$= \text{R}34\ 907{,}55$$

Solution 2

The first deposit of R9 400 was in the savings account for eight years and hence the accumulated amount at the end of eight years will be:

$$A_1 = 9\ 400\left(1 + \frac{0{,}09}{12}\right)^{8 \times 12} = \text{R}19\ 259{,}86$$

The second deposit of R5 800 was in the savings account for six years.

$$\therefore A_2 = 5\ 800\left(1 + \frac{0{,}09}{12}\right)^{6 \times 12} = \text{R}9\ 932{,}81$$

The third deposit of R12 600 was in the savings account for three years.

$$A_3 = 12\ 600\left(1 + \frac{0{,}09}{12}\right)^{3 \times 12} = \text{R}16\ 488{,}93$$

Therefore the total amount at the end of eight years will be:

$$19\ 259{,}86 + 9\ 932{,}81 + 16\ 488{,}93 = \text{R}45\ 681{,}60$$

This can be done in one calculation using a memory key to store $\left(1 + \frac{0{,}09}{12}\right)$

$$A = 9\ 400\left(1 + \frac{0{,}09}{12}\right)^{8 \times 12} + 5\ 800\left(1 + \frac{0{,}09}{12}\right)^{6 \times 12} + 12\ 600\left(1 + \frac{0{,}09}{12}\right)^{3 \times 12}$$

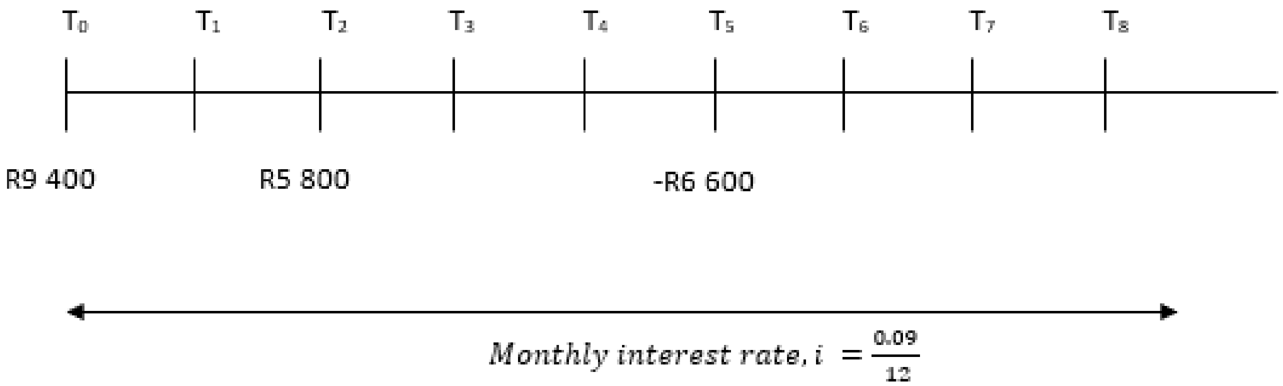
In all future problems we will use only the second method.

ACTIVITIES /ASSESSMENT
2.7.1. Godfrey deposits R3 500 into a savings account. Three years later he deposits a further R2 800. The interest rate for the first two years is 9% p.a. compounded monthly. How much does Godfrey have at the end of the 6th year? 2.7.2. Thomas invests R8 000. After 4 years he also deposits R3 500 into that account. The interest rate was quoted to be 11, 2% compounded semi-annually. How much money will be available at the end 7 years?

TOPIC: Growth and Decay (Lesson 8)	Weighting	15	Grade: 11

Example 1

A savings account is opened with a deposit of R9 400 and two years later a further R5 800 is added to the savings account. Five years after the saving account was opened, R6 600 is withdrawn from the account. The interest paid on the savings is 9% p.a. compounded monthly. Calculate the total amount saved at the end of the eight years.



At the end of the second year the total amount in the savings account is made up of:

- the initial deposit of R9 400 plus two years' interest on R9 400
- the second deposit of R5 800.

$$A_1 = 9\ 400\left(1 + \frac{0{,}09}{12}\right)^{2 \times 12} + 5\ 800$$
$$= R17\ 046{,}29$$

At the end of the fifth year the total amount accumulated in the savings account will be made up of:

- R17 046,29 plus the interest paid on R17 046,29 for three years
- the third deposit of R12 600.

$$A_2 = 17\ 046{,}29\left(1 + \frac{0{,}09}{12}\right)^{3 \times 12} + 12\ 600$$
$$= R34\ 907{,}55$$

ACTIVITIES /ASSESSMENT

2.8.1. Thomas invests R8 000. After 4 years he needs R3 500 for an emergency so withdraws from the account. The interest rate is 11, 2% compounded semi-annually for 7 years. How much money will be available at the end of the investment period?

2.8.2. Kamil inherits R60 000 from his grandfather. He invests the money in a savings account which earns him 10% p.a. compounded quarterly. After 4 years, Kamil withdraws R14000 from his account. How much money is in Kamil's account at the end of 8 years?

TOPIC: Growth and Decay (Lesson 9)	Weighting	15	Grade: 11

RESOURCES

- All text books
- Grade10 and 11 text books
- KZN DOE document (JIT term 4)

Example 1
Thembi invests R16 000 for a period of 7 years into an account that pays 9% p.a. compounded monthly for the first 3 years. The interest rate then changes to 9, 5% p.a. compounded semi-annually. Calculate the future value of her investment.
Solution

$$A = P(1+i)^n = 4500[\left(1+\frac{9}{1200}\right)^{36}.\left(1+\frac{9{,}5}{200}\right)^{8}] = R30351{,}08$$

Example 2
Xolile invested a certain sum of money for 8 years at 7,5% p.a. compounded semi-annually for the first 2 years and 8,5% p.a. compounded quarterly for the next 6 years. At the end of 8 years her money had grown to R8636, 44. Determine the amount that Xolile invested.
Solution

$$P = A(1+\_)^{-n} = 8636{,}44[\left(1+\frac{7{,}5}{200}\right)^{-4}.\left(1+\frac{8{,}5}{400}\right)^{-24}] = R4500{,}00$$

Example 3
Godfrey deposits R3 500 into a savings account. Three years later he deposits a further R2 800. The interest rate for the first two years is 9% p.a. compounded monthly. The interest rate changes to 9, 75% p.a. compounded quarterly for the final 4 years of the investment period. How much does Godfrey have at the end of the 6th year?

Solution

- For R3500: $A = P(1+i)^n = 3500[\left(1+\frac{9}{1200}\right)^{24}.\left(1+\frac{9,75}{400}\right)^{16}] = R6155,92$

- For R2800: $A = P(1+i)^n = 2800\left(1+\frac{9,75}{400}\right)^{12} = R3738,23$

Godfrey would have $R6155,92 + R3738,23 = R9894,15$ is his investment.

ACTIVITIES / ASSESSMENT

2.9.1. Zama invests R15 000 for five years. For the first three years it earns interest at 8.5% per annum compounded quarterly, and for the last two years, 7.5% per annum compounded semi-annually. What is the total interest earned over five years?

2.9.2. Thomas invests R8 000. After 4 years he needs R3 500 for an emergency so withdraws from the account. The interest rate is 11, 2% compounded semi-annually for 3 years then changes to 10, 5% compounded quarterly for the remaining 2 years of the investment. How much money will be available at the end of the investment period?

2.9.3. Kamil inherits R60 000 from his grandfather. He invests the money in a savings account which earns him 10% p.a. compounded quarterly for the first three years and then 8,8% compounded monthly for the next 5 years. After 4 years, Kamil withdraws R14 000 from his account and 3 years later, he deposits an amount of R6 000. How much money is in Kamil's account at the end of 8 years?

2.9.4. Ramil invested R12 000 into a savings account. He was given an interest rate of 10% per annum compounded quarterly for three years. Two years after the initial investment he deposited a further R5 000 into the savings scheme. The interest rate changed to 10.8% p.a. at the start of the 4th year compounded semi-annually. He kept his money for a further two years at an interest rate of 12% per annum compound interest. Determine the value of the investment after 6 years.

TOPIC: Growth and Decay (Lesson 10)	**Weighting**	15	**Grade:**	11

RESOURCES

KZN DOE document grade 11

All textbooks

Previous question papers.

Example 1:

Calculate the effective interest rate if interest is 9, 8% p.a. compounded monthly.

Solution:

$$1+i_{eff}=\left(1+\frac{i_{nom}}{m}\right)^m$$

$$1+i_{eff}=\left(1+\frac{0{,}098}{12}\right)^{12}$$

$$i_{eff}=10{,}25\%$$

Example 2:

John invested R120 000. He is quoted a nominal interest rate of 7, 2% p.a. compounded monthly.

a) Calculate the effective interest rate p.a. correct to three decimal places.
b) Use the effective interest rate to calculate the value of John's investment if he invested the money for three years.
c) Suppose John invests his money for a total of 4 years, but after 18 months he makes a withdrawal of R20 000. How much will he receive at the end of 4 years?

Solution :

(a) $1+i_{eff}=\left(1+\frac{i_{nom}}{n}\right)^n$

$$i_{eff}=\left(1+\frac{\frac{7{,}2}{100}}{12}\right)^{12}-1$$

$$i_{eff}=7{,}44\%$$

(b) $A=P(1+i)^n$

$$=120000\left(1+\frac{7{,}44}{100}\right)^3$$

$$=R148834,46^c$$

(c) For R120000 with no withdrawals: $A=P(1+i)^n$

$$=120000[\left(1+\frac{7{,}2}{1200}\right)^{48}$$

$$=R159913,20^c$$

ACTIVITIES /ASSESSMENT

2.10.1 Calculate the effective interest rate if interest is 12, 8% p.a. if is compounded quarterly.

2.10.2 Convert a nominal interest rate of 7, 8% p.a. compounded monthly to an annual interest rate compounded annually.

2.10.3 Convert an effective annual interest rate of 8, 45% p.a. compounded annually to a nominal interest rate compounded quarterly.

TOPIC: STATISTICS (Lesson 1)	**Weighting**	15	Grade	10

Data Handling & Probability		**Platinum**	
PAGE	EX	PAGE	EX
13	9	226	1
15	1.2	229	2

Past Question Paper: 2018 paper 2

Example 1

The following result are mathematics scores of 12 grade 10 learners in Mvaba high school
36;45;45;73;73;36;36;36;41;41;41;10
Organize a frequency table
Solution

Mathematics scores	**Frequency**
10	**1**
36	**4**
41	**3**
45	**2**
73	**2**

Example 2

Veronica did a survey of 10 of her friends she asked them how many siblings they had.
The frequency table below shows the results of her survey

Number of siblings	0	1	2	3
frequency	2	3	4	1

Determine the mode, median and mean

Solution
Mode

- The greatest frequency in the table is 4, this mean that 2 appears the most.
  Mode= 2

Median

- Note values in the table are already in order size
- Veronica has 10 friends, half of 10 is 5 this mean count along the frequency and reach term 5 and 6, that is where the median lies. **(remember this is a position of the median**)

$$median = \frac{1+2}{2} = \frac{3}{2}$$

Mean

- To find the total number of siblings you must take the frequency of each into account, we multiply each number of sibling by it frequency, and the add the numbers
- Change table and add in another column

Value **Number of siblings** **x**	**Frequency** **Number of veronica's friends**	**Frequency value × value** $\boldsymbol{f \times x}$
0	2	2×0 =0
1	3	3×1 =3
2	4	4×2 =8
3	1	1×3 =3
Total	**N =10**	$\boldsymbol{\sum f \times x = 14}$

$$\bar{x} = \frac{\sum x}{n} = \frac{14}{10} = 1{,}4$$

ACTIVITIES/ ASSESSMENT

3.1.1

9	**5**	**11**	**8**	**12**	**2**	**6**	**9**	**15**	**10**	**12**	**6**	**9**	**3**	**9**	**13**	**14**	**16**	**4**	**7**

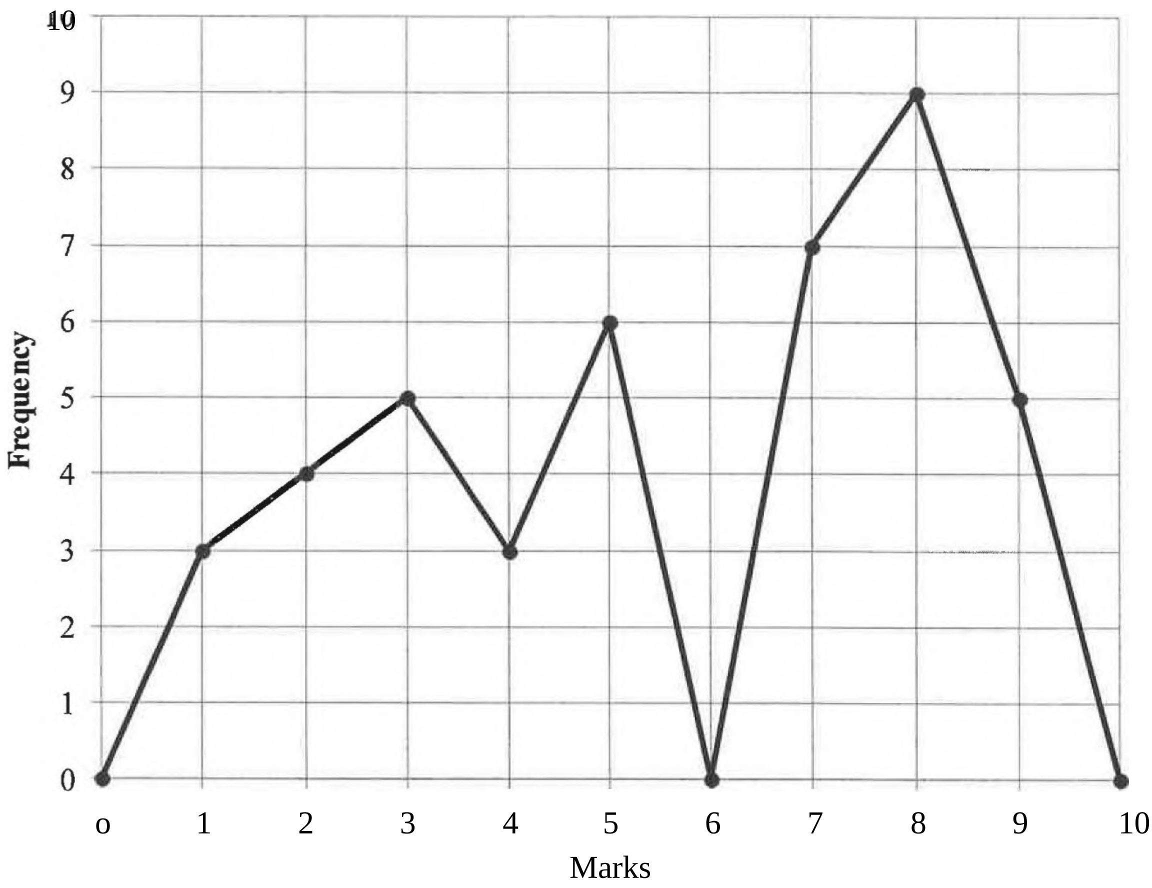
Determine:
3.1.1.1 Mean
3.1.1.2 Median
3.1.1.3 Mode

3.1.2 A certain school provides buses to transport the learners to and from a nearby village. A record is kept of the number of learners on each bus for 26 school days.

