

## education

Department:
Education
PROVINCE OF KWAZULU-NATAL

## NATIONAL SENIOR CERTIFICATE

## GRADE 12



MARKS: 150
TIME: $\quad 3$ hours

This question paper consists of 13 pages, Information Sheet and an answer book with 20 pages.

## INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.
2. Answer ALL questions.
3. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.
4. Answers only will not necessarily be awarded full marks.
5. An approved scientific calculator (non-programmable and non-graphical) may be used, unless stated otherwise.
6. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. Number the answers correctly according to the numbering system used in this question paper. Write neatly and legibly.

Mathenticspro aded from Stanmbrepfysics.co1september 2019 Preparatory Examination

## QUESTION 1

1. The following information represents the amount of maize exported to other countries over 11 years in 1000 tons.

| 39 | 42 | 48 | 54 | 62 | 68 | 78 | 78 | 82 | 91 | 93 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

1.1 Calculate the mean amount of maize exported over the 11 years.
1.2 Calculate the standard deviation of the data.
1.3 Calculate the number of years that are within one standard deviation of the mean.
1.4 Draw a box and whisker diagram to represent the data.
1.5 Comment on the skewness of the data.
1.6 There was an error in the data. The mean amount of maize exported over the 11 years should increase by 1,25 thousands of tons. What impact will this error have on the:
1.6.1 yearly data provided in the above table?
1.6.2 on the interquartile range of the given data above?

## QUESTION 2

The following information (in \%) represent contributions made by the Agricultural and Mining industries in order to evaluate the GDP(GROSS DOMESTIC PRODUCT) of a certain country.

| YEAR | 2002 | 2003 | 2004 | 2005 | 2006 | 2007 | 2008 | 2009 | 2010 | 2011 | 2012 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Agriculture (x) | 4.2 | 3,4 | 3,1 | 2,7 | 2,9 | 3,0 | 2,9 | 3,0 | 2,6 | 2,5 | 2,6 |
| Mining (y) | 19,2 | 19,4 | 19,2 | 18,5 | 17,5 | 17,0 | 16,8 | 15,2 | 14,2 | 12,8 | 12,4 |

2.1 Determine the equation of the least squares regression line.
2.2 Estimate the percentage that the mining industry will contribute if the agriculture industry dropped to $1,2 \%$.
2.3 Comment on the strength of the correlation between the contributions made by these two industries. Motivate your answer.

## QUESTION 3

In the diagram, $\mathrm{K}(-1 ; 4) ; \mathrm{L}(3 ; 6) ; \mathrm{M}$ and $\mathrm{N}(4 ; 1)$ are vertices of a parallelogram. R is the midpoint of LN. P is the $x$-intercept of the line MN produced.


### 3.1 Calculate the:

3.1.1 gradient of KL.
3.1.2 coordinates of R.
3.1.3 coordinates of M.
3.2 Determine the equation of NM in the form $y=m x+c$.
3.3 Calculate the:
3.3.1 coordinates of P .
3.3.2 size of $\theta$, the inclination of PM.
3.3.3 size of $\mathrm{K} \widehat{\mathrm{P}} \mathrm{N}$.

## QUESTION 4

In the diagram, T is the centre of two concentric circles. The larger circle has equation $x^{2}+y^{2}-4 y+2 x-27=0$. The smaller circle touches the straight line $y=-x+5$ at point B . BD is a tangent to smaller circle T . D is the $x$-intercept of the straight line. C is a point on the larger circle such that TBC is a straight line.

4.1 Calculate the coordinates of T .
4.2 Show that equation of TB is given by $y=x+3$.
4.3 Calculate the coordinates of B
4.4 Determine the equation of the smaller circle in the form $(x-a)^{2}+(y-b)^{2}=r^{2}$.
4.5 Calculate the area of quadrilateral OTBD.

## QUESTION 5

5.1 Use the diagram below to calculate, without the use of a calculator, the following

5.1.1 $\tan \theta$
5.1.2 $\sin (-\theta)$
5.1.3 $\sin \left(\theta-60^{\circ}\right)$
5.2 Determine the value of the following trigonometric expression:

$$
\begin{equation*}
\frac{\tan \left(180^{\circ}-\theta\right) \sin \left(90^{\circ}+\theta\right)}{\cos 300^{\circ} \sin \left(\theta-360^{\circ}\right)} \tag{6}
\end{equation*}
$$

5.3 Consider: $\frac{\cos 2 x-1}{\sin 2 x}=-\tan x$
5.3.1 Prove the identity
5.3.2 For which value(s) of $x, 0^{\circ}<x<360^{\circ}$, is this identity undefined?
5.3.3 Hence or otherwise, find the general solution of $\frac{\sin 4 x}{\cos 4 x-1}=4$.

