Basic Education

KwaZulu-Natal Department of Basic Education
REPUBLIC OF SOUTH AFRICA

MATHEMATICS P2

COMMON TEST

JUNE 2016

NATIONAL SENIOR CERTIFICATE

GRADE 12

MARKS: 125

TIME: 2½ hours

This question paper consists of 11 pages and an answer book of 16 pages.

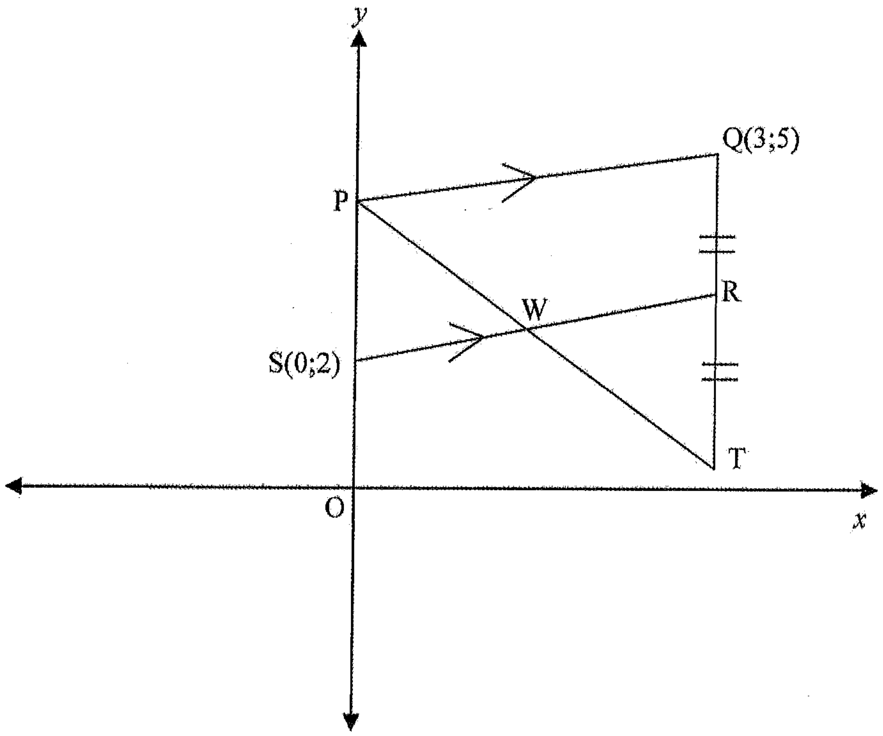
INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 9 questions.
2. Answer ALL the questions.
3. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.
4. Answers only will not necessarily be awarded full marks.
5. An approved scientific calculator (non-programmable and non graphical) may be used, unless stated otherwise.
6. If necessary, round off answers to TWO decimal places, unless stated otherwise.
7. Diagrams are not necessarily drawn to scale.
8. An information sheet with formulae is included at the end of this question paper.
9. Number the answers correctly according to the numbering system used in this question paper.
10. Write neatly and legibly.

QUESTION 1

In the diagram below, PQRS is a parallelogram. The vertices P and S lie on the y – axis. The side QR is produced to its own length to T, ie. QR = RT. Q (3;5) and S (0;2). PS= 2 units. The line segment PT intersects SR at W. PW = WT