27	25	27	29	31	24	25	27	28	29	24	26	30
28	31	25	25	27	28	28	28	26	28	31	24	30

3.1.2.1 Organise the data in a frequency table.
3.1.2.2 Use the table to calculate the total number of learners that were transported to school by bus.
3.1.2.2 Calculate the mean number of learners per trip, correct to one decimal place.
3.1.2.3 Determine the mode.
3.1.2.4 Determine the median.

3.1.3 The line graph below shows test marks out of 10 obtained by a Grade 10 class.



3.1.3.1 Complete the frequency column in the form of a table. (2)
3.1.3.2 How many learners wrote the test? (1)
3.1.3.3 Calculate mean for the test. (3)
3.1.3.4 Determine the median for the data. (3)

TOPIC: STATISTICS (Lesson 2)	**Weighting**	15	Grade	10

Data Handling & Probability		**Platinum**	
PAGE	EX	PAGE	EX
13	9	226	1
15	1.2	229	2

NOTES

Example 1

The mathematics percentage scores of grade 10 learners were represented as follows:

15	46	7	19	51	27	30	3	13	63
32	36	8	41	10	18	25	5	16	30
41	53	59	49	51	61	51	67	64	11

a) How many learners wrote the test?
b) Complete the table below:

Scores	Frequency
$0 < s \leq 10$	
$10 < s \leq 20$	
$20 < s \leq 30$	
$30 < s \leq 40$	
$40 < s \leq 50$	
$50 < s \leq 60$	
$60 < s \leq 70$	

c) Calculate the mean number of scores.
d) Determine the modal interval and the median test scores

Solution

a) Learners
b) Complete the table below:

Scores	Frequency
$0 < s \leq 10$	5
$10 < s \leq 20$	7
$20 < s \leq 30$	4
$30 < s \leq 40$	2
$40 < s \leq 50$	4
$50 < s \leq 60$	5
$60 < s \leq 70$	3

c) To find the mean of a grouped data calculate:

- the midpoint of each interval,

Scores	Frequency	Midpoint
$0 < s \leq 10$	5	5
$10 < s \leq 20$	7	15
$20 < s \leq 30$	4	25
$30 < s \leq 40$	2	35
$40 < s \leq 50$	4	45
$50 < s \leq 60$	5	55
$60 < s \leq 70$	3	65

- multiply the midpoints by the frequency and calculate the sum of the products

Scores	Frequency(f)	Midpoint(m)	Frequency x midpoint
$0 < s \leq 10$	5	5	25
$10 < s \leq 20$	7	15	105
$20 < s \leq 30$	4	25	100
$30 < s \leq 40$	2	35	70
$40 < s \leq 50$	4	45	180
$50 < s \leq 60$	5	55	275
$60 < s \leq 70$	3	65	195
	$n = 30$		$\sum f.x = 950$

- The mean that is calculated in a grouped data is called the estimated mean

$$\bar{x} = \frac{\sum f.x}{n} = \frac{950}{30} = 31{,}67$$

- **The use of CASIO calculator**

Grouped
Set frequency SHIFT→ MODE→ STAT →FREQ ON **Set stat mode** MODE→STAT →1-VAR→ ENTER DATA
<table><tr><th>X</th><th>FREQ</th></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr></table>
NB X is the midpoint of the intervals **Getting answer** →AC SHIFT 1→ VAR→ MEAN

d) In a grouped data modal interval is calculated,

- modal interval refers to the group of values that occurs most often (frequency)

modal interval is $10 < s \leq 20$

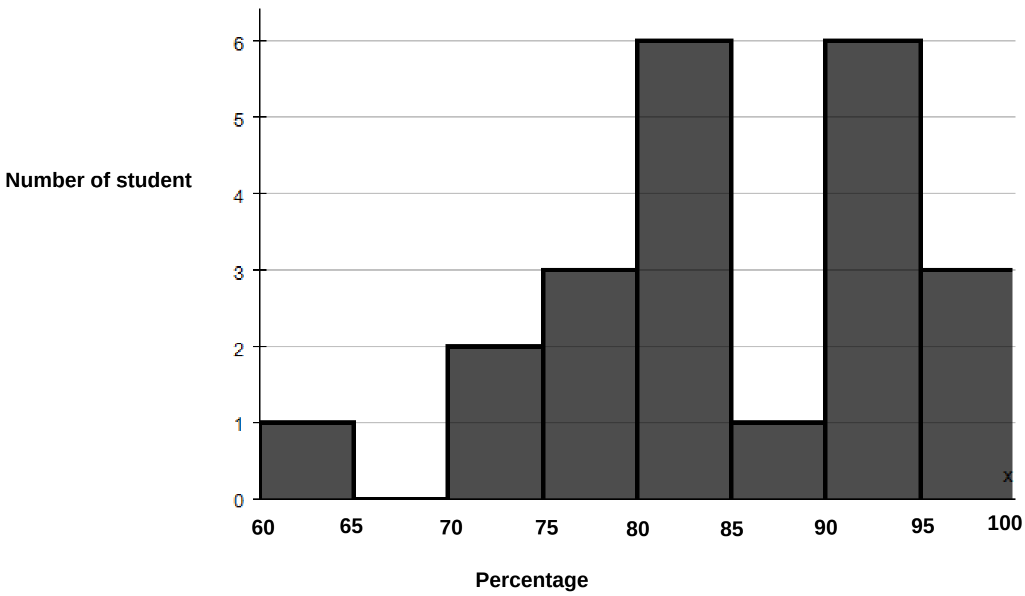
To calculate the estimated median in a group data, find the midpoint of the median interval

- the midpoint of the interval $20 < s \leq 30$ is 25

the median number of scores is approximately 25

ACTIVITIES/ ASSESSMENT

3.2.1 Drawn below is the histogram representing the percentage of student mathematics test results.



3.2.1.1 How many learners wrote a test
3.2.1.2 Use the information in the histogram provided to draw a frequency table.
3.2.1.3 Calculate the estimated mean.
3.2.1.4 Determine the modal interval and the median result.

3.2.2 The annual earnings (in pounds) of the top 20 soccer players during 2011 are represented as grouped data in the following table

Class intervals (in millions of pounds)	**Frequency (number of players)**
5≤x<10	9
10≤x<15	5
15≤x<20	2
20≤x<25	1
25≤x<30	3

3.2.2.1 Calculate the estimated mean of this data
3.2.2.2 Calculate the estimated median of this data
3.2.2.3 Write down the modal class interval of this data

3.2.3 The table below shows information about the number of hours 120 learners spent on their cell phones in the last week.

Number of hours (h)	**Frequency**
$0 < h \leq 2$	10
$2 < h \leq 4$	15
$4 < h \leq 6$	30
$6 < h \leq 8$	35
$8 < h \leq 10$	25
$10 < h \leq 12$	5

3.2.3.1 Identify the modal class for the data.
3.2.3.2 Determine the class that has the most number of values
3.2.3.3 Estimate the mean number of hours that these learners spent on their cell phones in the last week.
3.2.3.4 Draw histogram to represent the data

3.2.4 Alex timed 21 people in the sprint race, to the nearest second

59	**65**	**61**	**62**	**53**	**55**	**60**	**70**	**64**	**56**	**58**	**58**	**62**	**62**	**68**	**65**	**56**	**59**	**68**	**61**	**67**

Organise a group frequency table and determine the mean and the median

TOPIC: Statistics (lesson 3)	**Weighting**	15	Grade	11
RESOURCES				
Grade 10 & 11 Textbooks, Calculator				

Example
Given data set: 32; 37; 39; 41; 43; 45; 47
Determine

a) range
b) 30th percentile of the dataset
c) Lower and Upper quartiles
d) Interquartile and semi-interquartile range

****NB Make sure the data set is sorted**

Solution

32; 37; 39; 41; 43; 45; 47

a) Range = Max – Min = 47 – 32 = 15

b) 30% = $\frac{3}{10}$

Percentile$_{30}$= $\frac{3(n+1)}{10} = \frac{3(7+1)}{10} = 2,4$ OR $(n+1) \times 30\% = 2,4$

Position 2,4 is 37

c) Lower quartiles(Q_1) or 25th percentile

$\frac{n+1}{4} = \frac{7+1}{4} = 2$ position; Q_1= 37

Upper quartiles (Q3) or 75th percentile

$\frac{3(n+1)}{4} = \frac{3(7+1)}{4} = 6$ position; Q_3 = 45

d) Interquartile Range (IQR) = Q3 – Q1 = 45 – 37 = 8

Semi-interquartile Range = $\frac{IQR}{2} = \frac{8}{2} = 4$

ACTIVITIES/ASSESSMENT

3.3.1 The table contains the test results of 20 first year students at UJ

9 ; 10; 20; 30; 45; 50; 60; 65; 65; 65; 70; 70; 75; 82; 82; 89; 90; 93; 93; 95

Determine:
3.3.1.1 range
3.3.1.2 60th percentile of the dataset
3.3.1.3 Lower and Upper quartiles
3.3.1.4 What percentage of values lies between Q_1 and Q_3?
3.3.1.5 Interquartile and semi-interquartile range

3.3.2. The lower quartile = 2 and range = 10 of the following sorted data set:

1; a; 2; 3; 4; 6; 9; 10; b

3.3.2.1 Determine the value of a and b
3.3.2.2 Determine the interquartile range.

TOPIC : Statistics (lesson 4)	**Weighting**	15	Grade	11

RESOURCES

Textbook : Platinum grade 10 and Statistics South Africa grades 10,11 and 12

NOTES

Examples1

The data below represent the number of SMSs sent per day by 11 teenage girls.

12 ; 13 ; 13 ;15 ; 18 ; 19 ; 24 ; 25 ; 33 ; 38 ; 40

a) Write down the five number summary of the data set given above.
b) Calculate the range, IQR, Semi IQR
c) Draw the box and whisker plot /diagram.
d) Approximately what percentage of the data items lie within the IQR?
e) Comment on the spread of the data

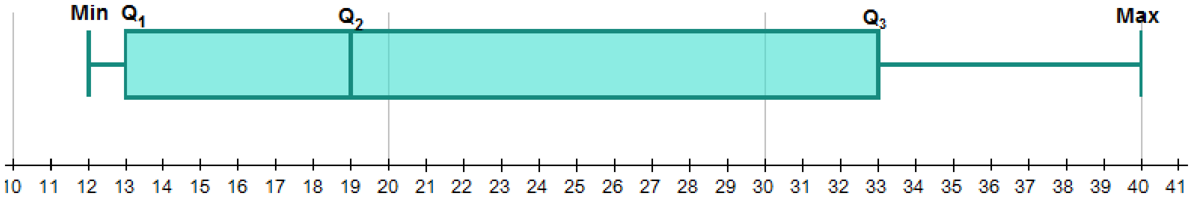
Solution

a) Five number summary
Min value = 12
Q1 = 13
Q2 = 19
Q3 = 33
Max value =40

b) Range = max value – min value
= 40 - 12 = 28

IQR = Q3 – Q2
= 33 – 13 = 20

c) Box and whisker diagram



d) Approximately 50% of the data items lie within the IQR.
e) Data is symmetrically skewed.

ACTIVITIES/ASSESSMENT

3.4.1 The percentage achieve by a learner for series of mathematics tests that he wrote throughout his Grade 9 year are as follows.

45 50 28 39 49 55 35 56 49
37 28 53 55 38 47 51 30 58

3.4.1.1 Calculate the five number summary
3.4.1.2 Draw a box-and-whisker diagram to illustrate the five number summary
3.4.1.3 Determine
3.4.1.3.1The range of set of data
3.4.1.4.2 Interquartile range

3.4.2 As part of 2009 Census@School, the 26 grade 10A learners measured their height.
The girls’ height (in centimetres) were:
150 150 153 155 156 158 160 161 164 164 166 170 170
The boys’ height (in centimetres) were:
140 142 151 157 158 159 160 162 165 180 180 180 180

3.4.2.1 Determine the five number summary and the interquartile range for the girls and for the boys
3.4.2.2 On the same number line draw two box-and-whisker diagrams to illustrate the girls' Height and the boys' height.
3.4.2.3 Use the five number summaries, the interquartile ranges and the box-and-whisker Diagrams to write down two conclusions you can make about the height of the girls and the boys

TOPIC: Statistics (lesson 5)	**Weighting**	20	Grade	11
RESOURCES				
Grade 11 November Past Papers				
ACTIVITIES/ASSESSMENT				

Class Test 1 Grade 11

Total Marks: 15 Duration: 18 Min

3.5.1 The heights of 20 children were measured (in centimetres) and the results were recorded.

The data collected is given in the table below.

