

# basic education

Department:  
Basic Education  
REPUBLIC OF SOUTH AFRICA

**NATIONAL  
SENIOR CERTIFICATE/  
NASIONALE  
SENIOR SERTIFFIKAAT**

**GRADE/GRAAD 12**

**MATHEMATICS P2/WISKUNDE P2  
NOVEMBER 2015  
MEMORANDUM**

**MARKS: 150  
PUNTE: 150**

*This memorandum consists of 27 pages./  
Hierdie memorandum bestaan uit 27 bladsye.*

**NOTE:**

- If a candidate answers a question TWICE, mark only the FIRST attempt.
- If a candidate crossed out an attempt of a question and did not redo the question, mark the crossed-out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.
- Penalty of only 1 mark for incorrect rounding throughout the paper (Q1.2.1)

**LET WEL:**

- Indien 'n kandidaat 'n vraag TWEE KEER beantwoord, sien slegs die EERSTE poging na.
- Indien 'n kandidaat 'n antwoord doodgetrek het en nie oorgegaan het nie, sien die doodgetrekte poging na.
- Volgehoue akkuraatheid word in ALLE aspekte van die memorandum toegepas. Hou op nastien by die tweede berekeningsfout.
- Om antwoorde/waardes om 'n probleem op te los, te veronderstel, word NIE toegelaat NIE.

QUESTION/RKAG1

Fat/Fet (in g)	9	14	25	8	12	31	28	14	29	20
Energy/Energie (in kJ)	1 100	1 300	2 100	300	1 200	2 400	2 200	1 400	2 600	1 600

1.1

**Scatter plot/Spreidiagram**

1.1.1

no marks:  
0 – 2 points correctly

✓✓ plotting  
3 – 5 points correctly

✓✓✓ plotting  
6 – 9 points correctly

✓✓✓ plotting  
all 10 points correctly

geen punte:  
0 – 2 punte korrek

✓ stip 3 – 5  
pte korrek

✓✓ stip 6 – 9  
pte korrek

✓✓✓ stip al  
10 pte korrek (3)

1.2.2

$\frac{1.2.2}{\checkmark y - \text{int}}$   
close to  
(0 ; 150)  
 $\checkmark$  one pt  
close to  
(25 ; 2100)  
or  
(20 ; 1700)  
(2)

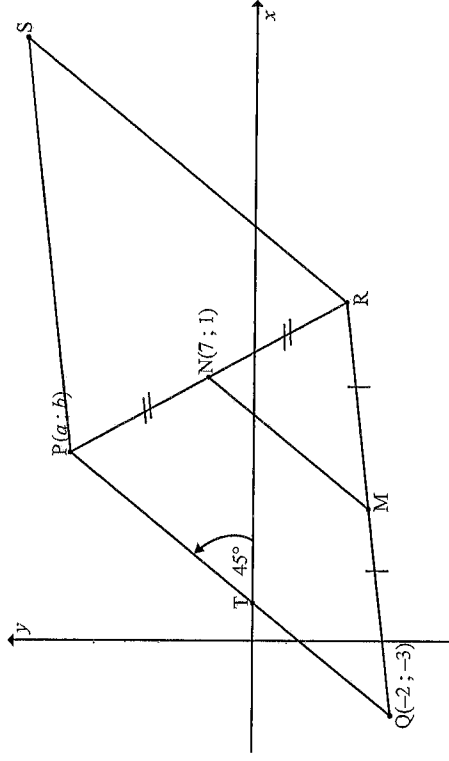
1.2.1	$\hat{y} = 154,60 + 77,13(18)$ $= 1\,542,94 \approx 1\,500$ kJ	✓ subst ✓ answ rounded off correctly/ antw korrek afgerond (2)
1.3	(8 ; 300)	✓ answ/antw (1)
1.4	$r = 0,9520 \dots \approx 0,95$	✓✓ answ/antw (2)
1.5	very strong positive relationship/ bate sterk positiewe verband	✓ strong/ sterk (1) [11]

QUESTION/VRAAG 2

Sum of the values on uppermost faces/ Som van die waardes op boonste vlakke	Frequency/ Frekwensie
2	0
3	3
4	2
5	4
6	4
7	8
8	3
9	2
10	2
11	1
12	1

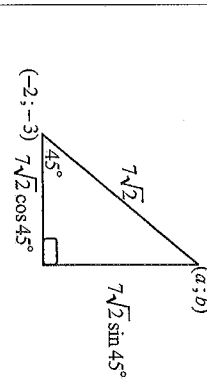
2.1	$\text{mean/gemiddelde} = \frac{2(0) + 3(3) + 4(2) + \dots + 12(1)}{30} = \frac{202}{30}$ $= 6,73$	✓ 202 ✓ answ/antw (2)
2.2	$\text{median/median} = \frac{T_{15} + T_{16}}{2} = \frac{7+7}{2} = 7$	✓ ✓ answ/antw (2)
2.3	$SD/SA = 2,264\dots \approx 2,26$	✓ ✓ answ/antw (2)
2.4	$(6,73 - 2,26; 6,73 + 2,26)$ $= (4,47; 8,99)$ <p>∴ 4 + 4 + 8 + 3 = 19 times/keer</p>	✓ lower boundary ✓ upper boundary ✓ answ/antw (3)