1.1 Determine the co-ordinates of the following points:

1.1.1 P (3)

1.1.2 R (3)

1.1.3 T (3)

1.1.4 W (3)

1.2

1.2.1 Calculate the gradient of PQ. (2)

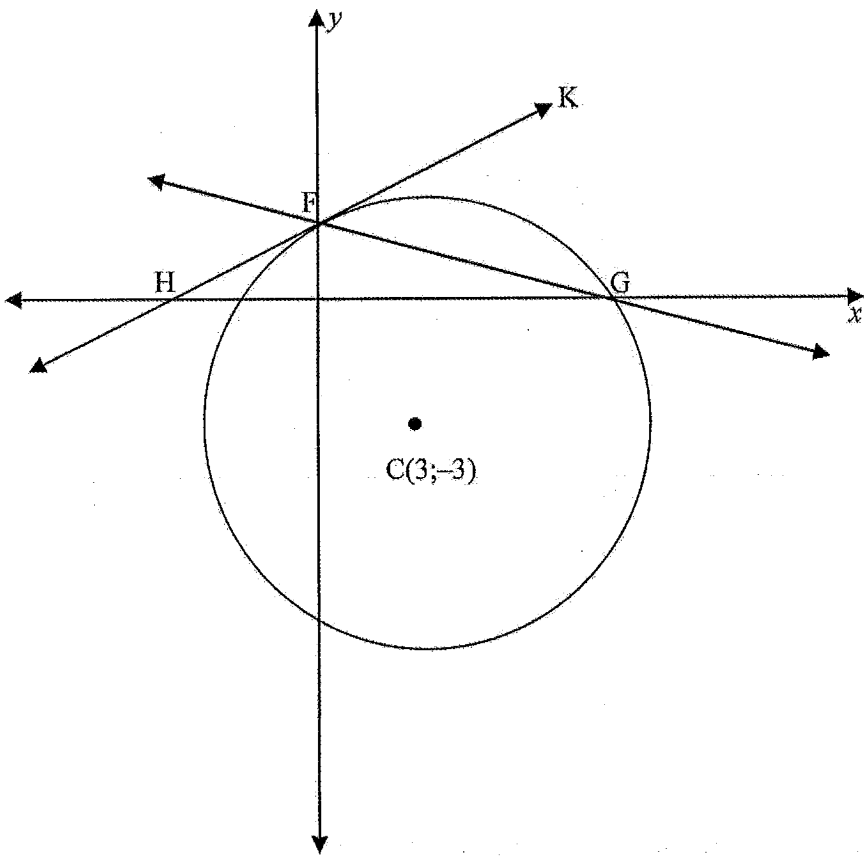
1.2.2 Hence, determine the equation of the line PQ. (2)

1.3 Determine the length of the line SQ. (3)

[19]

QUESTION 2

2.1 In the diagram below C(3; –3) is the centre of the circle with radius equal to 5 units. F and G are the points of intersection of the circle with y – axis and the x – axis respectively, HK is a tangent to the circle at F and cuts the x – axis at H.



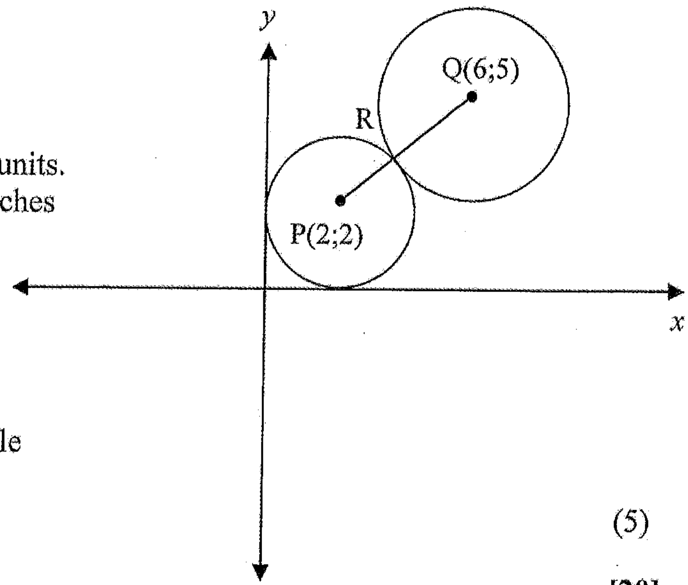
2.1.1 Determine the equation of the circle. (2)

2.1.2 Calculate the co-ordinates of F and G. (8)

2.1.3 Hence, determine the equation of the tangent HK. (5)

2.2 In the diagram alongside, a circle centre P has a radius of 2 units. It lies in the first quadrant and touches both axes.

The circle with the centre Q (6; 5) touches the circle with centre P externally at R. Determine the equation of the circle with centre Q.

