Name and Surname ....

Grade/Class   ....   Mathematics Teacher : ....

JUNE 2017

GRADE 11
MATHEMATICS
Paper 1

Marks

_____
100

Time 2 HOURS

INSTRUCTIONS
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1.   Illegible work, in the opinion of the marker, will earn zero marks.

2.   Number your answers clearly and accurately, exactly as they appear on the question paper.

3. NB   • Leave *2 lines* open between each of your answers.

4. NB   Fill in the details requested on the front of the Question Paper.
            Hand in your submission in the following manner :
               • Question Paper (on top)
               • Answer pages (below)

            Do not staple your Question Paper and Answer pages together.

5.   Employ relevant formulae and show all working out. Answers alone may not be awarded
            full marks.

6.   (Non-programmable and non-graphical) Calculators may be used, unless their usage is
            specifically prohibited.

7.   Round off answers to 2 decimal places, where necessary, unless instructed otherwise.

8.   If (Euclidean) Geometric statements are made, reasons must be stated appropriately.

QUESTION 1 [49 marks]

1.1. Solve for x :

1.1.1. $3x(x-4)=5$ <u>4</u>

1.1.2. $3.\sqrt{2-x}+2x=-5$ <u>6</u>

1.1.3. $\dfrac{3x+1}{7x-1}=\dfrac{2x-1}{3x+1}$ <u>4</u>

1.1.4. $2^x-5.2^{x-3}=3.\sqrt{2}$ (without the use of a calculator) <u>4</u>

1.1.5. $2x^{-3}-3x^{-\frac{3}{2}}=5$ <u>5</u>

1.1.6. $12>x(6x+1)$ <u>4</u>

1.1.7. $\dfrac{2^x.\sqrt{x-2}}{x^2+2}\geq 0$ <u>1</u> (28)

1.2. Solve for x and y :

$$x^2-xy-y^2=-1 \quad \text{and} \quad 2^x.32=4^y$$

(7)

1.3. Without solving the following equation

$$49x^2-42x+9=0$$

discuss the nature of its roots. (3)

1.4. Students are discussing the function f, whose equation is

$$y=x+\frac{1}{x}$$

in an attempt to determine its range.

An informed student suggests writing the equation in standard form and then to use the nature of roots.

It is decided to implement this suggestion.
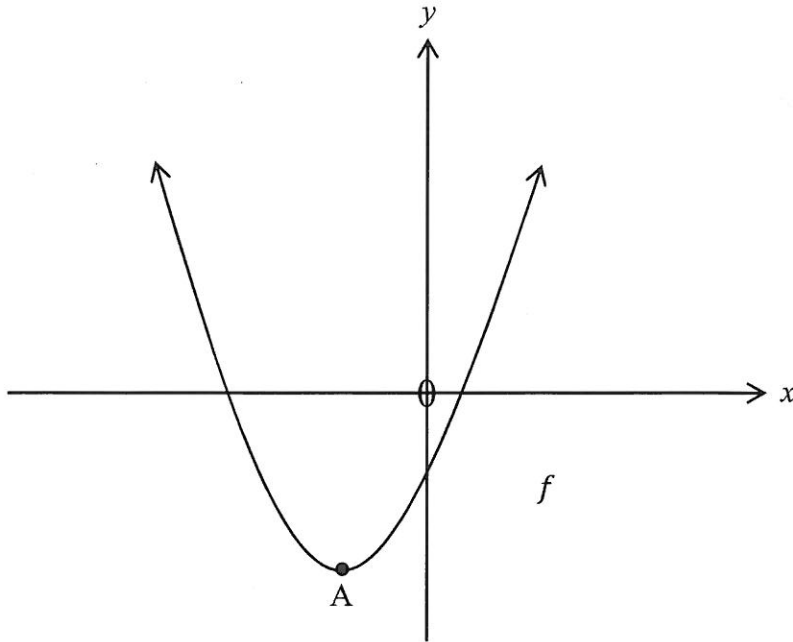
1.4.1. Write $y=x+\dfrac{1}{x}$ in standard form. <u>1</u>