QUESTION/VRAAG 3

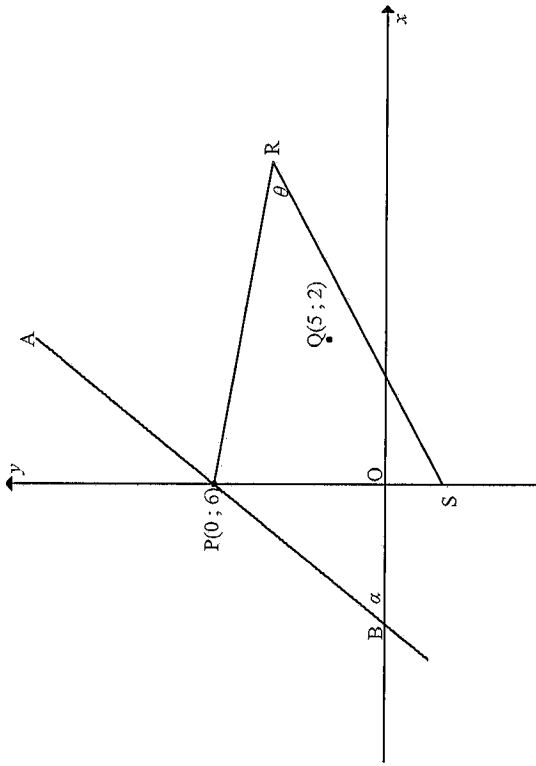


3.1	$m_{PQ} = \tan 45^\circ = 1$	✓ $m = \tan 45^\circ$ ✓ answ/antw (2)
3.2	$MN \parallel PQ$ $\therefore m_{MN} = 1$ $\therefore y - y_1 = m(x - x_1)$ $\therefore y - 1 = 1(x - 7)$ $\therefore y = x - 6$ <p><b>OR/OF</b></p> $MN \perp PQ$ $\therefore m_{MN} = 1$ $\therefore y = mx + c$ $\therefore 1 = 1(7) + c$ $-6 = c$ $\therefore y = x - 6$	✓ S OR R ✓ $m_{MN}$ ✓ subst $m$ and/en $N(7; 1)$ ✓ equation/vgl (4)
3.3	$MN = \frac{1}{2} PQ$ $\therefore MN = \frac{7\sqrt{2}}{2} \approx 4,95$	✓ S OR R ✓ $m_{MN}$ ✓ subst $m$ and/en $N(7; 1)$ ✓ equation/vgl (4)

3.5	<p>QN = NS [diag of  im/hoek  van  im]</p> $\frac{-2+x_5}{2} = 7 \quad \text{and/en} \quad \frac{-3+y_5}{2} = 1$ <p><math>\therefore x_5 = 16</math> and/en <math>\therefore y_5 = 5</math></p> <p><b>OR/OF</b>                  QN = NS [diag of  im/hoek  van  im]  <math>\therefore</math> by inspection/deur inspeksie: S(16; 5)</p>	<p>✓ method/metode                  ✓ x-value/waarde                  ✓ y-value/waarde (3)                  ✓ method/metode                  ✓ x-value/waarde                  ✓ y-value/waarde (3)</p>
3.6	<p>Equation of Vgl van PQ: <math>y = x + c</math>  <math>-3 = -2 + c</math>  <math>\therefore a = b + 1</math> .....(1)</p> <p>From distance formula/Van afstandformule:  <math>PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}</math>  <math>7\sqrt{2} = \sqrt{(a - (-2))^2 + (b - (-3))^2}</math>  <math>\therefore 98 = (a + 2)^2 + (b + 3)^2</math> .....(2)</p> <p>Subst (1) into (2):  <math>98 = (b + 1 + 2)^2 + (b + 3)^2</math>  <math>98 = b^2 + 6b + 9 + b^2 + 6b + 9</math>  <math>0 = 2b^2 + 12b - 80</math>  <math>0 = b^2 + 6b - 40</math>  <math>\therefore 0 = (b + 10)(b - 4)</math>  <math>\therefore b = 4</math> (since <math>b &gt; 0</math>)                  Subst <math>b = 4</math> into (1):  <math>\therefore a = 4 + 1 = 5</math>  <math>\therefore P(5; 4)</math></p> <p><b>OR/OF</b>                  Equation of Vgl van PQ: <math>y = x + c</math>  <math>-3 = -2 + c</math>  <math>y = x - 1</math>  <math>\therefore a = b + 1</math> .....(1)</p> <p>From distance formula/Van afstandformule:  <math>7\sqrt{2} = \sqrt{(a - (-2))^2 + (b - (-3))^2}</math>  <math>\therefore 98 = (a + 2)^2 + (b + 3)^2</math> .....(2)</p> <p>Subst (1) into (2):  <math>98 = (b + 1 + 2)^2 + (b + 3)^2</math>  <math>98 = 2(b + 3)^2</math>  <math>49 = (b + 3)^2</math>  <math>\pm 7 = b + 3</math>  <math>\pm 7 - 3 = b</math>  <math>\therefore b = 4</math> (since <math>b &gt; 0</math>)                  Subst <math>b = 4</math> into (1):  <math>\therefore a = 4 + 1 = 5</math>  <math>\therefore P(5; 4)</math></p>	<p>✓ eq of vgl van PQ                  ✓ subst Q &amp; <math>7\sqrt{2}</math> into/in distance formula/ afstandformule                  ✓ subst eq of vgl v. PQ                  ✓ st form/st vorm                  ✓ value of/waarde van <math>b</math>                  ✓ value of/waarde van <math>a</math> (6)</p> <p>✓ eq of vgl van PQ                  ✓ subst Q &amp; <math>7\sqrt{2}</math> into/in distance formula/ afstandformule                  ✓ subst eq of vgl v. PQ                  ✓ simplification/ vereenvoudig                  ✓ value of/waarde van <math>b</math>                  ✓ value of/waarde van <math>a</math> (6)</p>

<p><b>OR/OF</b>                  Equation of Vgl van PQ: <math>y = x + c</math>  <math>-3 = -2 + c</math>  <math>y = x - 1</math>  <math>\therefore a = b + 1</math> .....(1)</p> <p>From distance formula/Van afstandformule:  <math>7\sqrt{2} = \sqrt{(a - (-2))^2 + (b - (-3))^2}</math>  <math>98 = (a + 2)^2 + (a - 1 + 3)^2</math>  <math>= 2(a + 2)^2</math>  <math>\therefore a + 2 = 7</math> (since <math>a</math> angestien <math>a &gt; 0</math>)  <math>\therefore a = 5</math>                  Subst <math>a = 4</math> into (1):  <math>\therefore b = 5 - 1 = 4</math>  <math>\therefore P(5; 4)</math></p> <p><b>OR/OF</b></p>  <p><math>a = -2 + 7\sqrt{2} \cos 45^\circ = 5</math>  <math>b = -3 + 7\sqrt{2} \sin 45^\circ = 4</math></p>	<p>✓ eq of vgl van PQ                  ✓ subst Q &amp; <math>7\sqrt{2}</math> into/in distance formula/ afstandformule                  ✓ subst eq of vgl v. PQ                  ✓ simplification/ vereenvoudig                  ✓ value of/waarde van <math>a</math>                  ✓ value of/waarde van <math>b</math> (6)</p> <p>✓ ✓ ✓ ✓ ✓                  (6)                  [17]</p>
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QUESTION/VRAG 4