127	128	129	130	131	133	134	134	135	136
137	138	139	140	141	142	142	143	144	145

3.5.1.1 Write down the median height measured. (1)

3.5.1.2 Determine:

3.5.1.2.1 The mean height (2)

3.5.1.2.2 The range (1)

3.5.1.2.3 The interquartile range (3)

3.5.1.3 Draw a box and whisker diagram to represent the data. (2)

[9]

3.5.2 The intelligence quotient score (IQ) of a Grade 10 class is summarised in the table below.

IQ INTERVAL	FREQUENCY
$90 \leq x < 100$	4
$100 \leq x < 110$	8
$110 \leq x < 120$	7
$120 \leq x < 130$	5
$130 \leq x < 140$	4
$140 \leq x < 150$	2

3.5.2.1 Write down the modal class of the data. (1)

3.5.2.2	Determine the interval in which the median lies.	(2)
3.5.2.3	Estimate the mean IQ score of this class of learners.	(3)
		[6]

TOPIC: STATISTICS (lesson 6)	**Weighting**	20	**Grade**	11

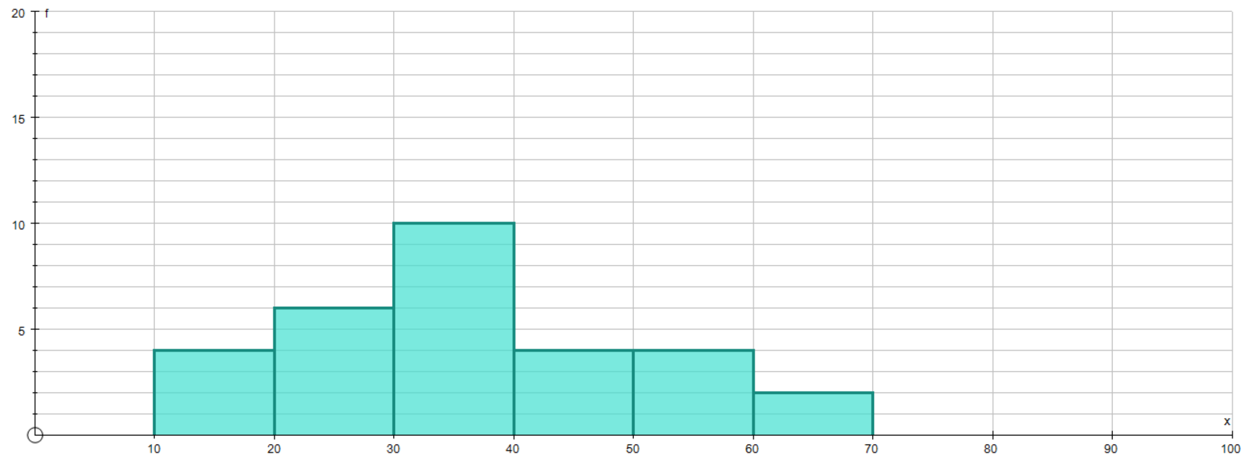
RESOURCES

Grade 11 Textbooks, Calculator
Textbook/s:Via Mathematics Gr 11, pg 302 -307
Maths Handbook and Study Guide Gr 11 pg 216 – 219
Clever Maths Simple Gr 11 pg 436 - 437

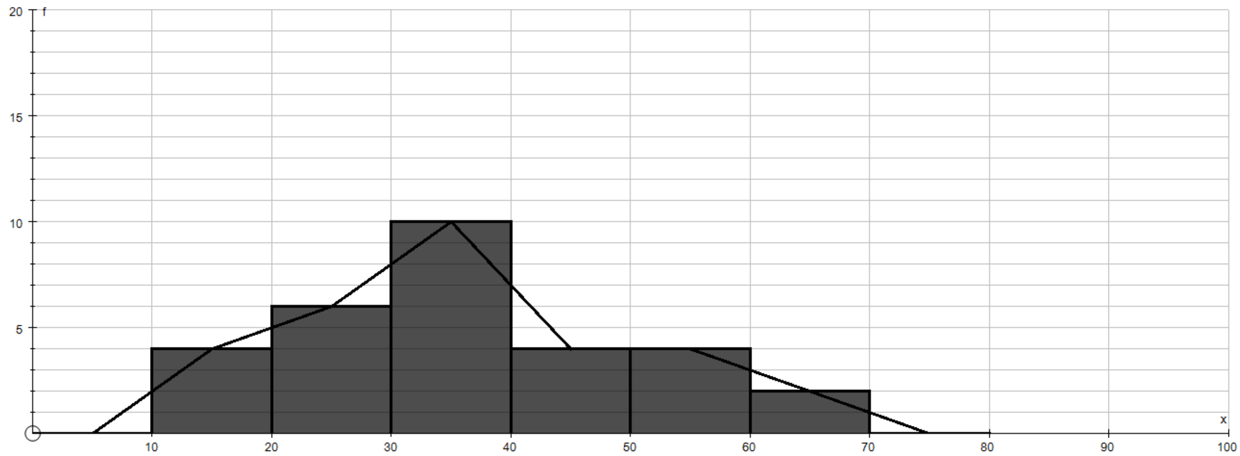
Example
A survey was conducted on the amount of time that a group of grade 11 learners spend on their homework. The results of the survey are show below.

Time (t)	$10 \le t < 20$	$20 \le t < 30$	$30 \le t < 40$	$40 \le t < 50$	$50 \le t < 60$	$60 \le t < 70$
Frequency	4	6	10	4	4	2

a) Draw a histogram for the above data.
b) Draw a frequency polygon on the histogram
solution



a)



b)

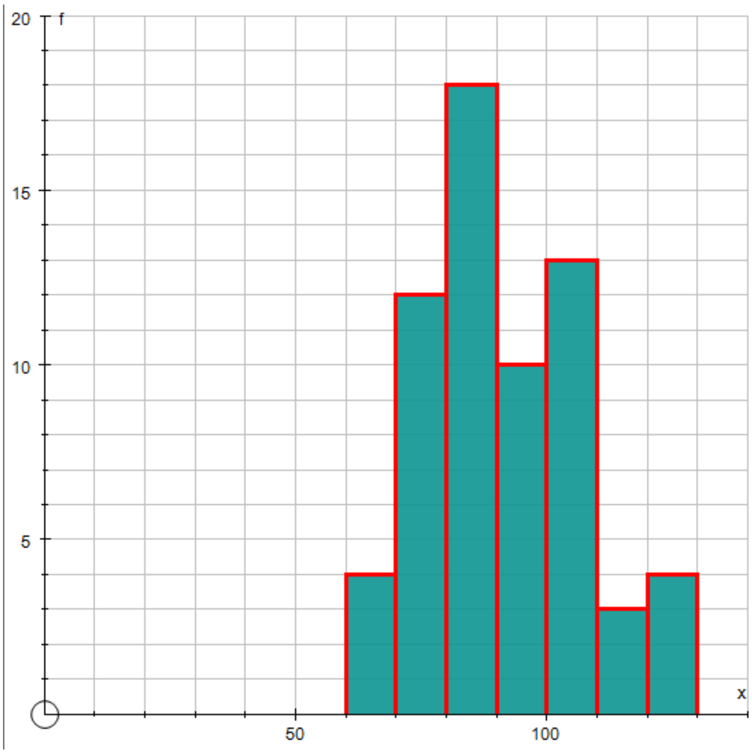
ACTIVITIES/ASSESSMENT

3.6.1 The table shows the grouped frequency distribution of number of text messages, m, sent by each learner per day in a grade 11 group.

No. of messages	$0 \le m < 2$	$2 \le m < 4$	$4 \le m < 6$	$6 \le m < 8$	$8 \le m < 10$	$10 \le m < 12$
Frequency	2	10	20	18	28	25

3.6.1.1 Draw a histogram to show the distribution of this data.
3.6.1.2 Draw a frequency polygon on the histogram
3.6.1.3 How many learners are there in this group?
3.6.1.4 How many learners send six or more text messages per day?
3.6.1.5 What percentage of the learners send less than four text messages per day?

3.6.2 A histogram below represents the distances that a salesman travels in 1 month on each trip.



Draw a frequency table of this data and use it to draw a frequency polygon

TOPIC: STATISTICS (Lesson 7)	**Weighting**	20	**Grade**	11

RESOURCES

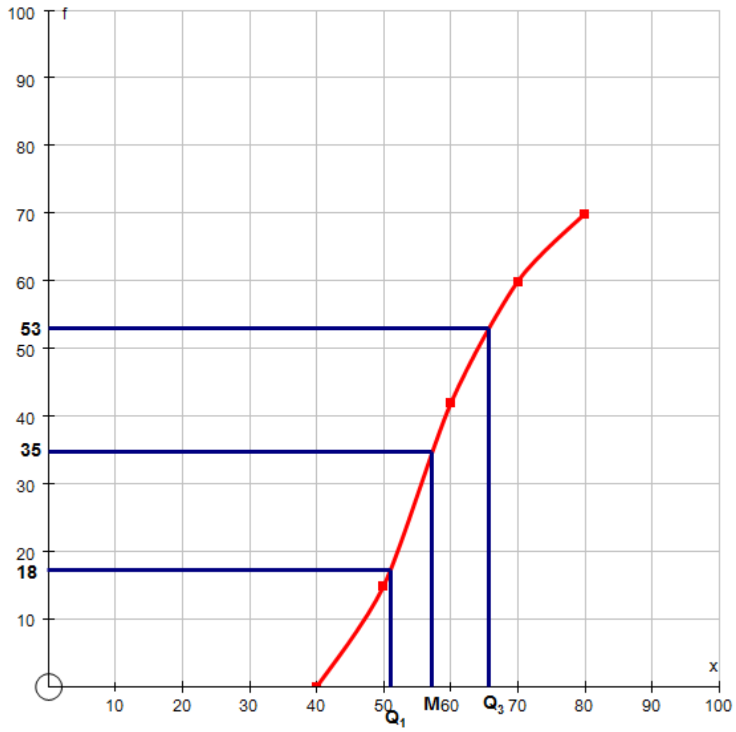
Grade 11 Textbooks, Calculator
Textbook/s:Via Mathematics Gr 11, pg 302 -307
Maths Handbook and Study Guide Gr 11 pg 220 – 223; 384
Clever Maths Simple Gr 11 pg 436 - 437

Example
Use the following data sets to construct an ogive. Label median, upper and lower quartiles.

Interval	Frequency
$40 \le x < 50$	15
$50 \le x < 60$	27
$60 \le x < 70$	18
$70 \le x < 80$	10

Solution

Interval	Frequency	Cumulative Freq
$40 \le x < 50$	15	15
$50 \le x < 60$	27	42
$60 \le x < 70$	18	60
$70 \le x < 80$	10	70



LEARNER ACTIVITIES

3.7.1 The following table shows the marks obtained by 220 learners in a Mathematics Examination.

Percentage	1 -10	11-20	21-30	31-40	41-50	51-60	61-70	71-80	81-90	91-100
Frequency	2	6	11	22	39	59	45	20	11	5

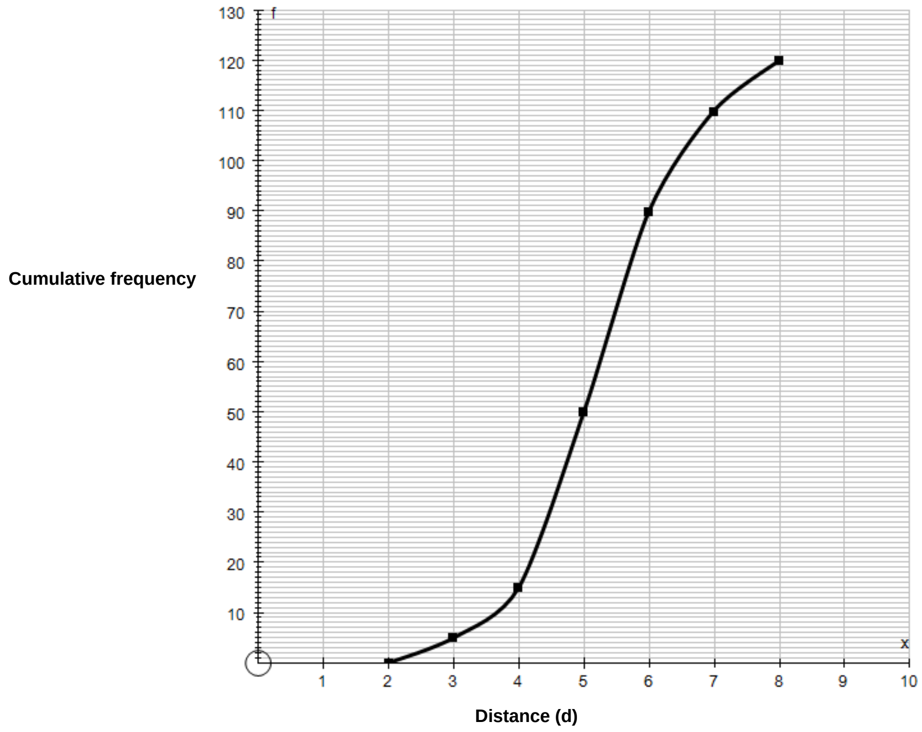
3.7.1.1 Redraw and complete the cumulative frequency for this data

Marks	Frequency	Cumulative frequency
$1 \le x \le 10$		
$11 \le x \le 20$		
$21 \le x \le 30$		
$31 \le x \le 40$		
$41 \le x \le 50$		
$51 \le x \le 60$		
$61 \le x \le 70$		
$71 \le x \le 80$		
$81 \le x \le 90$		
$91 \le x \le 100$		
Total		

3.7.1.2 Draw a cumulative frequency graph (ogive curve) for this data
3.7.1.3 Determine the quartiles.
3.7.1.4 Determine the 90^{th} percentile
3.7.1.5 Determine the 10^{th} percentile

TOPIC : Statistics (lesson 8)	Weighting	20	Grade	11

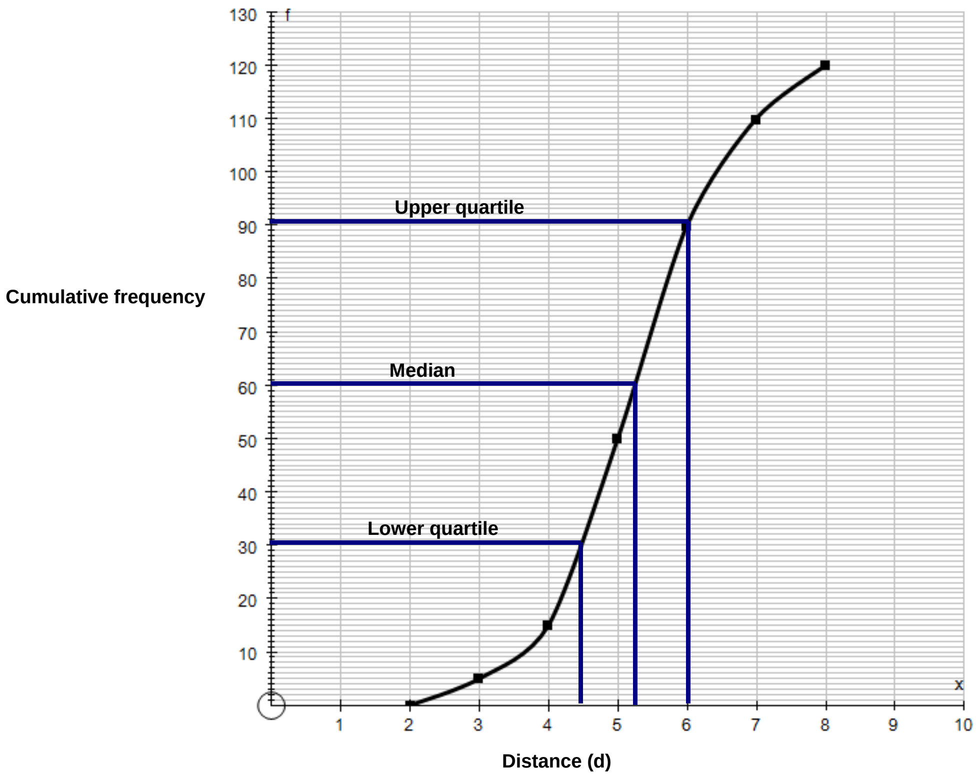
The ogive below shows distances (d) in kilometers that a person training for a race runs in 6 months.



a) How many time did this person ran over 6 months?
b) Determine median and inter-quartile range.
c) What is the model class?
d) What percentage that he runs less than 5km?