1.4.2. Now, determine the discriminant (Δ) of the equation in (1.4.1.). <u>2</u>

1.4.3. Hence, determine the range of f. <u>4</u> (7)

1.5. Without the use of a calculator, write $\dfrac{\left(2-3.\sqrt{5}\right)^2}{\sqrt{5}}$ in the form $a+b\sqrt{c}$, where $a,b\in\mathbb{Q}$ and $c\in\mathbb{N}$. (4)

QUESTION 2 [2 marks]

2. $A(-2; -4)$ is the turning point of $f(x) = ax^2 + bx + c$:

If $h(x) = f(x + 3) + 5$, write down the coordinates of the turning point of h. (2)

QUESTION 3 [8 marks]

3.   Given : $f(x) = -2.3^{x-1} + 5$

3.1.   Sketch the graph of f, showing all relevant details on the diagram. (5)

3.2.   Calculate the average gradient of f between $x = -1$ and $x = 1$. (3)

4. $f(x) = \dfrac{3x-8}{x-2}$ and ℓ is an axis of symmetry of f.

The asymptotes of f meet at point A. B and C are the y- and x-intercepts of f, respectively. P is a point of intersection of ℓ and f.
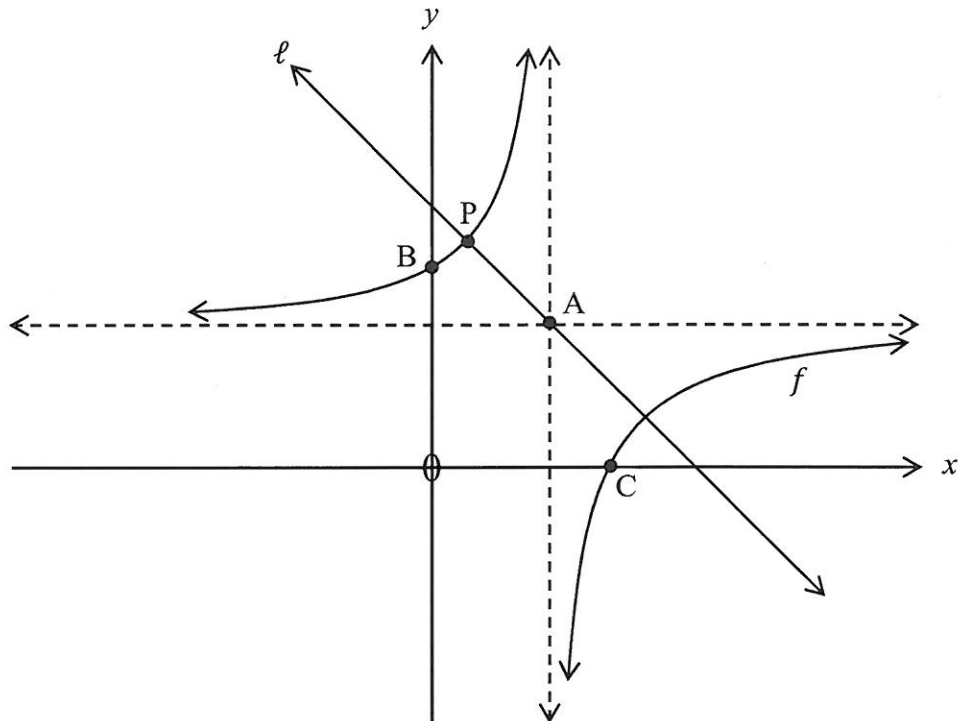
4.1.   Show that the equation of f can be written as

$$f(x) = -\frac{2}{x-2} + 3$$ (1)

4.2.   Calculate the coordinates of

4.2.1.   A   2

4.2.2.   B   1

4.2.3.   C   2   (5)

4.3.   Write down the

4.3.1.   equation of ℓ 2

4.3.2.   coordinates of P   2   (4)