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## QUESTION 6

In the diagram below, the graph of $f(x)=2 \sin x$ is drawn for the interval $x \in\left[-120^{\circ} ; 180^{\circ}\right]$.

6.1 Draw on the same system of axes the graph of $g(x)=\cos \left(x+30^{\circ}\right)$, for the interval $x \in\left[-120^{\circ} ; 180^{\circ}\right]$. Show all intercepts with the axes as well as the turning and end Points of the graph.
6.2 Write down the period of $f$.
6.3 For which values of $x$ in the interval $x \in\left[-120^{\circ} ; 180^{\circ}\right]$ is:
6.3.1 The graph of $g$ decreasing?
6.3.2 $f(x) . g(x)>0$ ?
6.4 If the graph of $g$ is moved $60^{\circ}$ to the left, determine the equation of the new graph which is formed, in its simplest form.

## QUESTION 7

In the diagram, PQRS is a cyclic quadrilateral with $\mathrm{QS}=\mathrm{QR}$ and $\mathrm{PQ}=\mathrm{PS}=d$ units. $\mathrm{Q} \mathrm{PS}=\theta$.


Use the diagram to prove that:
7.1. $\mathrm{QS}=d \sqrt{2(1-\cos \theta)}$
7.2 The area of $\Delta \mathrm{QRS}=-d^{2} \sin 2 \theta(1-\cos \theta)$

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## QUESTION 8

8.1 In the diagram, ABCD is a cyclic quadrilateral in the circle centered at O . ED is a tangent to the circle at D . Chord AB is produced to F . Radii OC and OD are drawn. $\mathrm{ADE}=40^{\circ}$ and $\hat{\mathrm{C}}_{2}=65^{\circ}$,


Determine, giving reasons, the size of each of the following angles:
8.1.1 $\hat{\mathrm{D}}_{2}$
8.1.2 FB̂C
8.2 In the diagram, O is the centre of the circle RMPS. OT bisects RM with T a point on RM .
$\mathrm{P} \hat{R} M=32^{\circ}$. SP, SM and radii OP and OM are drawn. $\mathrm{OMT}=15^{\circ}$.


Calculate, with reasons, the size of the angles:
8.2.1 $\hat{\mathrm{S}}$
8.2.2 $\hat{\mathrm{O}}_{2}$
8.2.3 $\hat{\mathrm{O}}_{1}$

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## QUESTION 9

In the diagram, PV and VQ are tangents to the circle at P and $\mathrm{Q} . \mathrm{PQ}$ is produced to S and chord PR is produced to T such that VTS $\| \mathrm{RQ} . \mathrm{VQ}$ and RT intersect at $\mathrm{W} . \quad \hat{\mathrm{P}}_{1}=\hat{\mathrm{P}}_{2}=x$.


Prove that:
9.1 $\hat{\mathrm{S}}=x$
9.2 PQTV is a cyclic quadrilateral
9.3 TQ is a tangent to the circle passing through $\mathrm{Q}, \mathrm{W}$ and P

## QUESTION 10

10.1 In the diagram below, $\triangle \mathrm{ABC}$ and $\triangle \mathrm{DEF}$ are drawn with $\hat{\mathrm{A}}=\hat{\mathrm{D}}, \hat{\mathrm{B}}=\hat{\mathrm{E}}$ and $\hat{\mathrm{C}}=\hat{\mathrm{F}}$. Prove the theorem that states that equiangular triangles are similar and therefore

$$
\begin{equation*}
\frac{\mathrm{AB}}{\mathrm{DE}}=\frac{\mathrm{AC}}{\mathrm{DF}} . \tag{7}
\end{equation*}
$$



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10.2 In the diagram, DE is a tangent to the circle at E and DFG is a secant intersecting the circle at F and $\mathrm{G} . \mathrm{DE}=\mathrm{EF}=\mathrm{FG} . \mathrm{H}$ is a point on EG such that $\mathrm{FH} \| \mathrm{DE}$.