<p>4.1</p> $(x-5)^2 + (y-2)^2 = r^2$ $(0-5)^2 + (6-2)^2 = r^2$ $25 + 16 = r^2$ $41 = r^2$ $\therefore (x-5)^2 + (y-2)^2 = 41$ <p><b>OR/OF</b></p> $PQ = \sqrt{(0-5)^2 + (6-2)^2}$ $= \sqrt{25+16}$ $r = \sqrt{41}$ $\therefore (x-5)^2 + (y-2)^2 = 41$	<p>✓ subst (5 ; 2) into circle eq/in sirkel/vgl</p> <p>✓ value of/waarde van <math>r^2</math></p> <p>✓ equation/vgl (3)</p> <p>✓ subst (5 ; 2) &amp; (0 ; 6) into dist. form/in afst. form</p> <p>✓ value of/waarde van <math>r</math></p> <p>✓ equation/vgl (3)</p> <p>✓ <math>x = 0</math></p>
<p>4.2</p> $(0-5)^2 + (y-2)^2 = 41$ $25 + (y-2)^2 = 41$ $25 + y^2 - 4y + 4 = 41$ $y^2 - 4y - 12 = 0$ $(y-6)(y+2) = 0$ <p><math>y \neq 6</math> or/of <math>y = -2</math></p> <p><math>\therefore S(0 ; -2)</math> or <math>y = -2</math></p>	<p>✓ st. form/st. vorm</p> <p>✓ answ/antw (neg value) (3)</p>

**OR/OF**

$$(0-5)^2 + (y-2)^2 = 41$$

$$25 + (y-2)^2 = 41$$

$$(y-2)^2 = 16$$

$$y-2 = \pm 4$$

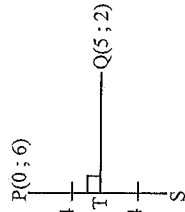
$$y = 2 \pm 4$$

$y \neq 6$  or/of  $y = -2$

$\therefore S(0 ; -2)$

**OR/OF**

Draw/Trek  $QT \perp PS$   
 PT = TS [line from centre  $\perp$  to chord/lyn van midpt.  $\perp$  koord]  
 $PT = y_P - y_Q = 6 - 2 = 4$   
 $y_Q - y_S = 4$   
 $y_S = 2 - 4 = -2$   
 $\therefore S(0 ; -2)$



4.3

$$m_{PQ} = \frac{6-2}{0-5} = -\frac{4}{5}$$

$m_{PQ} \times m_{AB} = -1$  [tan/raakl  $\perp$  radius]

$$\therefore m_{AB} = \frac{5}{4}$$

$$\therefore y = \frac{5}{4}x + 6$$

4.4

$$\tan \alpha = \frac{5}{4}$$

$$\therefore \alpha = 51,34^\circ$$

**OR/OF**

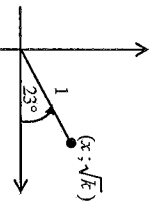
$B(4,8 ; 0)$

$$\therefore \tan \alpha = \frac{6}{4,8}$$

$$\therefore \alpha = 51,34^\circ$$

<p>4.5</p> <p><math>\theta = \text{B3S}</math>  <math>= 90^\circ - \alpha</math>  <math>= 90^\circ - 51,34^\circ</math>  <math>= 38,66^\circ</math></p> <p><b>OR/OF</b></p> <p><math>\text{PS} = 8</math>  <math>\text{PQ} = \text{SQ} = \sqrt{41}</math>  <math>\text{PS}^2 = \text{PQ}^2 + \text{SQ}^2 - 2 \cdot \text{PQ} \cdot \text{SQ} \cdot \cos \hat{\text{PQS}}</math>  <math>64 = 41 + 41 - 2 \cdot 41 \cdot \cos \hat{\text{PQS}}</math>  <math>\cos \hat{\text{PQS}} = \frac{18}{82}</math>  <math>\hat{\text{PQS}} = 77,32^\circ</math></p> <p><math>\theta = \frac{1}{2} \hat{\text{PQS}}</math>  <math>= 38,66^\circ</math></p> <p>[<math>\angle</math> at centre = <math>2 \times \angle</math> circum]</p>	<p>[tan-chord th/ruakel-koordst.]          [<math>\angle</math> sum in <math>\Delta \angle</math> som van <math>\Delta</math>]</p> <p>✓ S ✓ R          ✓ <math>90^\circ - \alpha</math>          ✓ answ/antw          (4)</p> <p>✓ correct subst into cosine rule          ✓ <math>\hat{\text{PQS}} = 77,32^\circ</math>          ✓ R          ✓ answ/antw          (4)</p>
<p>4.6</p> <p>Area <math>\Delta \text{PQS} = \frac{1}{2} \text{PS} \times \text{height/hoopte}</math>  <math>= \frac{1}{2} (8)(5)</math>  <math>= 20 \text{ sq units/vk eenh}</math></p> <p><b>OR/OF</b></p> <p><math>\hat{\text{PQS}} = 2 \times 38,66^\circ</math> [<math>\angle</math> at centre = <math>2 \times \angle</math> at circum/          midpts <math>\angle = 2 \text{ omtreks} \angle</math>]  <math>= 77,32^\circ</math></p> <p>Area <math>\Delta \text{PQS} = \frac{1}{2} \text{PQ} \cdot \text{QS} \cdot \sin \hat{\text{PQS}}</math>  <math>= \frac{1}{2} \cdot \sqrt{41} \cdot \sqrt{41} \cdot \sin 77,32^\circ</math>  <math>= 20 \text{ sq units/vk eenh}</math></p>	<p>✓ area formula/e: <math>\Delta \text{PQS}</math>          ✓ <math>\text{PS} = 8</math>          ✓ <math>\text{lh} = 5</math>          ✓ answ/antw          (4)</p> <p>✓ size of <i>grootte</i> v <math>\hat{\text{PQS}}</math>          ✓ area rule/reël:  <math>\Delta \text{PQS}</math>          ✓ subst correctly/          subst <i>korrek</i>          ✓ answ/antw          (4)</p> <p>[20]</p>