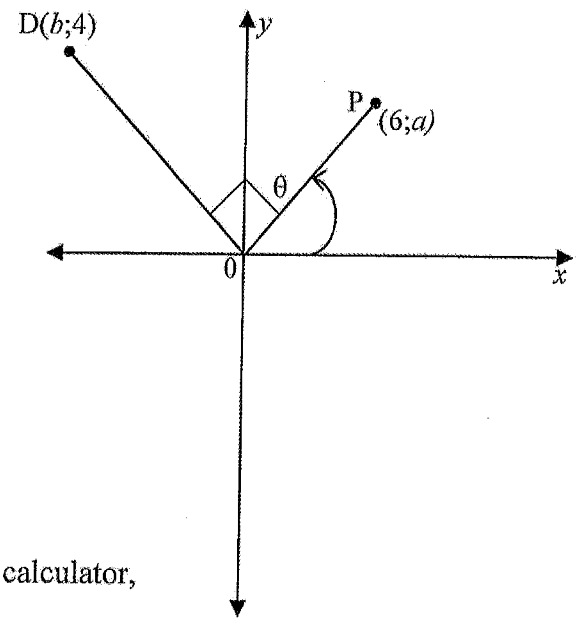


(5)

[20]

QUESTION 3

In the diagram alongside, $D\hat{O}P = 90°$. The angle which OP makes with the x – axis is θ. Given D (b;4) ; P (6;a) and $\sqrt{5}\cos\theta - 2 = 0$



3.1 Determine, without the use of a calculator, the numerical value of:

3.1.1 a (3)

3.1.2 b (4)

3.2 Simplify without the use of the calculator.

3.2.1 $\sin^2 20° + \sin^2 70°$ (3)

3.2.2 $\dfrac{\cos 330°.\sin 140°}{\sin(-160°).\tan 405°.\sin 290°}$ (10)

[20]

QUESTION 4

4.1 Prove the identity:

$$\frac{\cos x}{1+\sin x}+\tan x=\frac{1}{\cos x}$$ (4)

4.2 Solve for A:

$\sin 2A = \cos 48^\circ$; $0^\circ \leq A \leq 90^\circ$ (3)

4.3 Determine the general solution of:

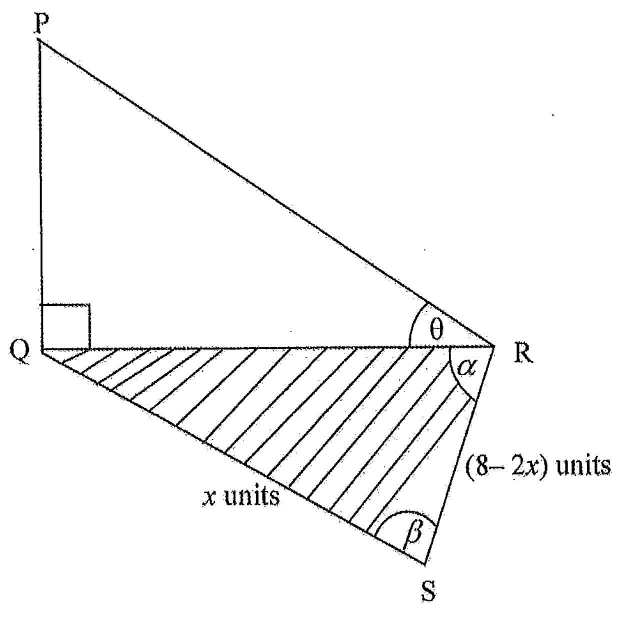
$$\cos 3\theta \,.\, \cos\theta + \sin 3\theta \,.\, \sin\theta = \frac{-\sqrt{3}}{2}$$ (6)

[13]

QUESTION 5

In the diagram below, PQ is a vertical mast. R and S are two points in the same horizontal plane as Q, such that :

$Q\hat{R}S=\alpha$; $Q\hat{S}R=\beta$, $SR=8-2x$, $QS=x$



5.1 Show that: $PQ=\dfrac{x\sin\beta\tan\theta}{\sin\alpha}$ (5)

5.2

5.2.1 If $\beta = 60^\circ$, show that the area of ΔQSR $= 2\sqrt{3}x - \dfrac{\sqrt{3}}{2}x^2$ (3)

5.2.2 Determine the value of x for which the area of ΔQSR will maximum. (3)

5.2.3 Calculate the length of QR if the area of Δ QSR is maximum. (3)

[14]