10.2.1 Determine, giving reasons, 3 angles each equal to DÊF.
10.2.2 Prove that:
a) $\triangle \mathrm{DEF}\|\| \mathrm{DGE}$
b) $\hat{\mathrm{D}}=72^{\circ}$.
10.2.3 If it is further given that $\mathrm{DF}=k$ units and $\mathrm{FG}=2$ units, prove that $k^{2}+2 k=4$.
10.2.4 Determine, giving reasons, the ratio of $\frac{\mathrm{GH}}{\mathrm{GE}}$ in terms of $k$.

## INFORMATION SHEET

$x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}$
$A=P(1+n i) \quad A=P(1-n i)$

$$
A=P(1-i)^{n}
$$

$$
A=P(1+i)^{n}
$$

$T_{n}=a+(n-1) d$ $\mathrm{S}_{n}=\frac{n}{2}[2 a+(n-1) d]$
$T_{n}=a r^{n-1}$

$$
S_{n}=\frac{a\left(r^{n}-1\right)}{r-1} ; r \neq 1
$$

$S_{\infty}=\frac{a}{1-r} ;-1<r<1$
$F=\frac{x\left[(1+i)^{n}-1\right]}{i}$
$P=\frac{x\left[1-(1+i)^{-n}\right]}{i}$
$f^{\prime}(x)=\lim _{h \rightarrow 0} \frac{f(x+h)-f(x)}{h}$
$d=\sqrt{\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2}}$
$\mathrm{M}\left(\frac{x_{1}+x_{2}}{2} ; \frac{y_{1}+y_{2}}{2}\right)$
$y=m x+c \quad y-y_{1}=m\left(x-x_{1}\right) \quad m=\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \quad m=\tan \theta$
$(x-a)^{2}+(y-b)^{2}=r^{2}$
$\operatorname{In} \triangle \mathrm{ABC}$ :

$$
\begin{aligned}
& \frac{a}{\sin \mathrm{~A}}=\frac{b}{\sin \mathrm{~B}}=\frac{c}{\sin \mathrm{C}} \\
& a^{2}=b^{2}+c^{2}-2 b c \cdot \cos A \\
& \text { area } \triangle \mathrm{ABC}=\frac{1}{2} a b \cdot \sin \mathrm{C}
\end{aligned}
$$

$$
\sin (\alpha+\beta)=\sin \alpha \cdot \cos \beta+\cos \alpha \cdot \sin \beta \quad \sin (\alpha-\beta)=\sin \alpha \cdot \cos \beta-\cos \alpha \cdot \sin \beta
$$

$$
\cos (\alpha+\beta)=\cos \alpha \cdot \cos \beta-\sin \alpha \cdot \sin \beta
$$

$$
\cos (\alpha-\beta)=\cos \alpha \cdot \cos \beta+\sin \alpha \cdot \sin \beta
$$

$\cos 2 \alpha=\left\{\begin{array}{l}\cos ^{2} \alpha-\sin ^{2} \alpha \\ 1-2 \sin ^{2} \alpha \\ 2 \cos ^{2} \alpha-1\end{array}\right.$

$$
\sin 2 \alpha=2 \sin \alpha \cdot \cos \alpha
$$

$$
\bar{x}=\frac{\sum f . x}{n}
$$

$$
\sigma^{2}=\frac{\sum_{i=1}^{n}\left(x_{i}-\bar{x}\right)^{2}}{n}
$$

$$
\mathrm{P}(\mathrm{~A})=\frac{n(\mathrm{~A})}{n(\mathrm{~S})}
$$

$$
P(A \text { or } B)=P(A)+P(B)-P(A \text { and } B) \hat{y}=a+b x
$$

$$
b=\frac{\sum(x-\bar{x})(y-\bar{y})}{\sum(x-\bar{x})^{2}}
$$



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This memorandum consists 15 of pages.