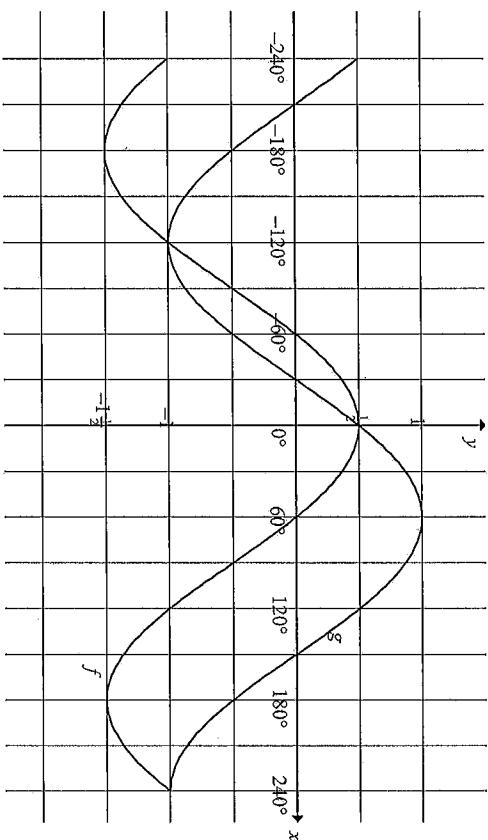
QUESTION/VRAGS

<p>5.1.1</p> <p><math>\sin 203^\circ</math>  <math>= -\sin 23^\circ</math>  <math>= -\sqrt{k}</math></p>	<p>✓ reduction/          reduksie          ✓ answ/antw          hv k          (2)</p>
<p>5.1.2</p> <p><math>\cos^2 23^\circ = 1 - \sin^2 23^\circ</math>  <math>= 1 - k</math>  <math>\cos 23^\circ = \sqrt{1-k}</math></p> <p><b>OR/OF</b></p> <p><math>x^2 + (\sqrt{k})^2 = 1</math>  <math>x^2 = 1 - k</math>  <math>x = \sqrt{1-k}</math>  <math>\cos 23^\circ = \frac{\sqrt{1-k}}{1} = \sqrt{1-k}</math></p> 	<p>✓ identity/identiteit          ✓ <math>\cos^2 23^\circ</math> into/inv k          ✓ answ/antw          (3)</p> <p>✓ <math>x^2 = 1 - k</math>          ✓ x into/inv k          ✓ answ/antw          (3)</p>
<p>5.1.3</p> <p><math>\tan (-23^\circ) = -\tan 23^\circ</math>  <math>= \frac{-\sin 23^\circ}{\cos 23^\circ}</math>  <math>= \frac{-\sqrt{k}}{\sqrt{1-k}} = -\sqrt{\frac{k}{1-k}}</math></p> <p><b>OR/OF</b></p> <p><math>\tan (-23^\circ) = -\tan 23^\circ</math>  <math>= \frac{-\sqrt{k}}{\sqrt{1-k}} = -\sqrt{\frac{k}{1-k}}</math></p>	<p>✓ reduction/          reduksie          ✓ answ/antw          hv k          (2)</p> <p>✓ reduction/          reduksie          ✓ answ/antw          hv k          (2)</p>
<p>5.2</p> <p><math>4 \cos x (-\sin x)</math>  <math>\sin(30^\circ - x + x)</math>  <math>= -4 \sin x \cos x</math>  <math>= \sin 30^\circ</math>  <math>= -4 \sin x \cos x</math>  <math>= \frac{1}{2}</math>  <math>= -8 \sin x \cos x</math>  <math>= -4(2 \sin x \cos x)</math>  <math>= -4 \sin 2x</math></p>	<p>✓ <math>\cos x</math> ✓ <math>-\sin x</math>          ✓ <math>\sin(\alpha + \beta)</math>          ✓ <math>\frac{1}{2}</math>          ✓ double sine form / <i>dubbel sin form</i>          ✓ answ/antw          (6)</p>