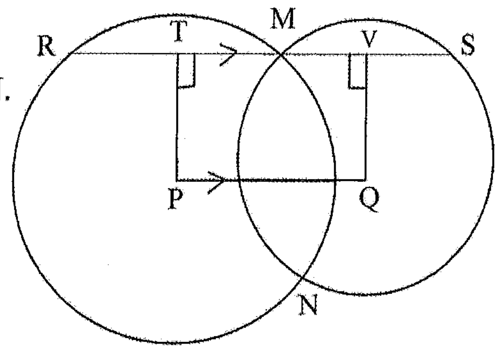
Give reasons for your answers in questions 6; 7; 8 and 9.

QUESTION 6

6.1 A line drawn from the centre of the circle perpendicular to a chord, --------- the chord. (1)

6.2 In the diagram alongside, circles with P and Q as centres intersect at M and N. RMS is a straight line such that RMS || PQ. PT ⊥ RM and QV ⊥ MS.

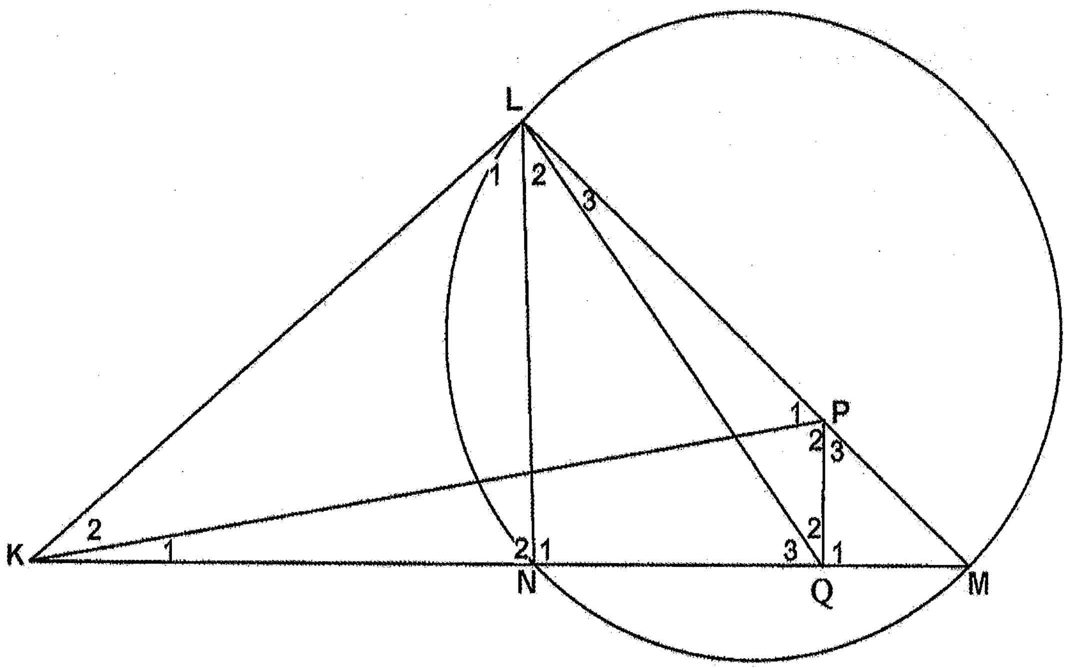
Show that RS = 2PQ.



(9)
[10]

QUESTION 7

In the diagram below, KL is a tangent to circle LMN. LM is a diameter.
MN is extended to K, P is a point on LM such that MP:PL = 1: 2.
MN = $3\sqrt{2}$ units and MQ = $\sqrt{2}$ units.