## QUESTION 1

| 39 | 42 | 48 | 54 | 62 | 68 | 78 | 78 | 82 | 91 | 93 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |



## QUESTION 2

| YEAR | 2002 | 2003 | 2004 | 2005 | 2006 | 2007 | 2008 | 2009 | 2010 | 2011 | 2012 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Agriculture (x)4.2 | 3,4 | 3,1 | 2,7 | 2,9 | 3,0 | 2,9 | 3,0 | 2,6 | 2,5 | 2,6 |  |
| Mining (y) | 19,2 | 19,4 | 19,2 | 18,5 | 17,5 | 17,0 | 16,8 | 15,2 | 14,2 | 12,8 | 12,4 |


| 2.1 | $a=5,65 \quad$ and $b=3,65$ <br> $y=5,65+3,65 x$ | A $\checkmark a-$ value <br> A $\checkmark b-$ value <br> CA $\checkmark$ equation | (3) |
| :--- | :--- | :--- | :--- |
| 2.2 | $y=5,65+3,65 x$ <br> $=5,65+3,65(1,2)$ <br> $=10,03 \%$ | A $\checkmark$ substitution of 1,2 <br> CA $\checkmark$ answer | (2) |
| 2.3 | strong positive correlation of the data <br> $r=0,7$ | CA $\checkmark$ strong positive <br> A $\checkmark$ value of correlation coefficient | $(2)$ |
|  |  |  | [7] |



## QUESTION 3




## 3.3

| 3.3.1 | $\begin{aligned} & 0=\frac{1}{2} x-1 \\ & x=2 \\ & \therefore \mathrm{P}(2 ; 0) \end{aligned}$ answer only: full marks | $\begin{aligned} & \mathrm{A} \checkmark \mathrm{y}=0 \\ & \mathrm{CA} \checkmark x \end{aligned}$ | (2) |
| :---: | :---: | :---: | :---: |
| 3.3.2 | $\begin{array}{\|ll} \hline \tan \theta=\frac{1}{2} & \text { answer only: full marks } \\ \theta=26,57^{\circ} & \\ \hline \end{array}$ | $\begin{aligned} & \text { CA } \checkmark \tan \theta=m \\ & \text { CA } \checkmark \text { answer } \end{aligned}$ | (2) |
| 3.3.3 | $\begin{aligned} & m_{K P}=\frac{0-4}{2-(-1)}=-\frac{4}{3} \\ & \tan ^{-1}\left(\frac{-4}{3}\right)=180^{\circ}-53,13^{\circ}=126,87^{\circ} \\ & \mathrm{K} \hat{\mathrm{P} N}=126,87^{\circ}-26,57^{\circ} \\ & \quad=100,3^{\circ} \end{aligned}$ | $\mathrm{CA} \checkmark m_{K P}$ <br> CA $\checkmark$ inclination of KP A $\checkmark$ method CA $\checkmark$ answer | (4) |
|  |  |  | [20] |



## QUESTION 4



\begin{tabular}{|c|c|c|c|}
\hline 4.1. \& \begin{tabular}{l}
\[
\begin{aligned}
\& x^{2}+2 x+y^{2}-4 y=27 \\
\& x^{2}+2 x+1+y^{2}-4 y+4=27+1+4 \\
\& (x+1)^{2}+(y-2)^{2}=32 \\
\& \therefore \text { T }(-1 ; 2)
\end{aligned}
\] \\
OR
\[
\begin{aligned}
\& x=\frac{2}{-2}=-1 \\
\& y=\frac{-4}{-2}=2
\end{aligned}
\]
\end{tabular} \& \begin{tabular}{l}
A \(\checkmark\) completing square LHS \\
A \(\checkmark\) LHS \\
CA \(\checkmark x\) CA \(\checkmark y\) \\
AA \(\checkmark \checkmark x\) \\
AA \(\checkmark \checkmark y\)
\end{tabular} \& (4)

(4) <br>

\hline 4.2 \& \[
$$
\begin{aligned}
& m_{T B} \times m_{B D}=-1 \quad \text { [tangent } \perp \text { radius] } \\
& m_{T B}=1 \\
& y=\mathrm{m} x+\mathrm{c} \\
& 2=1 \cdot(-1)+\mathrm{c} \\
& \therefore \mathrm{c}=3 \\
& \therefore \text { Equation of } \mathrm{TB} \text { is } y=x+3
\end{aligned}
$$

\] \& | $A \checkmark m_{T B}=1$ |
| :--- |
| A $\checkmark$ subst of $m$ andT |
| A $\checkmark$ value of $c$ | \& (3) <br>

\hline 4.3 \& equating TB and BD:

\[
$$
\begin{aligned}
& x+3=-x+5 \\
& 2 x=2 \\
& x=1 \\
& y=1+3 \text { or } y=-1+5 \\
& \therefore \mathrm{~B}(1 ; 4)
\end{aligned}
$$