<p><b>OR/OF</b></p> $\frac{4 \cos x (-\sin x)}{(\sin 30^\circ \cos x - \cos 30^\circ \sin x) \cos x + (\cos 30^\circ \cos x + \sin 30^\circ \sin x) \sin x}$ $= \frac{-4 \sin x \cos x}{\left(\frac{1}{2} \cos x - \frac{\sqrt{3}}{2} \sin x\right) \cos x + \left(\frac{\sqrt{3}}{2} \cos x + \frac{1}{2} \sin x\right) \sin x}$ $= \frac{-2(2 \sin x \cos x)}{-2(2 \sin x \cos x) \frac{1}{2} + \frac{1}{2} \sin^2 x - \frac{1}{2} \sin^2 x}$ $= \frac{1}{2} (\cos^2 x + \sin^2 x)$ $= \frac{-2(2 \sin x \cos x)}{1}$ $= -8 \cos x \sin x$ $= -4(2 \sin x \cos x)$ $= -4 \sin 2x$	<p>✓ <math>\cos x</math> ✓ <math>-\sin x</math></p> <p>✓ <math>\frac{1}{2} \cos^2 x + \frac{1}{2} \sin^2 x</math></p> <p>✓ <math>\frac{1}{2}</math></p> <p>✓ double sine form / <i>dubbel sin form</i></p> <p>✓ answ/antw (6)</p>
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<p>5.3</p> $\cos 2x - 7 \cos x - 3 = 0$ $2 \cos^2 x - 1 - 7 \cos x - 3 = 0$ $2 \cos^2 x - 7 \cos x - 4 = 0$ $(2 \cos x + 1)(\cos x - 4) = 0$ <p>∴ <math>\cos x = -\frac{1}{2}</math> or/of <math>\cos x = 4</math> (no solution)</p> <p>∴ <math>x = 120^\circ + n.360^\circ</math> or/of <math>x = 240^\circ + n.360^\circ</math>; <math>n \in \mathbb{Z}</math></p> <p><b>OR/OF</b></p> <p>∴ <math>x = \pm 120^\circ + n.360^\circ</math>; <math>n \in \mathbb{Z}</math></p>	<p>✓ expansion/uitbreiding</p> <p>✓ <math>2 \cos^2 x - 7 \cos x - 4 = 0</math></p> <p>✓ factors/faktore</p> <p>✓ <math>\cos x = -\frac{1}{2}</math></p> <p>✓ <math>120^\circ</math> &amp; <math>240^\circ</math></p> <p>✓ <math>+n.360^\circ</math></p> <p><b>OR/OF</b></p> <p>✓ <math>\pm 120^\circ</math></p> <p>✓ <math>+n.360^\circ</math></p> <p>(6)</p>
<p>5.4</p> $\sin 3\theta = \sin(2\theta + \theta)$ $= \sin 2\theta \cos \theta + \cos 2\theta \sin \theta$ $= 2 \sin \theta \cos \theta \cos \theta + (1 - 2 \sin^2 \theta) \sin \theta$ $= 2 \sin \theta (1 - \sin^2 \theta) + \sin \theta - 2 \sin^3 \theta$ $= 3 \sin \theta - 4 \sin^3 \theta$ $= 3\left(\frac{1}{3}\right) - 4\left(\frac{1}{3}\right)^3$ $= 1 - \frac{4}{27}$ $= \frac{23}{27}$	<p>✓ expansion of/uitbreiding van <math>\sin(2\theta + \theta)</math></p> <p>✓ expansions of <math>\sin 2\theta</math> AND <math>\cos 2\theta</math></p> <p>✓ <math>1 - \sin^2 \theta</math></p> <p>✓ subst</p> <p>✓ answ/antw (5)</p> <p>[24]</p>

QUESTION/VRAAG 6

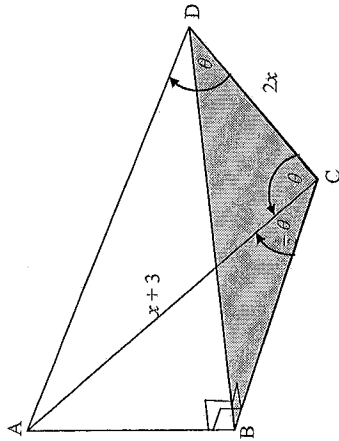


<p>6.1</p> <p><math>f(x) = \cos x - \frac{1}{2}</math> and/en <math>g(x) = \sin(x + 30^\circ)</math></p> <p><math>\therefore p = 30^\circ</math> and/en <math>q = -\frac{1}{2}</math></p> <p><b>OR/OF</b></p> <p><math>\sin(60^\circ + p) = 1</math> and/en <math>\cos 0^\circ + q = \frac{1}{2}</math></p> <p><math>\therefore p = 30^\circ</math> <span style="margin-left: 100px;"><math>\therefore q = -\frac{1}{2}</math></span></p>	<p><math>\checkmark f(x) = \cos x - \frac{1}{2}</math></p> <p><math>\checkmark g(x) = \sin(x + 30^\circ)</math></p> <p><math>\checkmark</math> value of/waarde v p</p> <p><math>\checkmark</math> value of/waarde v q</p> <p>(4)</p>
<p>6.2</p> <p><math>x \in (-120^\circ; 0^\circ)</math> <b>OR/OF</b> <math>-120^\circ &lt; x &lt; 0^\circ</math></p>	<p><math>\checkmark</math> critical values/ kritiese waardes</p> <p><math>\checkmark</math> correct interval/ korrekte interval</p> <p>(2)</p>

<p>6.3</p> <p>The graph of <math>g</math> has to shift <math>60^\circ</math> to the left and then be reflected about the <math>x</math>-axis./Die grafiek van <math>g</math> moet <math>60^\circ</math> na links skuif en dan om die <math>x</math>-as gereflekteer word.</p> <p><b>OR/OF</b></p> <p>The graph of <math>g</math> must be reflected about the <math>x</math>-axis and then be shifted <math>60^\circ</math> to the left./Die grafiek van <math>g</math> moet om die <math>x</math>-as gereflekteer word en dan met <math>60^\circ</math> na links geskuif word.</p> <p><b>OR/OF</b></p> <p>The graph of <math>g</math> has to shift <math>120^\circ</math> to the right./Die grafiek van <math>g</math> moet <math>120^\circ</math> na regs geskuif word.</p> <p><b>OR/OF</b></p> <p>The graph of <math>g</math> has to shift <math>240^\circ</math> to the left./Die grafiek van <math>g</math> moet met <math>240^\circ</math> na links geskuif word.</p>	<p><math>\checkmark</math> <math>60^\circ</math> left/links</p> <p><math>\checkmark</math> reflection about <math>x</math>-axis/refleksie om <math>x</math>-as</p> <p>(2)</p> <p><math>\checkmark</math> reflection about <math>x</math>-axis/refleksie om <math>x</math>-as</p> <p><math>\checkmark</math> <math>60^\circ</math> left/links</p> <p>(2)</p> <p><math>\checkmark</math> <math>\checkmark</math> <math>120^\circ</math> right/regs</p> <p>(2)</p> <p><math>\checkmark</math> <math>\checkmark</math> <math>240^\circ</math> left/links</p> <p>(2)</p> <p><b>8</b></p>
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QUESTION/VRAG 7