Prove that:

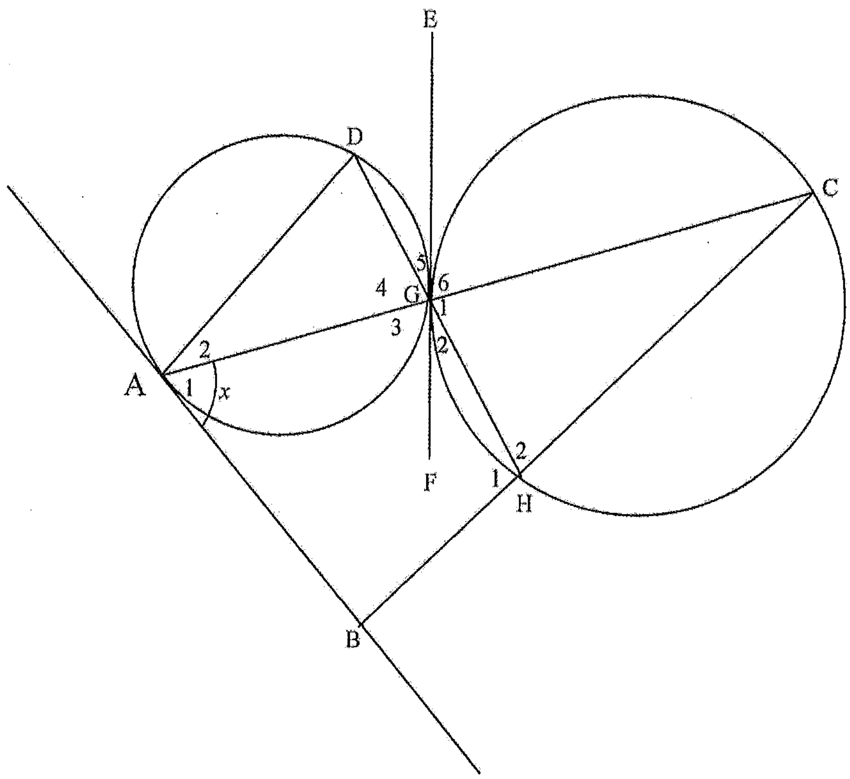
7.1 LN|| PQ (4)

7.2 LPQK is a cyclic quadrilateral. (5)

7.3 ΔQNL /// ΔPLK (4)
[13]

QUESTION 8

In the diagram below, two circles touch each other externally at G. EGF is a common tangent to both circles at G. AGC is a straight line. CH is a diameter of the larger circle. AB is a tangent to the smaller circle. CHB and DGH are straight lines.



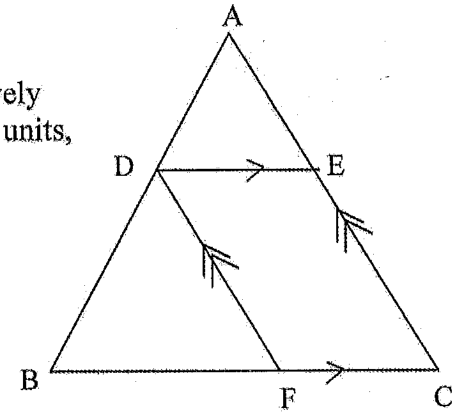
8.1 If $\hat{A}_1 = x$, name, stating reasons, <u>three</u> other angles each equal to x. (6)

8.2 Prove that AD is a diameter of the smaller circle. (3)

[9]

QUESTION 9

In the diagram alongside, ABC is a triangle with D,F and E on AB, BC and AC respectively sothat FDEC is a parallelogram. If AD = 10 units, AE = 9, BF = 12 and FC = 8.
Calculate the length of:



9.1 BD (4)

9.2 CE (3)
[7]

TOTAL: [125]

INFORMATION SHEET: MATHEMATICS
INLIGTINGSBLAD: WISKUNDE

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $A = P(1-ni)$ $A = P(1-i)^n$ $A = P(1+i)^n$

$T_n = a + (n-1)d$ $S_n = \frac{n}{2}(2a + (n-1)d)$

$T_n = ar^{n-1}$ $S_n = \frac{a(r^n - 1)}{r-1}$; $r \neq 1$ $S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $y - y_1 = m(x - x_1)$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $a^2 = b^2 + c^2 - 2bc.\cos A$ $area\ \Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum x}{n}$ $\sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ P(A or B) = P(A) + P(B) - P(A and B)

$\hat{y} = a + bx$ $b = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sum(x-\bar{x})^2}$

Maths P2 + M-Lit P2

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MEMORANDUM

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MARKS : 125

This memorandum consists of 10 pages.