Solution
a) 120 times

b)



Lower quartile position: $120 \times \frac{1}{4} = 30$; lower quartile = 4,5km (read off the graph)

Median position : $120 \times \frac{1}{2} = 60$; median = 5,2km (read off the graph)

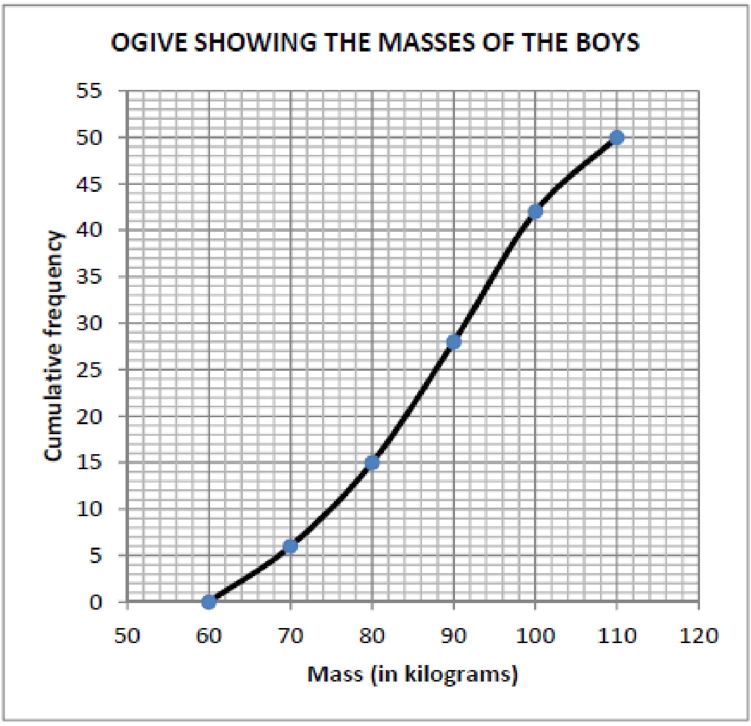
Upper quartile position: $120 \times \frac{3}{4} = 90$, upper quartile = 6km (read off the graph)

c) $5 \leq d < 6$

d) $\frac{50}{120} \times 100 = 41{,}7\%$

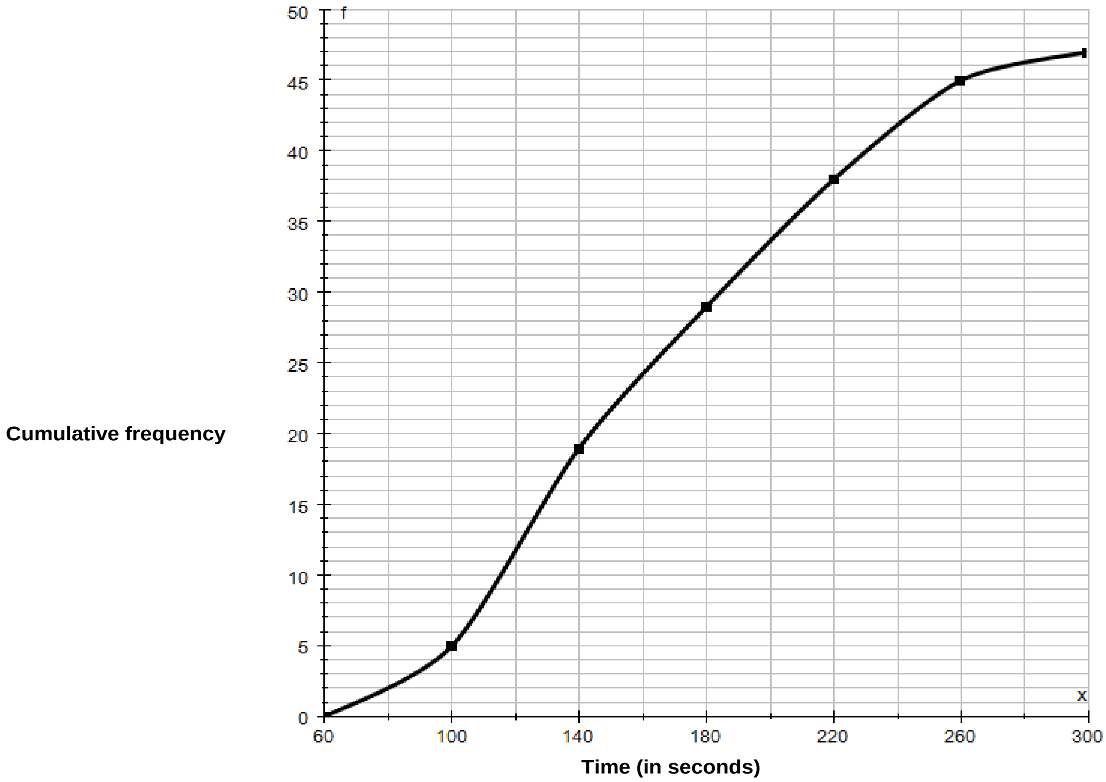
ACTIVITIES/ ASSESSMENT

3.8.1 The masses of a random sample of 50 boys in grade11 were recorded. This cumulative frequency graph (ogive) represents the recorded masses.



3.8.1.1 How many of the boys had a mass between 90 and 100 kilograms?
3.8.1.2 Estimate the median mass of the boys.
3.8.1.3 Estimate how many of boys had mass less than 80 kilograms?

3.8.2 The following cumulative frequency graph shows the times between planes landing at an airport.



3.8.2.1 Redraw and complete the following table

Times between planes (seconds)	Frequency	Cumulative frequency
$60 \leq t < 100$		5
$100 \leq t < 140$		
$140 \leq t < 180$		
$180 \leq t < 220$		
$220 \leq t < 260$		
$260 \leq t \leq 300$		

3.8.2.2 Determine the estimated median time between planes landing at the airport.
3.8.2.3 How many planes had a time between them of 220 seconds or more?

TOPIC: Statistics (lesson 9)	**Weighting**	20	Grade	11
RESOURCES				
Grade 11 November Past Papers				
ACTIVITIES/ASSESSMENT				

Class Test 2 Grade 11

Total Marks: 25 Duration: 22 Min

3.9.1 A student conducted a survey among his friends and relatives to determine the relationship between the age of a person and the number of marketing phone calls he or she received within one month. The information is given in the table below.

AGE OF PERSON IN SURVEY	FREQUENCY	CUMULATIVE FREQUENCY
$20 < x \leq 30$	7	7
$30 < x \leq 40$		27
$40 < x \leq 50$	25	
$50 < x \leq 60$		64
$60 < x \leq 70$		72
$70 < x \leq 80$	4	
$80 < x \leq 90$		80

3.9.1.1 Complete the frequency and cumulative frequency columns in the table given. (4)
3.9.1.2 How many people participated in this survey? (1)

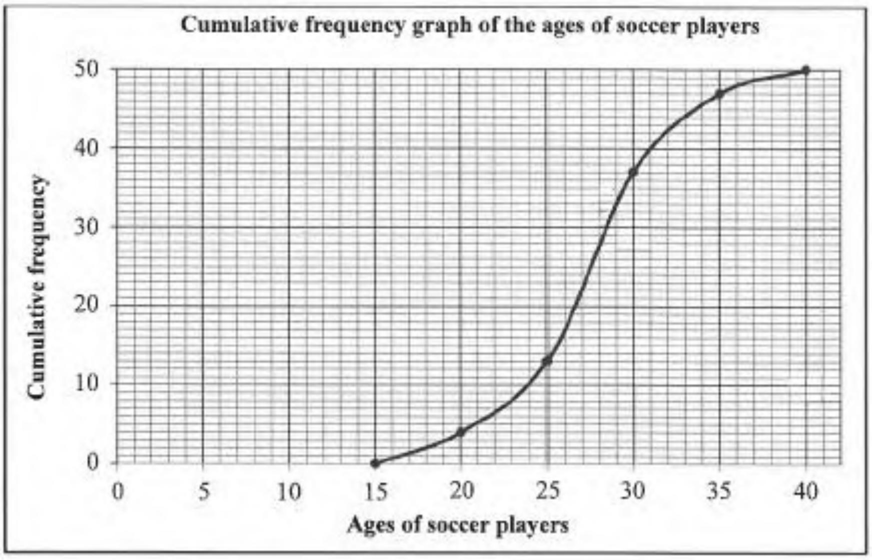
3.9.1.3 Write down the modal class. (1)

3.9.1.4 Draw an ogive (cumulative frequency graph) to represent the data. (3)

3.9.1.5 Determine the percentage of marketing calls received by people older than 54 years. (3)

[12]

3.9.2 A survey was conducted of the ages of players at a soccer tournament. The results are shown in the cumulative frequency graph(ogive) below.



3.9.2.1 How many players took part in the soccer tournament? (1)

3.9.2.2 Determine the number of players between the ages of 24 and 31 years old. (2)

3.9.2.3 Complete the frequency column of the table below. (3)

CLASS INTERVAL	FREQUENCY	CUMULATIVE FREQUENCY
$15 < x \leq 20$		4
$20 < x \leq 25$		13
$25 < x \leq 30$		37
$30 < x \leq 35$		47
$35 < x \leq 40$		50

3.9.2.4 Draw a frequency polygon for the data. (4)

[10]

TOPIC: Statistics (lesson 10)	**Weighting**	20	Grade	11

RESOURCES

- Textbooks: Platinum grade 11, Stat SA grades 10,11 and 12 and Mind action series
- Taken from textbook: Mind Action series, page 306
  Exercise 3, no. 3

Examples

Data set A: 182 ; 182 ; 184 ; 184 ; 185 ; 185 ; 186

Data set B: 152 ; 166 ; 176 ; 184 ; 194 ; 200 ; 216

Calculate the

a) Mean

b) Deviations from the mean i.e.
c) Squares of the deviations
d) Sum of the squares of the deviations i.e.
e) Variance
f) Standard deviation i.e.

Solution

a) Mean $\bar{x} = \frac{\sum 1288}{7}$
$= 184$

b) , (c) and (d)

Data set A	$(x-\bar{x})$	$(x-\bar{x})^2$
182	$182-184=-2$	$(-2)^2 = 4$
182	$182-184=-2$	$(-2)^2 = 4$
184	$184-184=0$	$(0)^2 = 0$
184	$184-184=0$	$(0)^2 = 0$
185	$185-184=1$	$(1)^2 = 1$
185	$185-184=1$	$(1)^2 = 1$
186	$186-184=2$	$(2)^2 = 4$
n = 7 items		d) $\sum(x-\bar{x})^2 = 14$

(e) Variance = $\frac{14}{7} = 2$

(f) Standard deviation = $\sqrt{2} = 1.414$

b), c) and d)

Data set B	$(x-\bar{x})$	$(x-\bar{x})^2$
152	$152-184=-32$	$(-32)^2 = 1024$
166	$166-184=-18$	$(-18)^2 = 324$
176	$176-184=-8$	$(-8)^2 = 64$
184	$184-184=0$	$(0)^2 = 0$
184	$184-184=0$	$(0)^2 = 0$
194	$194-184=10$	$(10)^2 = 100$
200	$200-184=16$	$(16)^2 = 256$
216	$216-184=32$	$(32)^2 = 1024$
$n=7$		$\sum(x-\bar{x})^2 = 2792$

e) variance = $\frac{\sum(x-\bar{x})}{n}$

$= \frac{2792}{7}$

$=$ 397,857….

f) standard deviation : $\sigma = \sqrt{variance}$

$= \sqrt{398{,}857\ \ldots\ldots}$

= 19,971

Comment

- The data set B has a higher standard deviation (σ) when compared to the data set A, this indicates that the data in data set B is far from the mean whereas the data set A is closer to the mean.

ACTIVITIES/ ASSESSMENT

3.10.1 The maximum daily temperature for Pretoria for the ten days in September are recorded in the following table:

18	17	24	28	27	20	23	25	22	25

3.10.1.1Calculate the standard deviation.
3.19.1.2Calculate the variance round off to one decimal place.
Show all your workings.

3.10.2 A teacher asked a group of learners how long (in minutes) it took them to complete their mathematics homework. They gave these answers:

12 19 ;33 ;40 ;24 ;25 ; 15 ;38

3.10.2.1 Determine the mean number of minutes taken by the learners to complete their homework.

3.10.2.2 Determine the variance and standard deviation correct to two decimal places by completing the table below using the formula:

$$\sqrt{\frac{\sum(x-\bar{x})^2}{n}}$$

Time taken in minutes	$(x-\bar{x})$	$(x-\bar{x})^2$
12		
15		
19		
24		
25		
33		
38		
40		

3.10.2.3 How many data values fall within one standard deviation of the mean ?

3.10.3 A learner does a survey on 23 cars travelling past him. He counts the number of passengers in each car. His results are recorded in the table.

Number of passengers	Frequency
1	2
2	9
3	5
4	4
5	3

Using your calculator, determine (correct to two decimal places):

3.10.3.1 the standard deviation
3.10.3.2 the variance
3.10.3.3 the mean

Textbook: Platinum grade 11, page 305
Exercise 2

TOPIC : Statistics (lesson 11)	**Weighting**	20 ± 3	Grade	11

RESOURCES

- Textbooks: Platinum grade 11, Stat SA grades 10,11 and 12 and Mind action series
- Calculator

NOTES

Examples

a) Data set A: 182 ; 182 ; 184 ; 184 ; 185 ; 185 ; 186

Calculate using a calculator the :

b) Standard deviation
c) Variance

Solution

- Show the learners how to set up the calculator to stat mode
- (write the steps)

a) mean : $\bar{x} = 184$

b) The data has already been entered in the calculator when calculating the mean. To get the standard deviation:

- Press AC to clear the screen of the calculator
- To get the answer for the standard deviation go to (write calculator steps)

Answer : standard deviation : $\sigma = 1,414$

Example (Platinum)
The table shows the number of questions answered correctly by a class of learners in a general knowledge test consisting of ten questions.

Number of questions answered correctly	Number of learners
0	3
1	2
2	1
3	5
4	7
5	6
6	2
7	4
8	5
9	2
10	1

Calculate, by using a calculator the:

a) Mean
b) standard deviation
c) variance

Solution

- Show the learners how to set up the calculator to stat mode
- (write the steps)

a) mean : $\bar{x} = 4{,}9$

b) The data has already been entered in the calculator when calculating the mean. To get the standard deviation:

- Press AC to clear the screen of the calculator
- To get the answer for the standard deviation go to (write calculator steps)

Answer : standard deviation : $\sigma = 2\,,63$

c) $\sigma = \sqrt{variance}$

$\sigma^2 = variance$
$(2{,}63)^2 = variance$
6 , 917 = variance

ACTIVITIES/ ASSESSMENT

3.11.1 A researcher recorded the growth in the number of bacteria over a period of 10 hours. The results are recorded below:

Time in hours	0	1	2	3	4	5	6	7	8	9	10	11
Number of bacteria	5	10	75	13	10	20	30	35	45	65	80	1

3.11.1.1 Calculate the mean, round off to one decimal place.
3.11.1.2 Determine the standard deviation.

Textbook: Mind Action Series

3.11.2

Marks	20	21	22	23	24	25	26	27	28	29
No. of learners	3	3	4	5	7	10	13	5	4	2

3.11.2.1 Calculate the standard deviation for this data by means of drawing a frequency table.
3.11.2.2 Then calculate the standard deviation by using a calculator.