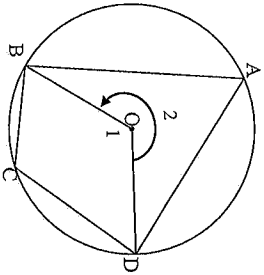


7.1	$\hat{C}\hat{A}\hat{D} = 180^\circ - 2\theta$ [∠s sum of Δ/∠e sòm van Δ]	✓ answ/antw	(1)
7.2	$\frac{\sin \theta}{x+3} = \frac{\sin(180^\circ - 2\theta)}{2x}$ $\frac{\sin \theta}{x+3} = \frac{\sin 2\theta}{2x}$ $\frac{\sin \theta}{x+3} = \frac{2 \sin \theta \cos \theta}{2x}$ $\cos \theta = \frac{2x \sin \theta}{2(x+3) \sin \theta}$ $\cos \theta = \frac{x}{x+3}$ <p><b>OR/OF</b>  <math>AD = x+3</math> [sides opp = ∠s/eye to = ∠e]  <math>AC^2 = AD^2 + CD^2 - 2AD \cdot CD \cdot \cos \theta</math>  <math>(x+3)^2 = (x+3)^2 + (2x)^2 - 2(2x)(x+3) \cos \theta</math>  <math>0 = 4x^2 - 4x(x+3) \cos \theta</math>  <math>\cos \theta = \frac{4x^2}{4x(x+3)}</math>  <math>= \frac{x}{x+3}</math></p> <p><b>OR/OF</b>                  Draw/Trek <math>AP \perp CD</math></p> $\cos \theta = \frac{x}{x+3}$	✓ correct subst into sine rule/korrekte subst in sin-reël ✓ $\sin 2\theta$ ✓ $2 \sin \theta \cdot \cos \theta$ ✓ $\cos \theta$ as subject/as onderwerp ✓ $AD = x+3$ ✓ correct subst into cosine rule/korrekte subst in cos-reël ✓ simplification/vereenvoudiging ✓ $\cos \theta$ as subject/as onderwerp ✓ constn/konstr ✓ sketch shown/toon skets	(4) (4)

7.3

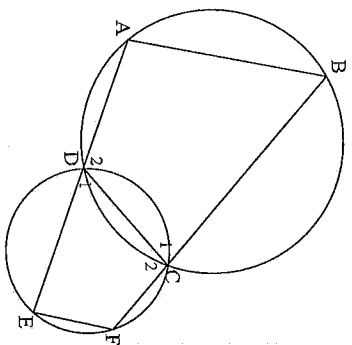
$\cos \theta = \frac{2}{5}$ $\therefore \theta = 66,42^\circ$ <p>In <math>\triangle ABC</math>:  <math>\sin \frac{1}{2}\theta = \frac{AB}{AC}</math>  <math>\sin 33,21^\circ = \frac{AB}{5}</math>  <math>\therefore AB = 5 \sin 33,21^\circ</math>  <math>= 2,74</math></p> <p><b>OR/OF</b>  <math>\sin \frac{\theta}{2} = \frac{AB}{5}</math>  <math>\therefore AB = 5 \sin \frac{\theta}{2}</math>                  but/maar:  <math>\cos \theta = \frac{2}{5}</math>  <math>1 - 2 \sin^2 \frac{\theta}{2} = \frac{2}{5}</math>  <math>\sin^2 \frac{\theta}{2} = \frac{3}{10}</math>  <math>\sin \frac{\theta}{2} = \sqrt{\frac{3}{10}}</math>  <math>\therefore AB = 5 \cdot \sqrt{\frac{3}{10}} = \sqrt{\frac{15}{2}} = 2,74</math></p>	✓ $\cos \theta = \frac{2}{5}$ ✓ size of/grootte v $\theta$ ✓ correct ratio/korrekte verh ✓ subst correctly/korrekt ✓ answ/antw ✓ $AB = 5 \sin \frac{\theta}{2}$ ✓ equation/vgl ✓ simplification/vereenvoudiging ✓ value of/waarde v $\sin \frac{\theta}{2}$ ✓ answ/antw	(5) (10)
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QUESTION/PRAAG 8



8.1.1	twice or double / twee keer of dubbels	✓ R	(1)
8.1.2	$\hat{O}_1 = 2\hat{A}$ [∠ at centre = 2×∠ at circ/midpts ∠ = 2×omtreks ∠] $\hat{O}_2 = 2\hat{C}$ [∠ at centre = 2×∠ at circ/midpts ∠ = 2×omtreks ∠] $\hat{O}_1 + \hat{O}_2 = 360^\circ$ [∠s in a revl/∠e in omw of om 'n pl] $2\hat{A} + 2\hat{C} = 360^\circ$ $\therefore \hat{A} + \hat{C} = 180^\circ$ <b>OR/OF</b> Let/Gesetel $\hat{O}_1 = 2x$ $\hat{A} = x$ [∠ at centre = 2×∠ at circ/midpts ∠ = 2×omtreks ∠] $\hat{O}_2 = 360^\circ - 2x$ [∠s in a revl/∠e in omw of om 'n pl] $\hat{C} = 180^\circ - x$ [∠ at centre = 2×∠ at circ/midpts ∠ = 2×omtreks ∠] $\therefore \hat{A} + \hat{C} = 180^\circ$	✓ S ✓ S ✓ S	(3)