QUESTION 1

1.1.1 P(0 ; 4) PS = 2 units	✓✓A(0 , 4) ✓R (3) Answer only full marks
1.1.2 R(3 ; 3) QR // y – axis and QR = 2 units	✓✓R (3 ; 3) ✓R (3) Answer only full marks
1.1.3 T(3 ; 1) QT // y – axis and RT = QR	✓✓ T(3 ; 1) ✓R (3) Answer only full marks
1.1.4 W is the mid-point $W\left(\frac{x_1+x_2}{2};\frac{y_1+y_2}{2}\right)$ $W\left(\frac{0+3}{2};\frac{4+1}{2}\right)$ $W\left(1\frac{1}{2};2\frac{1}{2}\right)$	✓ midpoint formula ✓substitution ✓answer (3)
1.2.1 $m_{PQ} = \frac{5-4}{3-0} = \frac{1}{3}$	✓substitution ✓$\frac{1}{3}$ (2)
1.2.2 $y - y_1 = m(x - x_1)$ $y - 4 = \frac{1}{3}(x-0)$ $y = \frac{1}{3}x + 4$	✓substitution ✓equating
1.3 $SQ = \sqrt{(3-0)^2 + (5-2)^2}$ $= \sqrt{9+9}$ $= \sqrt{18}$ $= \sqrt{9 \times 2}$ $= 3\sqrt{2}$	✓dist form ✓substitution ✓answer (3)

QUESTION 2

2.1.1 $(x-3)^2 + (y+3)^2 = 5^2$	✓substitution of centre ✓radius (2)
2.1.2 $(x-3)^2 + (0+3)^2 = 25$ $x^2 + 6x + 9 + 9 = 25$ $x^2 - 6x - 7 = 0$ $(x-7)(x+1) = 0$ $x = 7$ or $x = -1$ $\therefore x = 7$ G(7 ; 0) F(0 ; y) $(x-3)^2 + (y+3)^2 = 25$ $(0-3)^2 + (y+3)^2 = 25$ $y^2 + 6y + 18 - 25 = 0$ $y^2 + 6y - 7 = 0$ $(y+7)(y-1) = 0$ $y = -7$ or $y = 1$ $\therefore y = 1$ F (0; 1)	✓substitution $y = 0$ ✓factorising ✓$x = 7$ ✓co–ordinate of G ✓substitution $x = 0$ ✓factorizing ✓$y = 1$ ✓co-ordinate (8)
2.1.3 $m_{CF} = \frac{1-(-3)}{0-3}$ $= -\frac{4}{3}$ $m_{HK} \cdot m_{CF} = -1$ $\therefore m_{HK} = \frac{-1}{-\frac{4}{3}} = \frac{3}{4}$ $\therefore y = \frac{3}{4}x + 1$	✓substitution ✓$-\frac{4}{3}$ ✓product of gradient = - 1 ✓$\frac{3}{4}$ ✓equation (5)

2.2 PR = 2 PR + RQ = PQ $2 + RQ = \sqrt{(6-2)^2 + (5-2)^2}$ $2 + RQ = 5$ $\therefore RQ = 3$ Equation of the circle is $(x-6)^2 + (y-5)^2 = 3^2$ or $x^2 + y^2 - 12x - 10y + 52 = 0$	✓ PR = 2 ✓ substitution ✓ PQ = 5 ✓RQ = 3 ✓equating of the circle (5)

QUESTION 3

Due to printing error of θ in the diagram. Questions 3.1.1 and 3.1.2 will not be considered in the marking of the script. The paper will now be marked out of 118 marks.

3.2.1	$\sin^2 20° + \sin^2 70°$ $= \sin^2 20° + \sin^2(90° - 20°)$ $= \sin^2 20 + \cos^2 20°$ $= 1$	✓$70° = 90° - 20°$ ✓$\sin^2 20° + \cos^2 20°$ ✓answer