3.11.3 Five data values are represented as follows

$$2x; x+1; x+2; x-3; 2x-2$$

3.11.3.1 Determine the value of x if the mean of the data set is 15.
3.11.3.2 Draw a box and whisker plot for the data values.
3.11.3.3 Calculate the inter-quartile range.
3.11.3.4 Calculate the standard deviation for this data, round off to one decimal place.
3.11.3.5 Calculate the variance rounded off to one decimal place.

TOPIC : Statistics (lesson 12)	**Weighting**	20	Grade	11

RESOURCES

Mind Action Series		Platinum	
PAGE	EX	PAGE	EX
306		226	1
307		229	2

Example

Is the following data set symmetric, skewed to right (Positively skewed) or skewed to left (Negatively skewed)? Motivate your answer.

27	28	30	32	34	38	41	42	43	44	46	53	56	62

Solution

Mean =41,1
Lower quartile =33
Median =41,5
Upper quartile =45

It is skewed to the left; mean is less than median. The median is closer to upper quartile than the lower quartile

ACTIVITIES/ ASSESSMENT

3.12.1The results of four learners in a series of formative tests each out of 10 marks are recorded in the following table:

A	1	1	1	2	6	6	7	8	8	8	10	10	10
B	1	2	6	8	8	8	8	8	8	10	10	10	-
C	1	1	2	2	4	4	6	6	8	8	10	-	-
D	2	2	2	4	4	6	6	8	8	10	10	10	-

3.12.1.1 Calculate the mean for each of the learners.
3.12.1.2 List the Five Number Summary for each learner.
3.12.1.3 Draw a Box and Whisker plot for each learner.
3.12.1.4 Discuss each learner's distribution of scores in terms of the spread about the median and mean.
3.12.1.5 Compare the performance results for each learner by using the information obtained above.

3.12.2 The Science marks (out of 40) of Mrs Basson's learners are recorded below:

30	24	21	18	31	28	21	20	18	27	19	23	21
17	25	22	19	27 35	18	22	27	30	20	27	21	23

3.12.2.1 Complete the table below.

Science Marks	Frequency
$12 \leq x < 16$	
$16 \leq x < 20$	
$20 \leq x < 24$	
$24 \leq x < 28$	
……….	
……….	

3.12.2.2 Draw a histogram and frequency polygon for the above data
3.12.2.3 Describe the shape of the frequency polygon and then predict the relationship between the mean and median.
3.12.2.4 Now calculate the mean and the median and determine whether your prediction is correct
3.12.2.5 Draw a box and whisker diagram for the data in order to verify how the data is positively skewed?

TOPIC: Statistics (lesson 13)	**Weighting**	20	Grade	11
METHODOLOGY				

Examples

1	8	12	14	14	15	17	17	19	26	32

1) Calculate:
   a) mean
   b) median
   c) interquartile range (IQR)

2) Are any of the entries in the data set outliers?
   Solutions

   1) (a) $\bar{x} = 15{,}91$ (b) median = 15

      c) Q1 = 12 and Q3 = 19

   IQR = Q3 – Q1
   = 19 - 12 =7

2) = Q1 - 1,5 x IQR and = Q3 + 1,5 x 7
= 12 - 1.5 x 7 = 19 + 1,5 x 7
= 1.5 = 29.5
Therefore the outliers are 1 and 32

ACTIVITIES/ ASSESSMENT

Textbooks: Stat SA

3.13.1. Determine the interquartile range and then find the outliers (if there are any) for the following set of data:

10,2 ; 14,1 ; 14,4 ; 14,4 ; 14,5 ; 14,5 ; 14,6;
14,7 ; 14,7 ; 14,9 ; 15,1 ; 15,9 ; 16,4 ; 18,9

3.13.2.A class of 20 learners has to submit Mathematics assessment tasks over the course of the year. While
some learners were conscientious others were not.

The following table shows the number of assessment tasks each learner handed in:

9	5	11	8	12	2	6	9	15	10
12	6	9	3	9	13	14	16	4	7

3.13.2.1 Determine the IQR
3.13.2.2 Determine the outliers (if any)

3.13.3 The following are the ages of boys in one of the Grade 8 class of Dendron Secondary School:

12 12 13 14 14 13 12 15 15 14 12 19 14 12 9

3.13.3.1 Determine the five-number summary.
3.13.3.2 Determine the outliers, if any.

TOPIC: Statistics (lesson 14)	**Weighting**	20	Grade	11

ACTIVITIES/ASSESSMENT

Class Test 3 Grade 11

Total Marks: 20 Duration: 25 Marks

3.14.1 The table below shows the number of cans of food collected by 9 classes during a charity drive.

5	8	15	20	25	27	31	36	75

3.14.1.1 Calculate the mean of the data (2)

3.14.1.2 Calculate the standard deviation of the data (2)

3.14.1.3 How many classes collected cans of food outside of one standard deviation of the Mean? (3)

3.14.1.4 Determine the median of the data. (1)

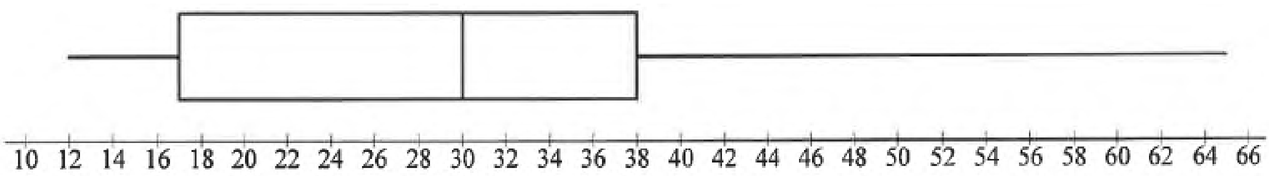
3.14.1.5 Determine the interquartile range of the data. (3)

3.14.1.6 Draw box and whisker diagram for the data above (3)

3.14.1.7 Describe the skewness of the data. (1)

3.14.1.8 Identify outliers, if any exist, for the above data

3.14.2 Mr Brown conducted a survey on the amount of airtime (in rands) EACH student had on his or her cellphone. He summarised the data in the box and whisker diagram below.



3.14.2.1 Write down the five-number summary of the data. (2)

3.14.2.2 Determine the interquartile range. (1)

3.14.2.3 Comment on the skewness of the data. (1)

TOPIC: PROBABILITY (LESSON 1)	**Weighting**	15	**Grade**	10
RESOURCES				
Transparent container, colored objects, coins, dice, etc.				

EXAMPLE 1

A coin is tossed once :

1.1 the sample space S = { H, T }

1.2 n (S) = 2

1.3 P(tail) = $\frac{1}{2} = 0{,}5$

EXAMPLE 2

A fair 6 - sided die is tossed once :

2.1 the sample space S = { 1, 2, 3, 4, 5, 6 }

2.2 n (S) = 6

2.3 P (6) = $\frac{1}{6}$

2.4 P (even number) = $\frac{3}{6} = \frac{1}{2}$

2.5 P (7) = $\frac{0}{6} = 0$

EXAMPLE 3

Research was conducted on a new flavour cold drink. The results are shown in the table :

Response	Like	Not like	Undecided	Total
Frequency	**160**	**70**	**20**	**(250)**

If a person is chosen at random find the probability that :

3.1 the person does not like the new flavour $\frac{70}{250}=\frac{7}{25}$

3.2 the person is undecided $\frac{20}{250}=\frac{2}{25}$

3.3 the person does not like the flavour or is undecided $\frac{70+20}{250}=\frac{90}{250}=\frac{9}{25}$

ACTIVITIES/ASSESSMENT

4.1.1 500 tickets were sold in a raffle. There is one prize. You buy 15 tickets.

What is the probability that you:

a) will win the prize?

b) will not win the prize?

4.1.2 Each letter of the word MATHEMATICS is placed on a card and then placed in a box. If a letter is drawn at random find the probability of:

a) drawing the letter E

b) drawing the letter M

c) drawing a vowel

d) not drawing a vowel

4.1.3 A bag contains 6 blue, 5 red and 7 green marbles. What is the probability of:

a) drawing a blue marble?

b) drawing no blue marbles?

c) drawing a blue marble or red?

d) drawing a pink marble?

4.1.4 A card is drawn from a pack of 52 playing cards. Find the probability of drawing:

a) a heart

b) a jack of clubs

c) an ace

d) a king or a queen

e) neither a heart nor a spade

f) not drawing the ace of spades

4.1.5 The probability of catching flu is 1 out of 1000.

a) Write the probability as a fraction and decimal and %:

b) In a town of 2583 000 people, how many people are likely to catch flu?

ACTIVITY 4.1.6

A traffic light shows green for 2 minutes, amber for 30 seconds and red for 1 minute. Calculate the probability that if you arrive at an intersection you will find the lights:

a) on red

b) on amber

c) on red or amber

d) not green

TOPIC: PROBABILITY (LESSON 2)	**Weighting**	±15	**Grade**	10
Term	3	**Week no.**		

RESOURCES

A4 or A3 white paper, ruler, pencil, scissors, markers, prestik, etc

NOTES

Examples:

VENN DIAGRAMS

SETS:

- the Sample Space S = all possible elements e.g. S = {a; b ; c …….. x; y ;z } = all letters of alphabet
- a sub-set of S is a set containing some of the sample space e.g. A = {a; b c; d; e}

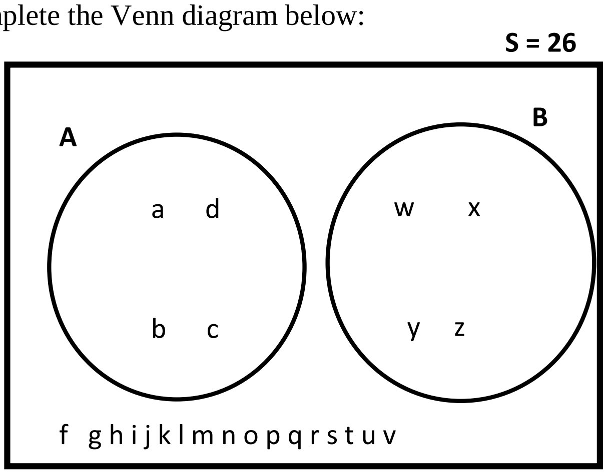
We use a Venn diagram to determine the probability for 2 or more events

MUTUALLY EXCLUSIVE EVENTS (DISJOINT SETS)

Example 1

S = {a; b ; c ………. x ; y ; z}   A = { a ; b ; c ; d ; e }   B = { w ; x ; y ; z}

Complete the Venn diagram below:



n (S) = 26

n (A) = 5

n (B) = 4

- A and B have no elements in common i.e. There is no intersection
- So the intersection A and B is the empty set : $A \cap B = \{ \quad \}$
- Therefore n (A and B) = $n(A \cap B) = 0$
- A or B $= A \cup B = \{ a, b, c, d, e, w, x, y, z\}$
- $n(\ A\ or\ B\ ) = n(A \cup B) = 9$
- $all\ elements\ in\ either\ A\ or\ B\ or\ both\ sets\ make\ up\ the\ union\ \ A\ \cup B$

If all letters are placed in a box and one letter is drawn randomly determine:

P(A) = $\frac{5}{9}$

P(B) = $\frac{4}{9}$

$P(\ A\ or\ B\ ) = P(\ A\ \cup\ B\ ) = \frac{9}{9} = 1$

$P(\ A\ and\ B\ ) = P(\ A\ \cap B\ ) = 0$

RULE for MUTUALLY EXCLUSIVE EVENTS : $\boldsymbol{P(\ A\ and\ B\ )} = 0$

$\boldsymbol{P(\ A\ or\ B\ )} = P(A) + P(B)$

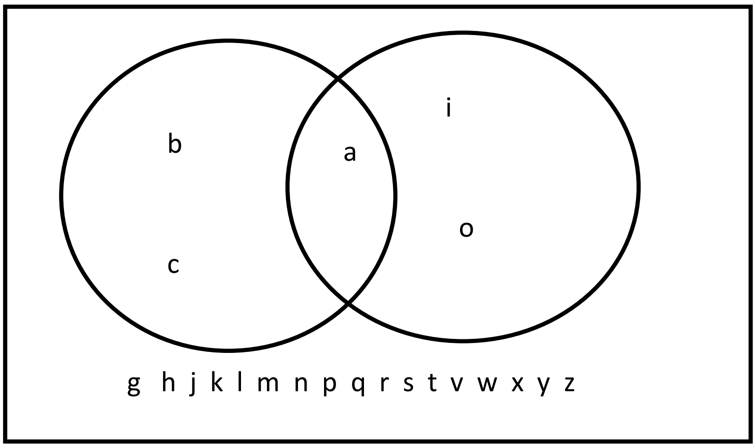
Example 2

NON-MUTUALLY EXCLUSIVE EVENTS (INCLUSIVE SETS)

S = {a ; b ; c ………. x ; y ; z} A = { a ; b ; c ; d ; e ; f } C = { a ; e ; i ; o ; u }

Complete the Venn diagram below:

S=26



n (S) = 26

n (A) = 6

n (C) = 5

- A and C have elements in common i.e. There is an intersection.
- the intersection A and C is $A \cap C = \{ a; e \}$
- Therefore n (A and C) = $n(A \cap C) = 2$
- A or C $= A \cup C = \{a; b; c; d; e; f; i; o; u\}$
- $n(A \text{ } or \text{ } C) = n(A \cup C) = 9$

If all letters are placed in a box and one letter is drawn randomly determine:

P(A) = $\frac{6}{26} = \frac{3}{13}$

P(C) = $\frac{5}{26}$

$P(A \text{ } and \text{ } C) = P(A \cap C) = \frac{2}{26} = \frac{1}{13}$

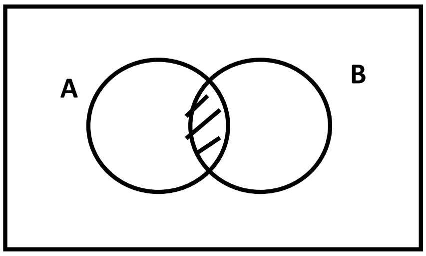
$P(A \text{ } or \text{ } C) = P(A \cup C) = \frac{9}{26}$

RULE for NON-MUTUALLY EXCLUSIVE EVENTS : $\boldsymbol{P(A \text{ } or \text{ } C) = P(A) + P(C) - P(A \text{ } and C)}$

Make sure you understand the difference between "and" and "or" in probability:

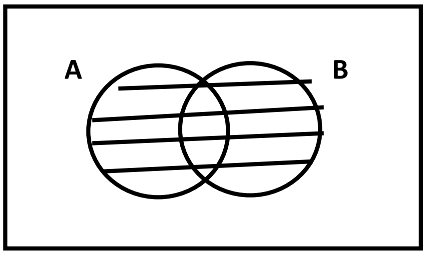
- AND = all the elements that belong to two sets A and B simultaneously (as shaded below) is called the INTERSECTION and is written as $A \cap B$:

$A \cap B$ (A intersection B):



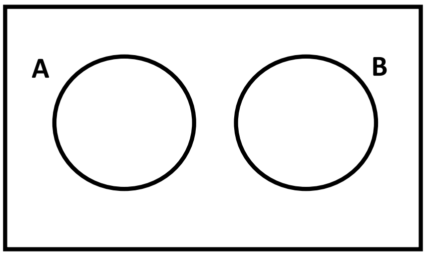
- OR = all the elements that belong to set A or B or both (as shaded below) is called the UNION and is written $A \cup B$:

  $A \cup B$ (A union B):



SUMMARY OF THE RULES:

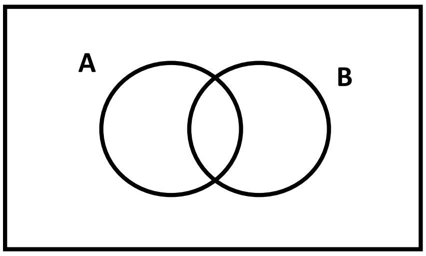
1. For mutually exclusive (disjoint events)**:**



$$\boldsymbol{P(A\ and\ B\ ) = P(A \cap B) = 0}$$

$$\boldsymbol{P(A\ or\ B) = P(A \cup B) = P(A) + P(B)}$$

For non-mutually exclusive**:**



$$\boldsymbol{P(A\ and\ B\ ) = P(A \cap B)\ \neq 0}$$

$$\boldsymbol{P(A\ or\ B) = P(A \cup B) = P(A) + P(B) -}$$

$\boldsymbol{P(AandB)}$

Note: You can even use the rule $\boldsymbol{P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)}$ for mutually exclusive events and make $\boldsymbol{P(A\ and\ B) = 0}$

EXHAUSTIVE EVENTS

Two events A and B are exhaustive if together they contain all the elements of the sample space

ie. if $P(A\ or\ B) = 1$

eg. S = { 1 ; 2 ; 3; ; 4 ; 5 ; 6}

A = { 1 ; 3 ; 5 } B = { 2 ; 4 ; 6 } C = { 2 }

A or B = { 1; 2; 3; 4; 5; 6 } $\therefore$ P(A or B) = $\frac{3+3}{6} = \frac{6}{6} = 1$

$\therefore$ A and B are exhaustive

A or C = {1; 2; 3; 5} $\therefore$ P(A or C) = $\frac{4}{6} = \frac{2}{3} \neq 1$

$\therefore$ A and C are NOT exhaustive events

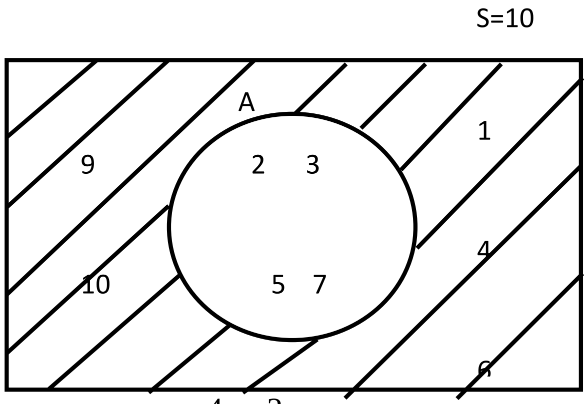
COMPLEMENTARY EVENTS

Mutually exclusive, exhaustive events are called complementary.

The complement of an event A is all the elements that **do not** appear in set A.

For example : S = { 1 ; 2 ; 3 ; 4 ; 5 ; 6 ; 7 ; 8 ; 9 ; 10 } and A = { 2 ; 3 ; 5 ; 7 }

Then the complement of A = A' = {1; 4; 6; 8; 9; 10} (as shaded below)



$n(S) = 10$

$n(A) = 4$

$n(A') = 6$

$P(A) = \frac{n(A)}{n(S)} = \frac{4}{10} = \frac{2}{5}$

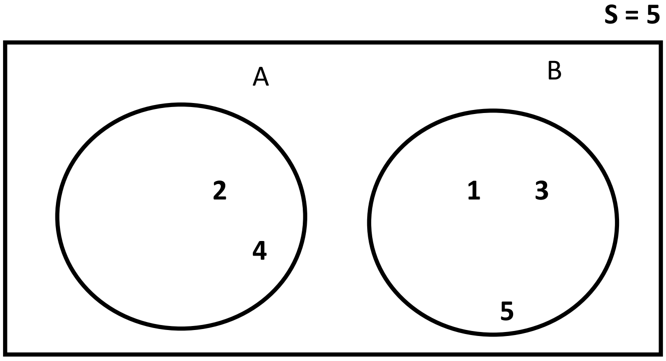
$P(A') = \frac{n(A')}{n(S)} = \frac{6}{10} = \frac{3}{5}$

RULE FOR COMPLEMENTARY EVENTS**:** $P(A) + P(A') = 1$

or $P(A') = 1 - P(A)$

Another example of COMPLEMENTARY EVENTS:

A and B are mutually exclusive



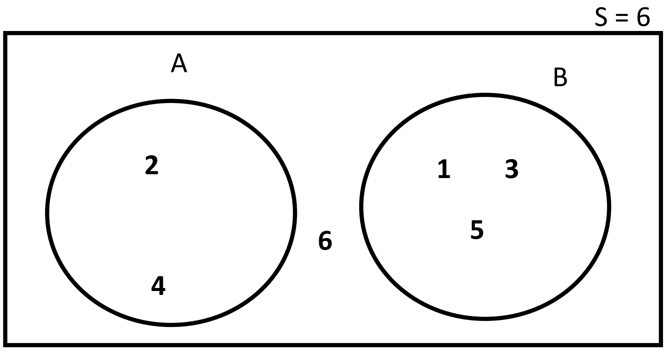
$P(A) = \frac{2}{5}$

$P(B) = \frac{3}{5}$

$\therefore P(A) + P(B) = \frac{2}{5} + \frac{3}{5} = 1$

Note : $A' = B$. However the events below are not COMPLEMENTARY EVENTS**:**

A and B are mutually exclusive:



$P(A) = \frac{2}{6} = \frac{1}{3}$

$P(B) = \frac{3}{6} = \frac{1}{2}$

$\therefore P(A) + P(B) = \frac{2}{6} + \frac{3}{6} = \frac{5}{6} \neq 1$

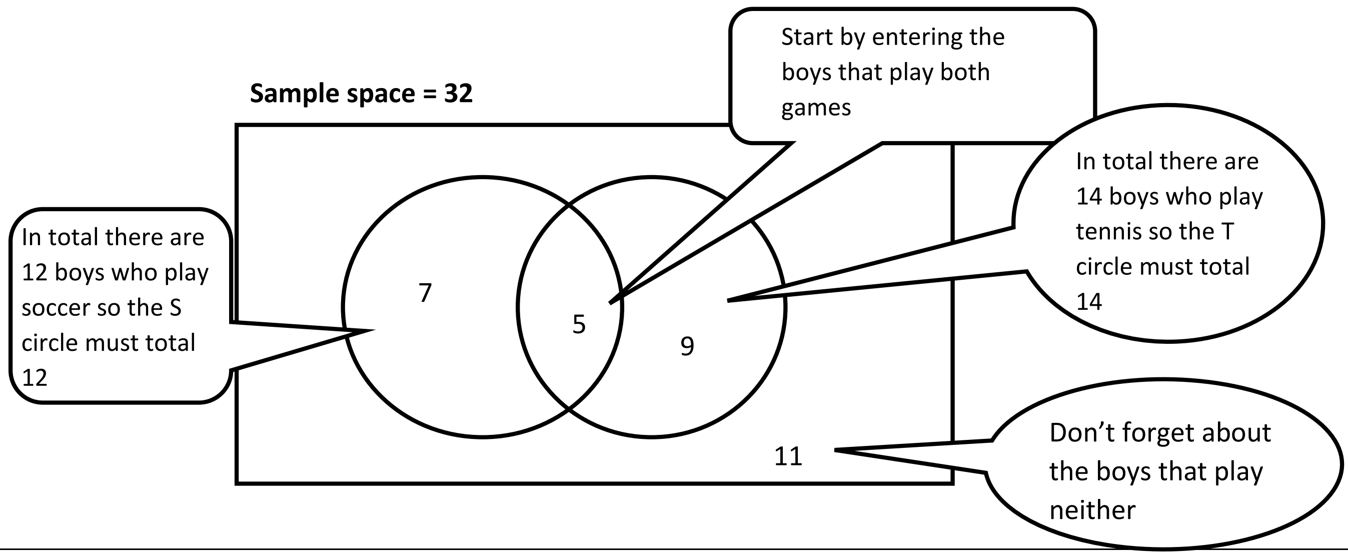
Note : $A' \neq B$

<u>Note</u> : All complementary events are mutually exclusive but not all mutually exclusive events are complementary!

Example 3

There are 32 boys in gr 10. 12 boys play soccer (S), 14 play tennis (T) and 5 play soccer and tennis.

Represent this information in a Venn diagram:



3.1 **How many** boys play neither soccer nor tennis? Don't forget to represent these boys on the diagram too. $=11$

3.2 What is the **probability** that a randomly chosen boy plays tennis and soccer? $= \frac{5}{32}$

3.3 What is the **probability** that a randomly chosen boy plays tennis or soccer? $\frac{7+5+9}{32} = \frac{21}{32}$

3.4 What is the **probability** that a randomly chosen boy plays neither tennis nor soccer? $= \frac{11}{32}$

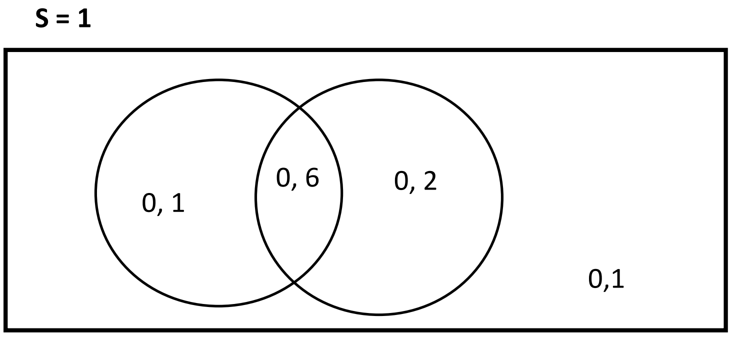
3.5 What is the **probability** that a randomly chosen boy plays tennis only? $= \frac{9}{32}$

3.6 What is the probability that a boy will play only one of these sports? $= \frac{7+9}{32} = \frac{16}{32} = \frac{1}{2}$

Example 4

. The probability that Joe will see a movie is 0,7. The probability that he will go to a restaurant is 0,8. The probability of him doing both is 0,6.

4.1 Complete the Venn diagram:



4.2 Find the probability that he doesn't go to a movie or restaurant. $= 0{,}1$

4.3 Find the probability that he only goes to a restaurant. $= 0{,}2$

4.4 Find the probability that he doesn't go to a movie. $P(notM) = 1 - P(M) = 1 - 0{,}7 = 0{,}3$

Example 5

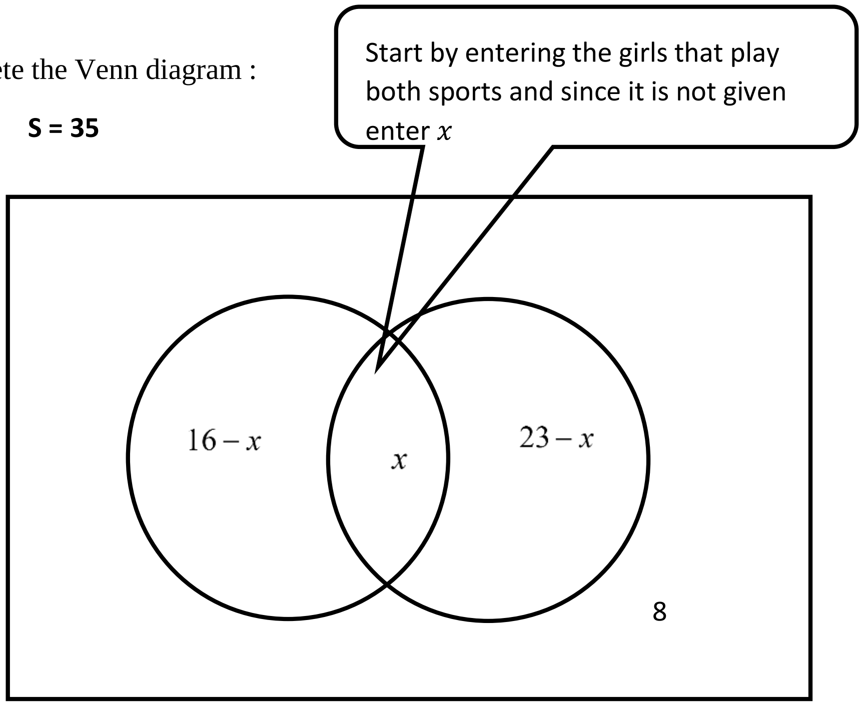
INTRODUCING A VARIABLE :

In a grade 10 class of 35 girls, 16 play hockey and 23 play netball. 8 play neither.

Let the number of girls that play both equal x

5.1 Complete the Venn diagram :

S = 35



5.2 Use the above information to calculate the value of x.

$$16-x+x+23-x+8=35$$
$$47-x=35$$
$$-x=-12$$
$$x=12$$

5.3 Determine the probability that a randomly chosen girl :

5.3.1 only plays netball $=\frac{11}{35}$

5.3.2 plays hockey only $=\frac{4}{35}$

5.3.3 plays netball or hockey $=\frac{27}{35}$

5.3.4 plays netball and hockey $=\frac{12}{35}$

5.3.5 plays none of these sports $=\frac{8}{35}$

5.3.6 plays at least one of these sports $=\frac{27}{35}$

ACTIVITIES /ASSESSMENT

ACTIVITY 4.2.1

4.2.1.1 If $P(A)=\frac{1}{2}$, $P(B)=\frac{4}{10}$ and $P(A\ and\ B)=\frac{1}{5}$

a) Are A and B mutually exclusive?

b) Determine $P(A\ or\ B)$

4.2.1.2 If $P(A) = 0{,}4$, $P(B) = 0{,}5$ and $P(A\ or\ B) = 0{,}7$ determine $P(A\ and\ B)$

ACTIVITY 4.2.2

a) If $P(B) = \frac{5}{9}$ $then\ P(not\ B) = P(B') =$

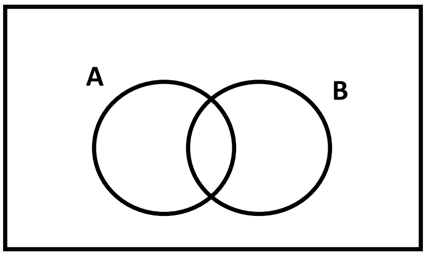
b) Determine the probability of not throwing a six on a die.

c) Determine the probability of not drawing an ace from a pack of cards.