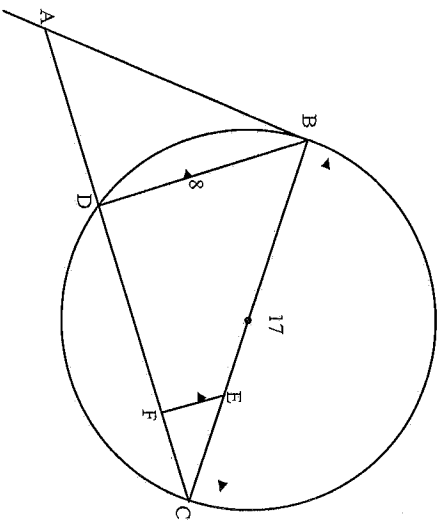
8.2



8.2	$\hat{A} = \hat{C}_2$ $\hat{E} = 180^\circ - \hat{C}_2$ $\therefore \hat{E} = 180^\circ - \hat{A}$ $\therefore EF \parallel AB$ <b>OR/OF</b> $\hat{B} = \hat{D}_1$ $\hat{F} = 180^\circ - \hat{D}_1$ $\therefore \hat{F} = 180^\circ - \hat{B}$ $\therefore EF \parallel AB$	[ext ∠ of cyclic quad/buite ∠ v kakh] [opp ∠s of cyclic quadros/∠e v kakh] [co-interior ∠s 180°/ko-binne ∠e 180°] [ext ∠ of cyclic quad/buite ∠ v kakh] [opp ∠s of cyclic quadros/∠e v kakh] [co-interior ∠s 180°/ko-binne ∠e 180°]	✓ S ✓ R ✓ S ✓ R ✓ R ✓ S ✓ R ✓ S ✓ R ✓ R	(5) [9]
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QUESTION/VR44G 10



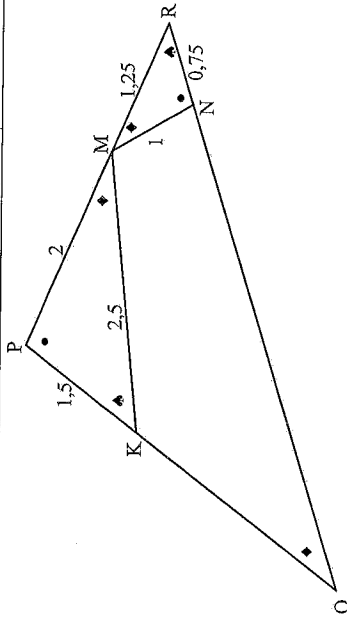
10.1	$\hat{B}\hat{D}C = 90^\circ$ $DC^2 = 17^2 - 8^2$ $= 225$ $\therefore DC = 15$	[∠ in semi circle / ∠ in halfstreekel] [Th ofstelling v Pythagoras]	✓ S ✓ using/gebruik Pyth korrek/correctly ✓ answ/antw	(3)
10.2.1	$\frac{CF}{CD} = \frac{CE}{CB}$ $\frac{CF}{15} = \frac{1}{4}$ $\therefore CF = 3,75$	[line    one side of Δ / lyn    een sy van Δ] OR/OF ACEF     ΔCBD	✓ S/R ✓ subst correctly/korrek ✓ answ/antw	(3)
10.2.2	$\hat{B}\hat{D}C = 90^\circ$ $\hat{E}\hat{F}C = \hat{B}\hat{D}C$ $\hat{A}\hat{B}C = 90^\circ$ In $\triangle ABC$ and/en $\triangle FEC$ : $\hat{A}\hat{B}C = \hat{E}\hat{F}C$ $\hat{C} = \hat{C}$ $\therefore \triangle BAC \parallel \triangle FEC$	[∠ in semi circle / ∠ in halfstreekel] [corresp ∠s / ooreenk ∠e; EF    BD] [tan ∠ diameter / radl ∠ middellyn] In $\triangle ABC$ and/en $\triangle FEC$ : [proven/bewys] [common/gemeen] [∠ in semi circle / ∠ in halfstreekel] [corresp ∠s / ooreenk ∠e; EF    BD] [tan ∠ diameter / radl ∠ middellyn] In $\triangle ABC$ and/en $\triangle FEC$ : [proven/bewys] [common/gemeen]	✓ S/R ✓ S ✓ R ✓ S ✓ R	(5)

10.2.3	$\hat{B}\hat{A}C = \hat{E}\hat{F}C$ $\therefore \triangle BAC \parallel \triangle FEC$	[∠ sum in Δ / ∠ som van Δ]	✓ S	(5)
10.2.3	$EC = \frac{1}{4} \times 17 = 4,25$ $\frac{AC}{EC} = \frac{BC}{FC}$ $\frac{AC}{4,25} = \frac{17}{3,75}$ $\therefore AC = 19,27$ or/of $19 \frac{4}{15}$	[ABAC     ΔFEC]	✓ length of/lengte v EC ✓ S ✓ subst correctly/korrek ✓ answ/antw	(4)
	OR/OF $\cos \hat{C} = \frac{CF}{CE} = \frac{BC}{AC}$ $\therefore \frac{3,75}{4,25} = \frac{17}{AC}$ $\therefore AC = 19,27$ or/of $19 \frac{4}{15}$		✓ ✓ correct ratios/korrette verhs ✓ subst correctly/korrek ✓ answ/antw	(4)
	OR/OF $ABCA \parallel \triangle DBC$ $CB^2 = CD \cdot AC$ $AC = \frac{BC^2}{DC}$ $= \frac{17^2}{15}$ $= 19,27$ or/of $19 \frac{4}{15}$		✓ S OR Pyth in correct ratio ✓ subst	(4)
	OR/OF $\hat{C} = \hat{A}\hat{B}D$ $\frac{AD}{8} = \tan \hat{A}\hat{B}D$ $= \tan \hat{C}$ $= \frac{8}{15}$ $\therefore AD = \frac{64}{15}$ $\therefore AC = 19,27$ or/of $19 \frac{4}{15}$	[tan-chord theorem / ht-katstelling]	✓ S ✓ correct ratio ✓ subst ✓ answ/antw	(4)

10.2.4	<p>AC is diameter of the circle passing through A, B and C                  [chord subtends 90° <b>OR</b> converse ∠ in semi circle ]                  AC is middellyn van die sirkel wat deur die punte A, B en C gaan                  [koord onderspan 90° <b>OF</b> omgek ∠ in halfsirkel ]                  ∴ radius = <math>\frac{1}{2} \times 19,27 = 9,63</math> or/of <math>9 \frac{19}{30}</math> or/of <math>\frac{1}{2} AC</math></p>	<p>✓ S/R                  ✓ answ/antw (2)                  [17]</p>
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**QUESTION/VRAG 11**