3.2.2	$\dfrac{\cos 330° \ . \ \sin 140°}{\sin(-160°) \ . \ \tan 405° \ . \ \sin 290°}$ $= \dfrac{\cos(360° - 30°) \ . \ \sin(180° - 40°)}{-\sin 160° \ . \ \tan(360° + 45°) \ . \ \sin(360° - 70°)}$ $= \dfrac{\cos 30° \ . \ \sin 40°}{-\sin 20° \ . \ \tan 45° \ . \ -\sin 70°}$ $= \dfrac{\sqrt{3}/2 \ . \ 2\sin 20° \cos 20°}{-\sin 20° \ . \ 1 \ . \ -\cos 20°} = +\sqrt{3}$	✓$\cos 30°$✓$\sin 40°$✓$-\sin 20°$ ✓$\tan 45°$✓$-\sin 70°$ ✓$\sqrt{3}/2$ ✓$2\sin 20° \cos 20°$ ✓$-\cos 20°$ ✓ $\tan 45° = 1$ ✓ $\sqrt{3}$ (10)

QUESTION 4

4.1	$\dfrac{\cos x}{1+\sin x} + \tan x = \dfrac{1}{\cos x}$ LHS $= \dfrac{\cos x}{1+\sin x} + \tan x$ $= \dfrac{\cos x}{1+\sin x} + \dfrac{\sin x}{\cos x}$ $= \dfrac{\cos^2 x + \sin x(1+\sin x)}{(1+\sin x)\cos x}$ $= \dfrac{\cos^2 x + \sin x + \sin^2 x}{(1+\sin x)\cos x}$ $= \dfrac{1+\sin x}{(1+\sin x)\cos x}$ $= \dfrac{1}{\cos x}$ $=$ RHS	✓$\dfrac{\sin x}{\cos x}$ ✓common denominator ✓simplification ✓$\sin^2 x + \cos^2 x = 1$ (4)
4.2	$\sin 2A = \cos 48°$ $= \sin(90° - 42°)$ $= \sin 42°$ $2A = 42°$ or $2A = (180° - 42° = 138°)$ $\therefore A = 21°$ or $A = 69°$	✓$\sin 42°$ ✓$A = 21°$ ✓$A = 69°$ Calculator use full marks (3)
4.3	$\cos 3\theta \ . \ \cos\theta + \sin 3\theta \ . \ \sin\theta = \dfrac{-\sqrt{3}}{2}$ $\cos(3\theta - \theta) = \dfrac{-\sqrt{3}}{2}$ Ref angle $= 30°$ $2\theta = 150° + n.360°$ or $2\theta = 210° + k.360°$ $\theta = 75° + n.180°$ or $\theta = 105° + k.180°$ Where $k \in Z$	✓$\cos(3\theta - \theta)$ ✓ref angle = 30° ✓solution of 2θ for 2 quadrants ✓✓solution of θ for each quadrant ✓ $k \in Z$ (6)

OR $\cos 3\theta \cdot \cos\theta + \sin 3\theta \cdot \sin\theta = \frac{-\sqrt{3}}{2}$ $\text{Cos}(2\theta) = \frac{-\sqrt{3}}{2}$ $2\theta = \pm 150° + k.360°$ $\theta = \pm 75° + k.180° \quad , k \in Z$	✓$\cos(2\theta)$ ✓✓±150° ✓✓solution of θ for each quadrant ✓$k \in Z$

QUESTION 5

5.1 $\tan\theta = \frac{PQ}{QR}$ $QR = \frac{PQ}{\tan\theta}$ ……… (1) $\frac{\sin\beta}{QR} = \frac{\sin\alpha}{x}$ $\therefore QR = \frac{\text{Sin}\,\beta \cdot x}{\sin\alpha}$ ……… (2) $(1) = (2): \frac{PQ}{\tan\theta} = \frac{\sin\beta \cdot x}{\sin\alpha}$ $\therefore PQ = \frac{x\sin\beta \cdot \tan\theta}{\sin\alpha}$	✓ $QR = \frac{PQ}{\tan\theta}$ ✓ applying sine rule ✓ making QR the subject ✓ equating ✓ making PQ the subject (5)
5.2.1 Area $= \frac{1}{2}$ SR . QS sin y $= \frac{1}{2}.(8-2x)(x)\sin 60°$ $= (4x - x^2).\frac{\sqrt{3}}{2}$ $= 2x\sqrt{3} - \frac{\sqrt{3}}{2}x^2$	✓ Area rule ✓ subst ✓ Area (3)

