



education

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**NATIONAL
SENIOR CERTIFICATE**

GRADE 12

MATHEMATICS P1

September 2019

MARKS: 150

TIME: 3 hours

This question paper consists of 9 pages and 1 information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 11 questions.
2. Answer ALL the questions.
3. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining the answers.
4. Answers only will not necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round off answers to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. An information sheet with formulae is included at the end of this question paper.
9. Number the answers correctly according to the numbering system used in this question paper.
10. Write neatly and legibly.

QUESTION 1

1.1 Solve for x , correct to TWO decimal places where necessary:

$$1.1.1 \quad 2x^2 - 7x + 3 = 0 \quad (3)$$

$$1.1.2 \quad 17x - 8 = 3x^2 \quad (5)$$

$$1.1.3 \quad (2x - 3)(4 - x) \geq 0 \quad (3)$$

1.2 Given: $4^{a+b} = 2^{b+4}$

$$1.2.1 \quad \text{Show that: } b = 4 - 2a \quad (2)$$

1.2.2 Hence, solve for a and b simultaneously if it is further given that

$$2a^2 - 3ab = -4 \quad (5)$$

1.3 Determine the value of $(0,04)^{0,5}$ without using a calculator. (2)

1.4 If $x = \sqrt{3 - 2\sqrt{2}}$ and $y = \sqrt{2} - 1$, prove that $y = x$ without using a calculator. (3)

[23]

QUESTION 2

2.1 Evaluate:

$$\sum_{n=3}^{20} (15 - 4n) \quad (4)$$

2.2 A water tank contains 216 litres of water at the end of day 1. Because of a leak, the tank loses one-sixth of the previous day's contents each day.

How many litres of water will be in the tank by the end of:

2.2.1 the 2nd day? (2)

2.2.2 the 7th day? (3)

2.3 Consider the geometric series: $2(3x - 1) + 2(3x - 1)^2 + 2(3x - 1)^3 + \dots$

2.3.1 For which values of x is the series convergent? (3)

2.3.2 Calculate the sum to infinity of the series if $x = \frac{1}{2}$. (4)

- 2.4 $2; x; 12; y; \dots$ are the first four terms of a quadratic sequence. If the second difference is 6, calculate the values of x and y . (5)
[21]

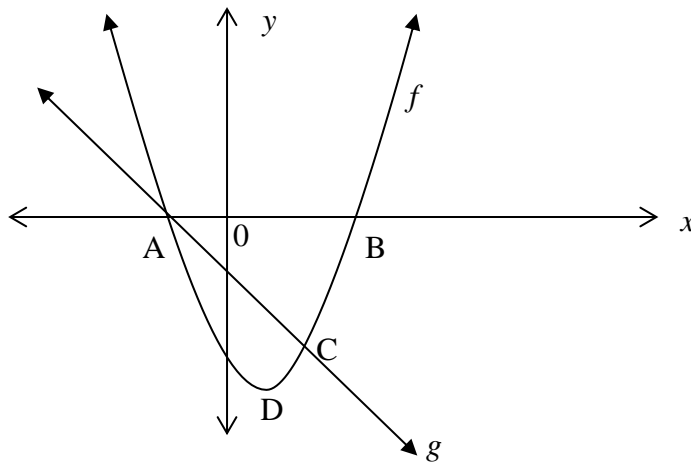
QUESTION 3

Given: $h(x) = \frac{12}{x-4} + 6$ for $x > 0$

- 3.1 Draw a neat sketch graph of h in your ANSWER BOOK. Show all intercepts with the axes and asymptotes. (4)
- 3.2 Write down the equation of k if k is the reflection of h about the x -axis. (3)
[7]

QUESTION 4

- 4.1 Sketched below are the functions: $f(x) = 2x^2 - 6x - 20$ and $g(x) = -2x + k$.