ACTIVITY 4.2.3

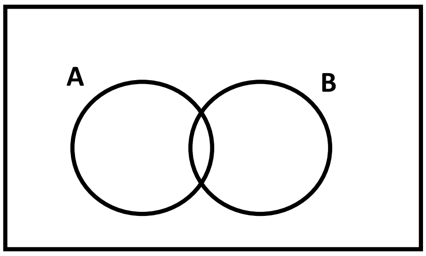
SHADING REGIONS: Shade the region described in the following Venn diagrams

A **:**

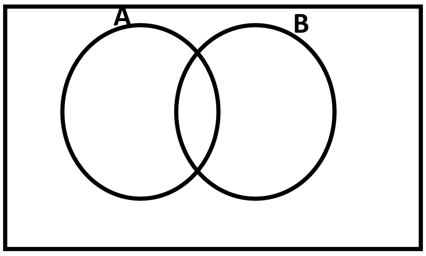


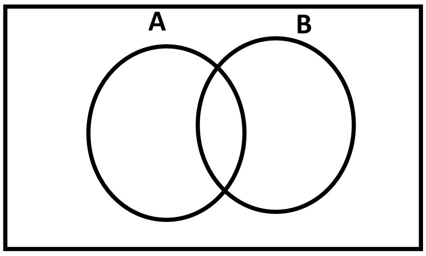
$\boldsymbol{A \cup B}$ **(A union B) :**

B **:**

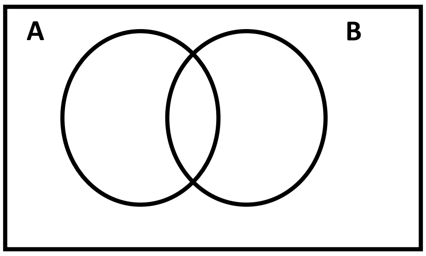


$\boldsymbol{A \cap B}$ **(A intersection B):**

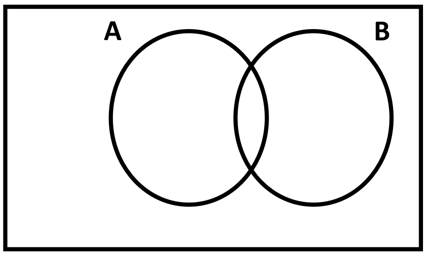




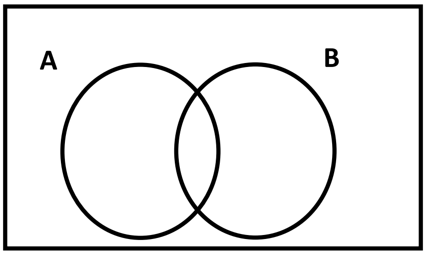
A' (complement **of A):**



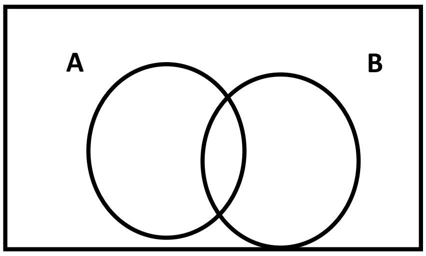
$\boldsymbol{A\ only}$ $(\boldsymbol{A \cap B'})$:



$\boldsymbol{B\ only}$:



Neither A nor B $(\boldsymbol{A \cup B})'$ $\boldsymbol{or\ A' \cap B'}$:



ACTIVITY 4.2.4

S = { 1 ; 2 ; 3 ; 4 ; 5 ; 6 ; 7 ; 8 ; 9 ; 10 } A = { 1 ; 2 ; 3 ; 4 ; 5 ; 6 } B = { 1 ; 3 ; 7 ; 8 }

a) Draw the Venn diagram to represent the sets :

S =10

b) List $A \cap B = \{$ $\}$

c) List $A \cup B = \{$ $\}$

d) List A'

e) Determine n (A ∩ B)

f) Determine n (A ∪ B)

g) If a number from the sample space is randomly chosen find the following:

1. P(A)
2. P(B)
3. $P(A \cap B)$
4. $P(A \cup B)$
5. $P(A) + P(B) - P(A \cap B)$
6. $P(A')$
7. $P(A) + P(A')$

ACTIVITY 4.2.5

Cards numbered from 1 to 12 are placed in a box.

Set A is the event of drawing a card with a number less than 5

Set B is the event of drawing a card with a number greater than 7

a) List S

b) List A

c) List B

d) Draw the Venn diagram:

S =12

e) Determine $A \cap B$

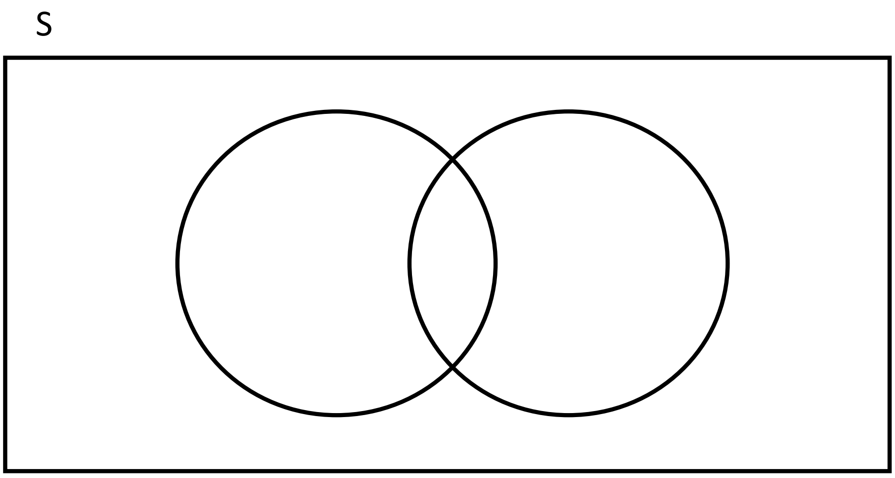
f) Determine $P(A \cap B)$

g) Are the events A and B mutually exclusive or not? Explain your answer.

h) Are the events A and B complementary? Substantiate your answer using the rules.

ACTIVITY 4.2.6

The manager of a repair shop noticed that a car will require a tune-up (T) with a probability of $\frac{6}{10}$. The probability of repairing the brakes (B) is $\frac{2}{10}$ and neither is $\frac{30}{100}$.



Find the following:

a) P(T and B)

b) P(T or B)

c) P (T but not B)

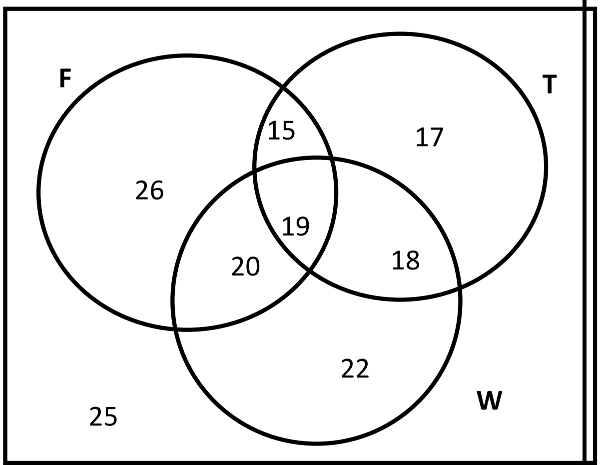
d) P(T′)

e) P(B only)

TOPIC : PROBABILITY(LESSON 3)	**Weighting**	20	**Grade**	11
RESOURCES				
A4 or A3 white paper, ruler, pencil, scissors, markers, prestik, etc				
NOTES				

Example 1

1. The Venn diagram below shows the number of learners who use Facebook (F), twitter (T) and Whatsapp (W):



1.1 How many learners were surveyed?

i.e. Sample Space =162

Find the probability that a learner chosen at random :

1.2 uses Facebook $= \frac{80}{162} = \frac{40}{81}$

1.3 uses all 3 social media networks $= \frac{19}{162}$

1.4 uses Facebook or twitter or WhatsApp $=\frac{137}{162}$

1.5 uses Facebook and Twitter $=\frac{34}{162}=\frac{17}{81}$

1.6 uses none of the social media networks $=\frac{25}{162}$

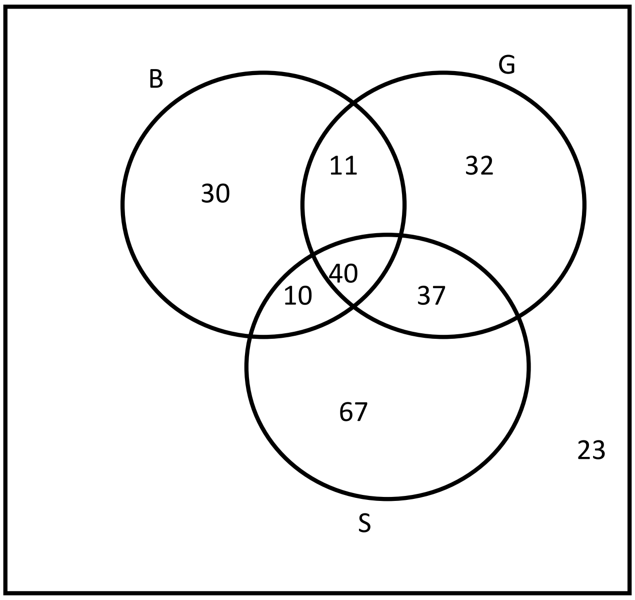
1.7 uses Facebook only $=\frac{26}{162}=\frac{13}{81}$

1.8 uses Facebook and twitter but not WhatsApp $=\frac{15}{162}=\frac{5}{54}$

Example 2

Of 250 learners at a school, all but 23 wrote one or more of Biology, Geography and Science.

- 91 wrote Biology
- 120 wrote Geography
- 154 wrote Science
- 51 wrote Biology and geography
- 77 wrote Geography and Science
- 67 wrote Science but not Geography or Biology
- 40 wrote all three subjects



2.1 Complete the Venn Diagram (Refer to the diagram)

2.2 Determine the probability that a leaner does geography only. $=\frac{32}{250}=\frac{16}{125}$

2.3 Determine the probability that a leaner does Biology and Science and not Geography. $=\frac{10}{250}=\frac{1}{25}$

2.4 Determine the probability that a leaner does not do Geography or Science. $=\frac{53}{250}$

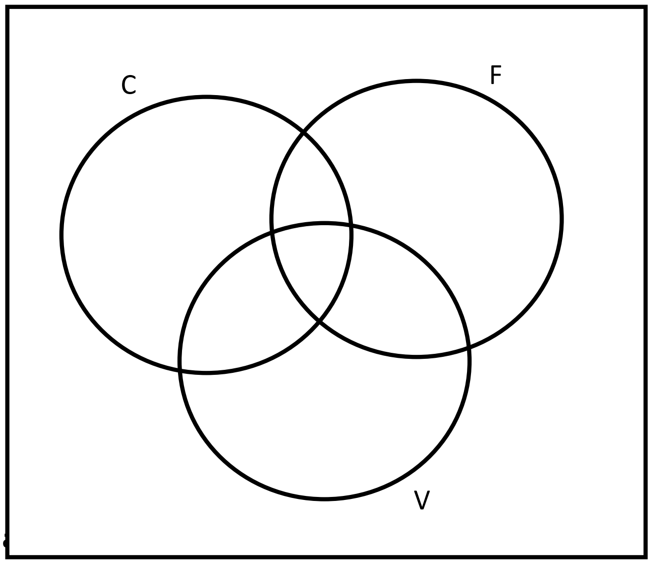
ACTIVITIES /ASSESSMENT

ACTIVITY 4.3.1

A school arranged a camp for 103 grade 11 learners. The learners were asked to indicate their food preferences. They could choose from chicken, vegetables and fish.

The following information was collected:

- 2 learners do not eat chicken, fish or vegetables
- 5 learners eat only vegetables
- 2 learners only eat chicken
- 21 learners do not eat fish
- 3 learners eat only fish
- 66 learners eat chicken and fish
- 75 learners eat vegetables and fish



Let the number of learners who eat chicken , vegetables

a) Complete the Venn diagram to represent the information

b) Calculate x

c) Determine the probability that a learner chosen at random

1. eats chicken and fish, and no vegetables

2. eats any **two** of the given food choices

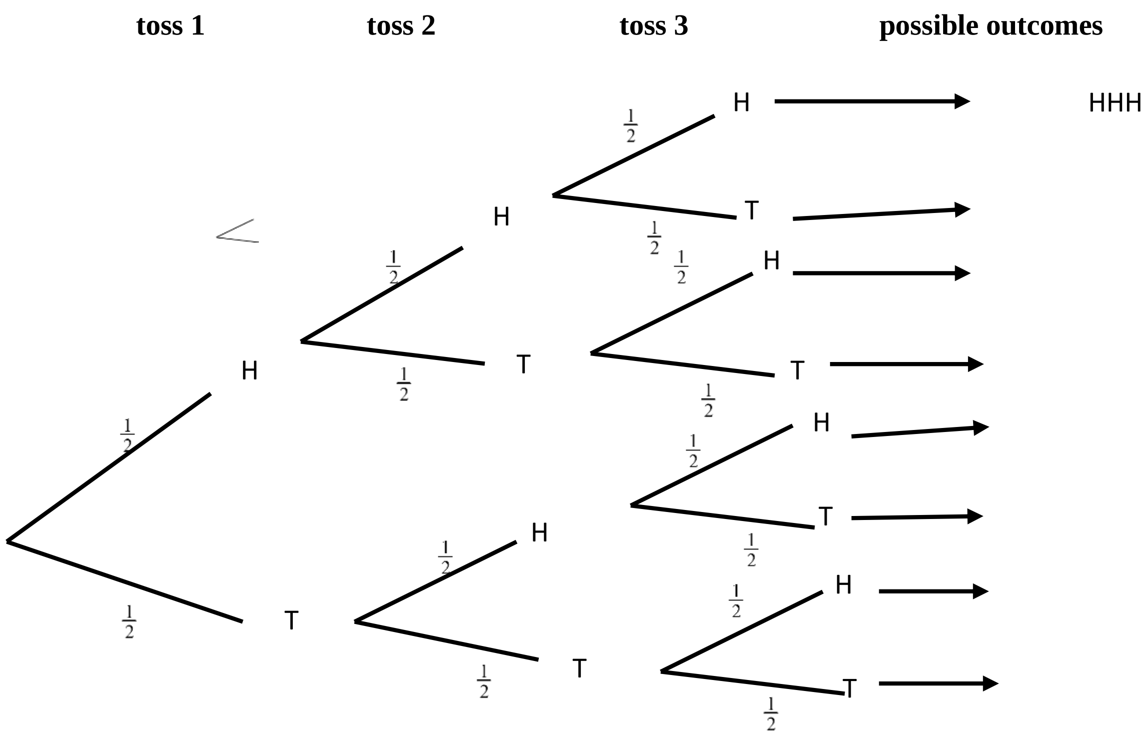
3. eats at least two of the food choices

TOPIC : PROBABILITY (LESSON 4)	**Weighting**	20	**Grade**	**11**
RESOURCES				
Pack of cards, dice, colored discs, box of smarties etc.				
NOTES				
USING TREE DIAGRAMS FOR SUCCESSIVE EVENTS A tree diagram is a graphical method for obtaining all possible outcomes (the sample space) of an experiment **when repetition** of the experiment occurs.				