11.1	equiangular or similar/gehyktoekig of gehykvormig	✓ answ/antw (1)
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11.2.1	<p><math>\frac{KP}{RN} = \frac{1,5}{0,75} = 2</math>; <math>\frac{PM}{NM} = \frac{2}{1} = 2</math>; <math>\frac{KM}{RM} = \frac{2,5}{1,25} = 2</math>  <math>\therefore \frac{KP}{RN} = \frac{PM}{NM} = \frac{KM}{RM}</math>  <math>\therefore \Delta KPM \parallel \Delta RNM</math> [Sides of <math>\Delta</math> in prop./syte v <math>\Delta</math> eweredig]  <b>OR/OF</b>  <math>\frac{RN}{KP} = \frac{0,75}{1,5} = \frac{1}{2}</math>; <math>\frac{NM}{PM} = \frac{1}{2}</math>; <math>\frac{RM}{KM} = \frac{1,25}{2,5} = \frac{1}{2}</math>  <math>\therefore \frac{RN}{KP} = \frac{NM}{PM} = \frac{RM}{KM}</math>  <math>\therefore \Delta KPM \parallel \Delta RNM</math> [Sides of <math>\Delta</math> in prop./syte v <math>\Delta</math> eweredig]  <b>OR/OF</b>                  In <math>\Delta MNR</math>:  <math>1,25^2 = 1^2 + 0,75^2 = 1,5625</math>  <math>\therefore \hat{MNR} = 90^\circ</math> [converse Pyth theorem]                  In <math>\Delta PKM</math>:  <math>2,5^2 = 1,5^2 + 2^2 = 6,25</math>  <math>\therefore \hat{P} = 90^\circ</math> [converse Pyth theorem]  <math>\cos \hat{PKM} = \frac{1,5}{2,5} = \frac{3}{5}</math> and <math>\cos \hat{R} = \frac{0,75}{1,25} = \frac{3}{5}</math>  <math>\therefore \hat{PKM} = \hat{R}</math>                  In <math>\Delta KPM</math> and <math>\Delta RNM</math>  <math>\hat{PKM} = \hat{R}</math> [proved]  <math>\hat{P} = \hat{MNR}</math> [proved]  <math>\therefore \Delta KPM \parallel \Delta RNM</math> [<math>\hat{C}; \hat{C};</math> OR 3<sup>rd</sup> <math>\hat{C}</math>]</p>	<p>✓✓✓                  all 3 statements/                  al 3 bewerings (3)</p> <p>✓✓✓                  all 3 statements/                  al 3 bewerings (3)</p> <p>✓ <math>\hat{P} = \hat{MNR}</math></p> <p>✓ <math>\hat{PKM} = \hat{R}</math></p> <p>✓ [<math>\hat{C}; \hat{C};</math> OR 3<sup>rd</sup> <math>\hat{C}</math>] (3)</p>
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11.2.2	<p><math>P\hat{K}M = \hat{R}</math> [AKPM     ARNM]</p> <p><math>\therefore \hat{P}</math> is common/<i>gemeen</i> [LLL]</p> <p><math>\therefore \Delta RPQ \parallel \Delta KPM</math> [ARPQ     AKPM]</p> <p><math>\frac{RP}{KP} = \frac{RQ}{KM}</math> [ARPQ     AKPM]</p> <p><math>\therefore \frac{3,25}{1,5} = \frac{RQ}{2,5}</math></p> <p><math>\therefore RQ = \frac{2,5 \times 3,25}{1,5} = 5,42</math> or <math>5\frac{5}{12}</math></p> <p><math>\therefore NQ = 5,42 - 0,75 = 4,67</math> or <math>4\frac{2}{3}</math></p> <p><b>OR/OF</b></p> <p><math>P\hat{K}M = \hat{P}</math> [AKPM     ARNM]</p> <p><math>\therefore \hat{R}</math> is common/<i>gemeen</i> [LLL]</p> <p><math>\therefore \Delta RNM \parallel \Delta RPQ</math> [ARNM     ARPQ]</p> <p><math>\frac{RN}{RM} = \frac{RQ}{RM}</math> [ARNM     ARPQ]</p> <p><math>\therefore \frac{3,25}{0,75} = \frac{RQ}{1,25}</math></p> <p><math>\therefore RQ = 5,42</math> or <math>5\frac{5}{12}</math></p> <p><math>\therefore NQ = 5,42 - 0,75 = 4,67</math> or <math>4\frac{2}{3}</math></p> <p><b>OR/OF</b></p> <p>In <math>\Delta MNR</math>:  <math>1,25^2 = 1^2 + 0,75^2 = 1,5625</math>  <math>\therefore M\hat{N}R = 90^\circ</math> [converse Pyth theorem]          In <math>\Delta PKM</math>:  <math>2,5^2 = 1,5^2 + 2^2 = 6,25</math>  <math>\therefore \hat{P} = 90^\circ</math> [converse Pyth theorem]          In <math>\Delta MNR</math> and <math>\Delta QPR</math>  <math>\angle R</math> is common  <math>M\hat{N}R = \hat{P} = 90^\circ</math>  <math>\therefore \Delta MNR \parallel \Delta QPR</math> [LLL]          [ARNM     ARPQ]</p> <p><math>\frac{RP}{RN} = \frac{RQ}{RM}</math> [ARNM     ARPQ]</p> <p><math>\therefore \frac{3,25}{0,75} = \frac{RQ}{1,25}</math></p> <p><math>\therefore RQ = 5,42</math> or <math>5\frac{5}{12}</math></p> <p><math>\therefore NQ = 5,42 - 0,75 = 4,67</math> or <math>4\frac{2}{3}</math></p>	<p>✓ S</p> <p>✓ <math>\Delta RPQ \parallel \Delta KPM</math></p> <p>✓ S</p> <p>✓ subst correctly/ <i>korrek</i></p> <p>✓ <math>RQ = 5\frac{5}{12}</math></p> <p>✓ <math>NQ = \text{answ/antw}</math> (6)</p> <p>[10]</p>
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TOTAL/TOTAAL: 149