Determine:

- 4.1.1 the coordinates of turning point D. (2)
- 4.1.2 the coordinates of A and B. (3)
- 4.1.3 the value of k . (2)
- 4.1.4 the values of p if $2x^2 - 6x + p = 0$ has no real roots. (2)
- 4.1.5 for which values of x is $f(x) \cdot g(x) \leq 0$. (2)
- 4.1.6 the value of t if $y = -2x + t$ is a tangent to f . (4)

4.2 Consider the following two functions: $p(x) = x^2 + 1$ and $r(x) = x^2 + 2x$.

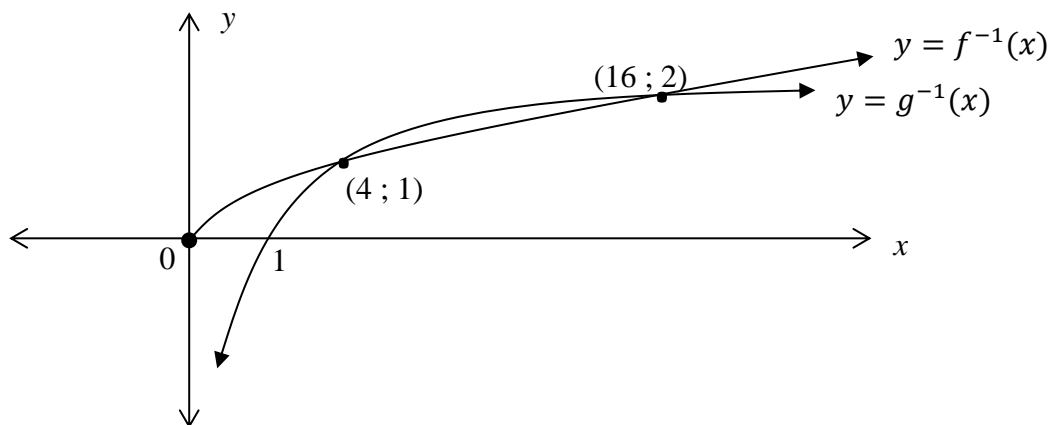
4.2.1 How will you shift p to become the function r ? (3)

4.2.2 Write down the range of p . (1)

[19]

QUESTION 5

The sketch below represents the inverses of $g(x) = 4^x$ and $f(x) = ax^2$; $x \geq 0$.



5.1 Write down the coordinates of ONE point through which both f and g will pass. (1)

5.2 Determine the equation of f . (3)

5.3 Calculate x if $g(x + 2) = 16$. (3)

5.4 If $h(x) = g^{-1}(x - 2)$, for which values of x will $h(x) \leq 0$? (2)

[9]

QUESTION 6

6.1 Convert an interest rate of 14% p.a. compounded monthly to an interest rate per annum compounded quarterly. (3)

6.2 Nelson deposits R3 500 into a savings account. Three years later he adds R5 700 to the account. The interest for the first two years is 7% p.a. compounded quarterly. The interest for the last three years is 8% p.a. compounded monthly. Calculate the balance in the savings account at the end of five years. (4)

[7]

QUESTION 7

Mr Daniels wants to take out a loan for a house over twenty years. He has approached two financial institutions and was offered two different options. The two options are shown in the table below:

VARIABLES	OPTION 1	OPTION 2
Loan amount	R950 000	R950 000
Interest rate (compounded monthly)	12%	11,8%
Repayments	R x per month	R10 328,16 per month
Bank charges	R0	R200 per month
Commissions	R6 000	R0

7.1 Determine the total cost for Option 1. (7)

7.2 Which option is the best? Provide relevant calculations to justify your answer. (3)
[10]

QUESTION 8

8.1 Given: $f(x) = 3 - x^2$. Determine $f'(x)$ from first principles. (5)

8.2 Determine $\frac{dy}{dx}$ if:

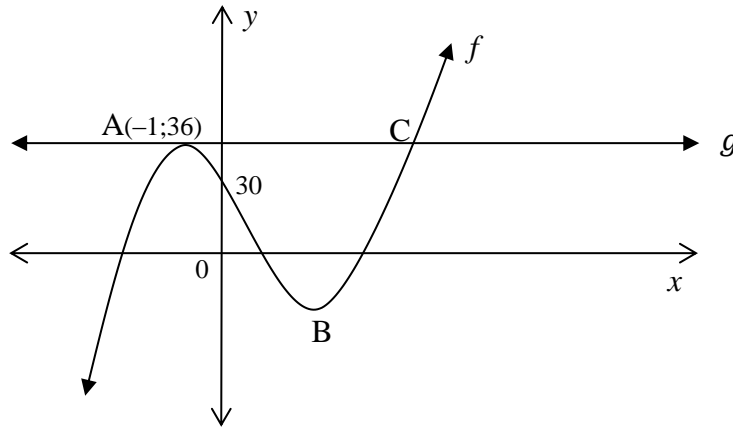
8.2.1 $y = \frac{2}{\sqrt[3]{x}} - \pi x$ (3)

8.2.2 $xy - y = x^2 - 1$ (4)

8.3 Determine the value of x where the gradient of the tangent to $f(x) = (x - 5)^2$ is equal to -8 . (4)
[16]

QUESTION 9

- 9.1 The function defined by $f(x) = x^3 + px^2 + qx + 30$ is represented by the sketch below. A $(-1; 36)$ and B are the turning points of f , while g is a tangent to f at A which cuts f at point C.