4.4.   If f is reflected in the x-axis to become g, write down the equation of g in y-form.   (2)

4.5.   Use the graph to solve for x if $x.f(x) \geq 0$.   (2)

QUESTION 5 [17 marks]

5.   The equation of the f and g are

$$y = ax^2 + bx + c \quad \text{and} \quad y - 2x - 10 = 0$$

respectively. HL is a vertical line, with H a point on f and L a point on g.
f and g intersect at A and C, with C being the y-intercept of both f and g.
M and $x + 2 = 0$ are the turning point and axis of symmetry of f, respectively.
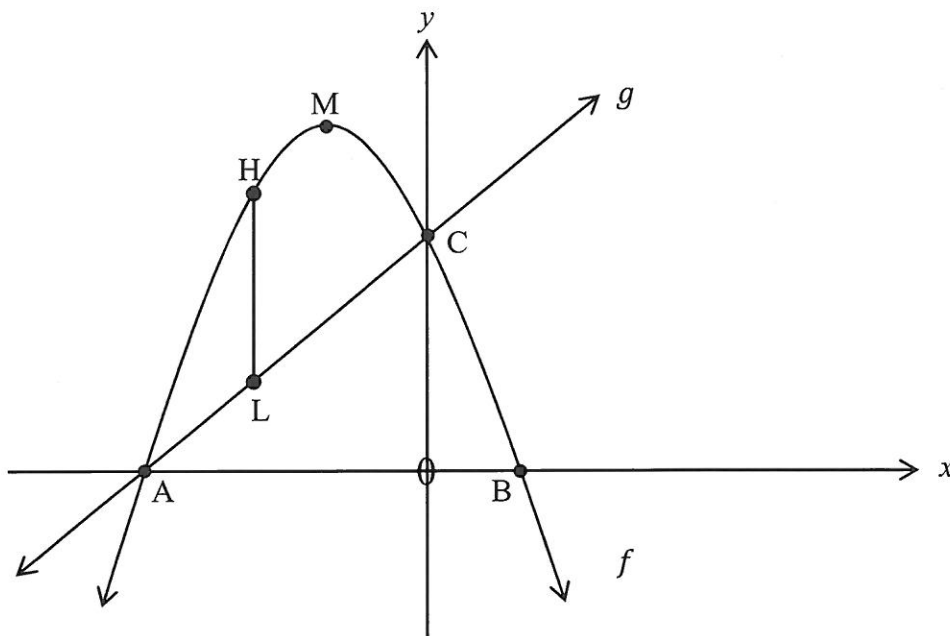
5.1.   Determine the coordinates of

5.1.1.   C   1

5.1.2.   A   1

5.1.3.   B   1   (3)

5.2.   Show that the equation of f is $y = -2x^2 - 8x + 10$.   (4)

5.3.   Calculate the coordinates of M.   (1)

5.4.   Use the graphs to solve for x if $f(x).g(x) \leq 0$.   (2)

5.5.1.   Show that the length of HL is given by $HL = -2x^2 - 10x$ 2

5.5.2.   At which value of x will HL attain its maximum value ?   1

5.5.3.   What is the maximum value of HL ?   1   (4)
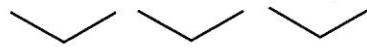
5.6.   For which value(s) of k will

$$2k - 5 = 2x^2 + 8x$$

have two distinct negative real roots ?   (3)

6.   Given :

A quadratic sequence :   _____ ; _____ ; _____ ; _____ ; ⋯

First differences   : -7 -15 -23

6.1.   Determine an expression for D_n, the general term of the
        first differences. Simplify your answer.   (2)

6.2.   What are the positions of the terms in the quadratic sequence for which
        the first difference is -1199 ?   (3)

6.3.   If the 38th term of the quadratic sequence is -5576, determine an
        expression for T_n, the general term of the quadratic sequence.   (5)

TOTAL [100]