Example 1.

A fair coin is tossed **THREE TIMES or 3 coins are tossed SIMULTANEOUSLY (together)**

All outcomes can be shown in a tree diagram as follows:



1.1 The Sample Space of all possible outcomes for this event is = 8

1.2 Determine the probability of tossing 3 heads = P (H , H , H) = $\frac{1}{8}$

1.3 Determine the probability of not getting 3 heads = $\frac{7}{8}$

1.4 Determine the probability of tossing 2 heads then a tail = $\frac{1}{8}$

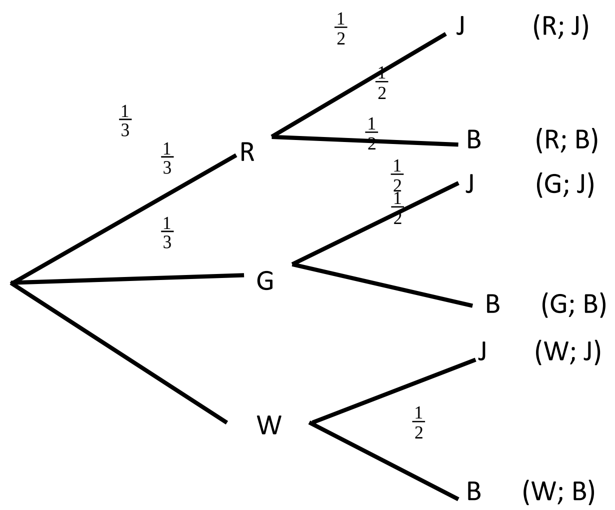
1.5 Determine the probability of tossing 2 heads and a tail = $\frac{3}{8}$

1.6 Find the probability of getting **at least** one head = $\frac{7}{8}$

Example 2.

Gavin has 3 T – shirts (red, green and white) and 2 pairs of pants (jeans and baggies).

2.1 Draw a tree diagram to show all possible combinations



2.2 What is the probability of him wearing a black T – shirt **and** jeans? = 0

2.3 What is the probability of him wearing a red T – shirt? $= \frac{1}{3}$

2.4 What is the probability of him wearing a red T-shirt **or** jeans? $\frac{4}{6} = \frac{2}{3}$

Example 3

RULE FOR INDEPENDENT EVENTS (product rule) :

Two successive events A and B are independent if the outcomes of the first event do not influence the outcomes of the second event.

RULE : If the events A and B are independent then P(A and B) = P (A) $\times$ P (B)

If you toss 3 coins then P (H , H , H) = P (H and H and H) = $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

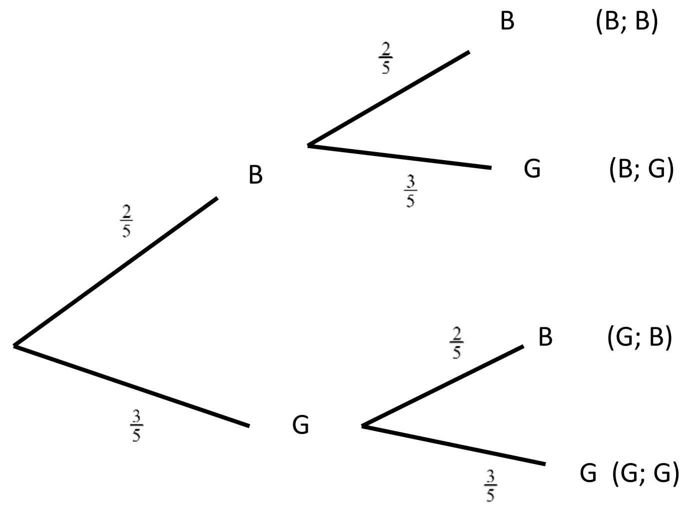
You select a card from a pack and **replace** it and select another card

then $P(king\ and\ queen) = \frac{4}{52} \times \frac{4}{52} = \frac{16}{2704}$

Example 3(Continuation)

A box contains 2 blue and 3 green smarties. A smartie is withdrawn and then **replaced** and another is chosen.

3.1 Draw a tree diagram to show all possible outcomes



3.1 Determine the probability of drawing 2 blue smarties. $P(B)\times P(B)=\frac{2}{5}\times\frac{2}{5}=\frac{4}{25}$

3.2 Determine the probability of not drawing 2 blue smarties. $P(B')=1-P(B)=1-\frac{4}{25}=\frac{21}{25}$

3.3 Determine the probability of drawing a blue then a green smartie. $P(B)\times P(G)=\frac{2}{5}\times\frac{3}{5}=\frac{6}{25}$

3.4 Determine the probability of drawing a blue then a green smartie or green then blue.

$$=\left(\frac{2}{5}\times\frac{3}{5}\right)+\left(\frac{3}{5}\times\frac{2}{5}\right)=\frac{12}{25}$$

3.5 Determine the probability of not drawing a blue smartie on the first **and** second draw.

$$P(not(BandB))=1-P(BandB)=1-\frac{4}{25}=\frac{21}{25}$$

3.6 Determine the probability of not drawing a blue smartie on the first **or** second draw $=\frac{3}{5}\times\frac{3}{5}=\frac{9}{25}$

Note : Not all probability questions can be answered using the product rule for independent events – sometimes you have to draw a tree diagram (e.g. Q3.5 and 3.6)

DEPENDENT EVENTS

Two successive events A and B are dependent if the outcomes of the first event influence the outcomes of the second event.

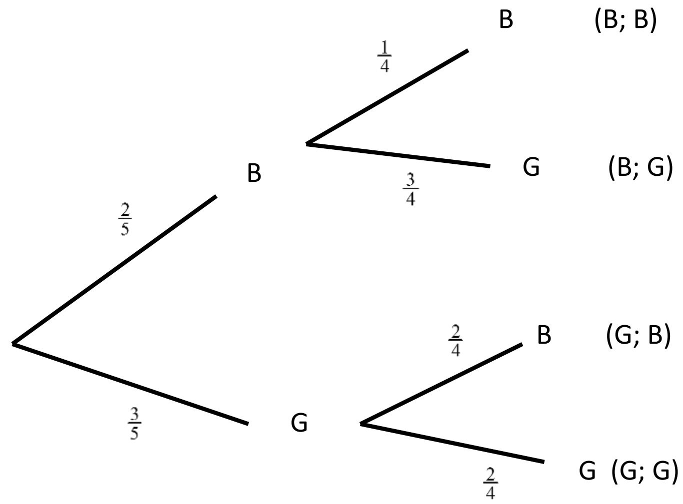
For example :

- selecting a card from a pack and not replacing it and selecting another card.
- Drawing 2 cards simultaneously

Example 4

A box contains 2 blue and 3 green smarties. A smartie is withdrawn and **not replaced (eaten)** and another is chosen or two smarties are **taken out simultaneously.**

4.1 Draw a tree diagram to show all possible outcomes



4.2 Determine the probability of drawing 2 blue smarties. $= \frac{2}{5} \times \frac{1}{4} = \frac{1}{10}$

4.3 Determine the probability of not drawing 2 blue smarties $= \frac{9}{10}$

4.4 Determine the probability of drawing a blue then a green smartie $= \frac{3}{10}$

4.5 Determine the probability of drawing a blue and a green smartie $= \left(\frac{2}{5} \times \frac{3}{4}\right) + \left(\frac{3}{5} \times \frac{2}{4}\right) = \frac{3}{5}$

Note 1 : For dependent events : $P(A \text{ and } B) \neq P(A) \times p(B)$

Note 2 : If events are mutually exclusive or non-mutually exclusive, you cannot assume they are independent.

Note 3: Sometimes independence is implied in the question 1 (when an item is not replaced or when an item cannot be 'used' again).

ACTIVITIES /ASSESSMENT

ACTIVITY 4.4.1

Consider 3 successive soccer matches. What is the probability that the captain will win the coin toss: (hint : draw a tree diagram to answer b and c)

a) every time?

b) only once?

c) at least once?

ACTIVITY 4.4.2

a) If you roll a die three times then the probability of getting three sixes is $P(6{,}6{,}6) =$

b) Find the probability of not getting three sixes:

c) Find the probability of getting at least one six:

ACTIVITY 4.4.3

Bag A contains 3 red and 2 white discs. Bag B contains 5 red and 4 white discs.

One bag is chosen at random **and then** a disc is chosen.

a) Draw a tree diagram to show all possible outcomes:

b) Determine the probability of getting a red disc.

c) Determine the probability of getting a white disc.

ACTIVITY 4.4.4

a) A and B are independent events. $P(A) = 0{,}2$ and $P(B) = 0{,}3$.

Find $P(A\ and\ B)$

b) A and B are independent events. $P(A) = 0{,}5$ and $P(A\ and\ B) = 0{,}2$. Find $P(B)$

c) $P(A) = 0{,}3$ and $P(B) = 0{,}4$ and $P(A \cap B) = 0{,}12$

Are A and B independent events? Substantiate your answer.

ACTIVITY 4.4.5

At a factory 2 smoke detectors work independently.

$P(smoke\ detector\ A\ finds\ smoke) = 0{,}6$ $P(smoke\ detector\ B\ finds\ smoke) = 0{,}7$

a) Find the probability that smoke is detected by both A and B:

ie. $P(A\ and\ B) =$

b) Find the probability that smoke is detected:

ACTIVITY 4.4.6

In a maths quiz two teams A and B work independently on a problem. They are allowed a maximum of 10 minutes to solve the problem. The probability that each team will solve the problem is $\frac{1}{2}$ and $\frac{1}{3}$ respectively. Calculate the probability that the problem will be solved in the 10 minutes allowed.

ACTIVITY 4.4.7

In a class of 20 boys and 15 girls, 3 learners are chosen to make a speech.

a) Draw a tree diagram to find all possible outcomes:

b) Find:

1) the probability of 3 boys being chosen

2) the probability of 3 girls being chosen

3) the probability of 2 boys and a girl being chosen
4) the probability of 3 girls not being chosen
5) the probability that at least one learner chosen is a boy

ACTIVITY 4.4.8

A raffle is held. 1000 tickets are sold. Michael buys 5 tickets. There are 2 prizes.

a) Find the probability of him winning both prizes
b) Find the probability of him winning 1 prize
c) Find the probability of him winning no prizes

(hint: draw a tree diagram)

ACTIVITY 4.4.9

A pack of 52 cards is shuffled. Two cards are drawn from the pack simultaneously. (the first is not replaced)

Determine the probability that you will draw:

a) an ace on the first draw
b) two aces
c) an ace on the second draw if the first card is not an ace
d) an ace on the second draw
e) an ace then a king

ACTIVITY 4.4.10

a) Weather experts state that the probability that it will rain tomorrow if it is raining today is 0,7 and the probability that it will rain tomorrow if it is fine today is 0,4. If it is raining on Saturday, what is the probability that it will rain on Monday? (hint: draw a tree diagram)

b) The probability that it will be sunny tomorrow is $\frac{1}{3}$. If it is sunny the probability that Peter will cycle is $\frac{4}{5}$. If it is not sunny, the probability that he will cycle is $\frac{2}{5}$. Determine the probability that Peter will cycle tomorrow. (hint: draw a tree diagram)

TOPIC : PROBABILITY (LESSON 5)	**Weighting**	20	**Grade**	**11**
RESOURCES				
Maths made easy, Textbook.				
NOTES				
Example 1 The sports director at a school analyzed data to determine how many learners play sport and what the gender of each learner is. The data is presented in the table below.				

	DO NOT PLAY SPORT	PLAY SPORT	TOTAL
Male	51	69	120
Female	49	67	116
Total	100	136	236

1.1 Determine the probability that a learner, selected at random, is:

1.1.1 Male.

$= \frac{120}{236}$

$= \frac{30}{59}$

1.1.2 Female and plays sport.

$= \frac{67}{236}$

1.2 If a person is selected at random and they are males, determine the probability that the person plays sport.

$= \frac{30}{59}$

1.3 Are the events 'male' and 'do not play sport' mutually exclusive? Use the values in the table to justify your answer.

- No. From the table, P (male and do not plays sport) = $\frac{51}{236}$, which is greater zero. Since the probability of the intersection of these two events is greater than zero, these events are not mutually exclusive.

1.4 Are the events 'male' and 'do not play sport' independent? Show ALL calculations to support your answer.

- P(male) = $\frac{120}{236}$
- P(NS) = $\frac{100}{236}$
- P(male) x P(NS) = $\frac{120}{236} \times \frac{100}{236}$

$= \frac{750}{3481}$

$= 0,22$

P(male and NS) = $\frac{51}{236}$

$= 0,22$

So, P(male) x P(NS) = P(male and NS)

ACTIVITIES /ASSESSMENT

ACTIVITY 5.5.1 The hair colour of 30 learners was recorded. The results of the survey were as follows:

	Boys	Girls	TOTAL
Black	6	2	8
Brown	8	4	12
Blonde	4	6	10
TOTAL	18	12	30

Calculate the probability that a learner chosen at random:

a) will have black hair.

b) will be a boy.

c) will have brown hair given that a girl is chosen.

d) will be a girl given that the hair colour is blonde.

ACTIVITY 5.5.2 A school investigated 100 learners to see who arrived on time, late or were absent during a particular week. The results were recorded in the table below:

	Boys	Girls	TOTAL
On time	40	25	a
Late	18	b	25
Absent	c	3	10
TOTAL	65	35	d

a) Determine the values of a , b , c and d.

b) Determine the probability of a learner chosen at random arriving late for school

c) Determine the probability of a learner being absent if the learner is a boy.

d) Determine the probability of a learner arriving late or being absent.

e) Is being on time independent of gender? Remember the Test for Independence: Events A and B are independent if: ______________________________

ACTIVITY 5.5.3 The sports director at a school analysed data to determine how many learners play sport and what gender of each learner is. The data is represented in the table below:

	Do not play sport	Play sport	TOTAL
Male	51	69	120
Female	49	67	116
TOTAL	100	136	236

Determine the probability that a learner, selected at random, is:

a) male

b) female and plays sport

c) Are the events 'male' and 'do not play sport' **mutually exclusive?**

Use the values in the table to justify your answer.

d) Are the events ' male ' and ' do not play sport' **independent ?**

Show all calculations to support your answer.

ACTIVITY 5.5.4

The table below shows the results of a survey:

	Male	**Female**	**TOTAL**
HIV +	61	49	a
HIV -	b	31	c
TOTAL	100	d	e

a) Find a , b , c , d and e.

b) Is a person's gender independent of a person's HIV status?

Show all calculations to support your answer.