5.2.2 At maximum Area; $A'(x) = 0$ $A'(x) = 2\sqrt{3} - \sqrt{3}.x$ $0 = 2\sqrt{3} - \sqrt{3}\,x$ $x = 2$	✓ $A'(x) = 0$ ✓ derivative ✓ answer (3)
5.2.3 At maximum Area QS = 2 SR = 8 – 2(2) = 8 – 4 = 4 $QR^2 = QS^2 + SR^2 - 2QS.\ SR\ \cos 60°$ $= (4)^2 + (2)^2 - 2.4.2.\ \frac{1}{2}$ $= 16 + 4 - 16.\ \frac{1}{2}$ $= 20 - 8$ $= 12$ $\therefore QR = \sqrt{12}$ $= 4 \times 3$ $= 2\sqrt{3}$	✓ SR = 4 ✓ substitution into the correct formula ✓ QR (3) [14]

QUESTION 6

6.1 bisects	✓ bisects (1)
6.2 Let RT = x Then RT = TM = x … (line from centre ⊥ chord….) Let MV = y then MV = VS = y … (line from centre ⊥ chord…..) ∴RS = 2RT + 2MV = 2 (RT + MV) = 2 (x + y) TP // VQ … (co-interior angles are supplementary) and TV // PQ … given ∴ PQ = TV = $x+y$ (TPQV is a rectangle) 2 PQ = 2 (x + y) RS = 2PQ	✓ S ✓R ✓ S ✓R ✓ S ✓ S ✓ S ✓ S ✓ multiplying by both 2 sides (9) [10]

QUESTION 7

7.1	$\frac{MQ}{MN}=\frac{\sqrt{2}}{3\sqrt{2}}=\frac{1}{3}$ $\frac{MP}{ML}=\frac{x}{3x}=\frac{1}{3}$ $\therefore \frac{MP}{MN}=\frac{MP}{ML}$ $\therefore$ LN // PQ (Line Divides 2 sides of triangle in proportion)	✓ $\frac{MQ}{MN}=\frac{1}{3}$ ✓ $\frac{MP}{ML}=\frac{1}{3}$ ✓ equating ✓ reason (4)
7.2	$\hat{N}_1=90°$ … $\angle$ in semi-circle $\hat{Q}_1=\hat{N}_1=90°$ … corresponding $\angle$'s LN // PQ $K\hat{L}M=90°$ … diameter LM $\perp$ KL $\therefore$ LPQK … is cyclic … ext $\angle$ of cyclic quad = interior opp $\angle$	✓ S ✓ R ✓ S/R ✓ S/R ✓ reason (5)
7.3	In Δ QNL and Δ PLK 1) $\hat{Q}_3=\hat{P}_1$ … $\angle$'s in the same segment 2) $\hat{N}_1=\hat{L}=90°$ … proved above 3) $\hat{L}_2=\hat{K}_2$ … remaining angle $\therefore \Delta$ QNL /// Δ PLK … ($\angle\angle\angle$)	✓ S/R ✓ S/R ✓ S/R ✓ S/R (4)

QUESTION 8

8.1	$D=\hat{A}_1=x$ … tan-chord theorem $\hat{G}_3=\hat{D}=x$ … tan-chord theorem $\hat{G}_3=\hat{G}_6=x$ … Vert. Opp. Angles	✓ S✓ R ✓ S✓ R ✓ S✓ R (6)
8.2	$\hat{G}_1=90°$ … (angle in the semi circle) $\hat{G}_4=\hat{G}_1=90°$ … vert. opp. $\angle$'s $\therefore$ AD is a diameter (conv. Angle in the semi circle)	✓ S ✓ S ✓ R (3) [9]

QUESTION 9

9.1	$\frac{BD}{AD}=\frac{BF}{FC}$ … Prop theorem DE // AC $\frac{BD}{10}=\frac{12}{8}$ $\therefore BD=\frac{120}{8}$ $=15$	✓ S ✓ R ✓ substitution ✓ BD = 15 (4)
9.2	$\frac{AD}{BD}=\frac{AE}{EC}$ … Prop theorem DE // BC $\frac{10}{15}=\frac{9}{EC}$ $\therefore$ EC = 13,5	✓ S/R ✓ substitution ✓ EC = 13,5 (3) [7]

TOTAL: [125]