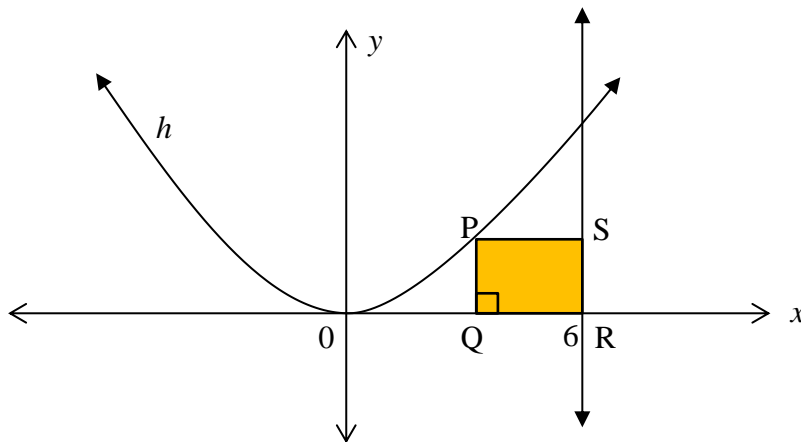


- 9.1.1 Show that $p = -4$ and $q = -11$. (7)
- 9.1.2 Determine the coordinates of C. (3)
- 9.1.3 Write down the coordinates of a turning point of k , if $k(x) = f(x) - 10$. (2)
- 9.2 The turning points of the graph of a cubic polynomial $h(x)$ are $(2; -3)$ and $(5; 4)$. Draw a sketch graph of the derivative function $h'(x)$, clearly showing the x -intercepts. (3)

[15]

QUESTION 10

PQRS is a rectangle with P on the curve $h(x) = x^2$ and with the x -axis and the line $x = 6$ as boundaries.



10.1 Show that the area of rectangle PQRS can be expressed as:

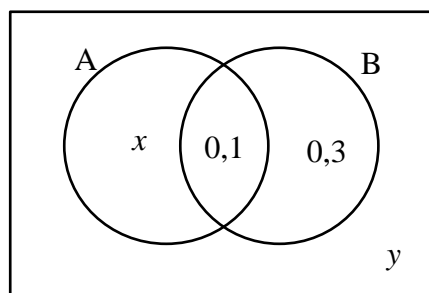
$$A = 6x^2 - x^3. \quad (3)$$

10.2 Determine the largest possible area for rectangle PQRS. Show all your calculations. (4)
[7]

QUESTION 11

11.1 The probability of getting the first answer in a quiz correct, is 60%. If the first answer is correct, the probability of getting the second answer correct, rises to 70%. However, if the first answer is wrong, the probability of getting the second answer correct, is only 40%. Determine the probability of getting the second answer correct. (4)

11.2 A and B are independent events.



Determine the values of x and y . All calculations must be shown. (5)

11.3 Ping-Pong balls are numbered 1 to 12 and placed in a hat. Every time a ball is drawn, it is placed on a rack, one next to the other.

11.3.1 How many different arrangements of the 12 balls are possible? (2)

11.3.2 Assume that the numbers 8 and 11 must be placed next to each other, in any order. In how many different ways can the numbers then be arranged? (3)

11.3.3 The numbers 2, 5, 6 and 9 are taken from the group of balls. Two-digit numbers must be formed out of these four numbers. How many two-digit numbers can be formed? (2)

[16]

TOTAL: 150

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$A = P(1 - i)^n$$

$$A = P(1 + i)^n$$

$$\sum_{i=1}^n 1 = n$$

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}$$

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}; \quad r \neq 1$$

$$S_\infty = \frac{a}{1 - r}; \quad -1 < r < 1$$

$$F = \frac{x[(1+i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

$$\text{In } \triangle ABC: \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$\text{area } \triangle ABC = \frac{1}{2} ab \cdot \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2\sin \alpha \cdot \cos \alpha$$

$$\bar{x} = \frac{\sum fx}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$