\] \& | A $\checkmark$ equating |
| :--- |
| CA $\checkmark x=1$ |
| CA $\checkmark y=4$ CA provided first quadrant. | \& (3) <br>

\hline
\end{tabular}

| 4.4 | $\begin{aligned} (x+1)^{2}+(y-2)^{2} & =r^{2} \\ (1+1)^{2}+(4-2)^{2} & =r^{2} \\ 8 & =r^{2} \\ (x+1)^{2}+(y-2)^{2} & =8 \end{aligned}$ | CA $\checkmark$ subst $T$ in circle eq <br> CA $\checkmark$ subst B <br> CA $\checkmark$ equation | (3) |
| :---: | :---: | :---: | :---: |
| 4.5 | Draw a horizontal line TE with E on BD: <br> Point $\mathrm{D}(5 ; 0)$ and Point E: $2=-x+5$ $\therefore \mathrm{E}(3 ; 2)$ <br> Area of trap TEDO $=\frac{1}{2}(\mathrm{TE}+\mathrm{OD}) \times \perp h$ $=\frac{1}{2}(4+5) \times 2$ $=9 \text { units }$ <br> Area of $\triangle \mathrm{TBE}=\frac{1}{2}(\mathrm{~TB} \times \mathrm{BE})$ $\begin{aligned} & =\frac{1}{2}(\sqrt{8})(\sqrt{8}) \\ & =4 \text { units } \end{aligned}$ <br> Area of OTBD $=9+4$ $=13 \text { units }^{2}$ | A $\checkmark$ coordinates of $D$ $\mathrm{CA} \checkmark$ coordinates of E <br> CA $\checkmark$ length of TE <br> CA $\checkmark$ subst in area of trap <br> CA $\checkmark$ subst in correct area of $\Delta$ formula <br> A $\checkmark$ method CA $\checkmark$ answer |  |
|  |  |  | [20] |

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## QUESTION 5

| 5.1.1 | $\tan \theta=\frac{-\sqrt{3}}{1}=-\sqrt{3}$ | A $\square$ answer | (1) |
| :---: | :---: | :---: | :---: |
| 5.1.2 | $\begin{aligned} & r^{2}=(1)^{2}+(-\sqrt{3})^{2} \quad \text { Pyth } \\ & r^{2}=4 \\ & r=2 \\ & \begin{aligned} & \sin (-\theta)=-\sin \theta \\ & \quad=-\left(\frac{-\sqrt{3}}{2}\right) \text { or } \frac{\sqrt{3}}{2} \end{aligned} \end{aligned}$ | $\mathrm{A} \square r=2$ <br> $\mathrm{A} \square$ reduction <br> $\mathrm{CA} \square$ answer | (3) |
| 5.1.3 | $\begin{aligned} & \sin \left(\theta-60^{\circ}\right) \\ & =\sin \theta \cos 60^{\circ}-\cos \theta \sin 60^{\circ} \\ & =\left(-\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right)-\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) \\ & =\frac{-\sqrt{3}}{4}-\frac{\sqrt{3}}{4}=\frac{-2 \sqrt{3}}{4}=\frac{-\sqrt{3}}{2} \end{aligned}$ | $A \square$ expansion <br> CA $\square$ both special ratios <br> $C A \square$ value of $\cos \theta$ <br> $\mathrm{CA} \square$ answer | (4) |
| 5.2 | $\begin{aligned} & \frac{\tan \left(180^{\circ}-\theta \sin \left(90^{\circ}+\theta\right.\right.}{\cos 300^{\circ} \sin \left(\theta-360^{\circ}\right)} \\ & =\frac{-\tan \theta \cdot \cos \theta}{\cos 60^{\circ} \cdot \sin \theta} \\ & =\frac{-\frac{\sin \theta}{\cos \theta} \cdot \cos \theta}{\frac{1}{2} \cdot \sin \theta} \\ & =-2 \end{aligned}$ | $\mathrm{A} \square-\tan \theta \mathrm{A} \square \cos \theta$ <br> $\mathrm{A} \square \cos 60^{\circ} \mathrm{A} \square \sin \theta$ <br> $\mathrm{A} \square$ identity <br> CAD answer | (6) |
| 5.3.1 | $\begin{aligned} \text { LHS } & =\frac{\cos 2 x-1}{\sin 2 x} \\ & =\frac{1-2 \sin ^{2} x-1}{2 \sin x \cos x} \\ & =-\frac{\sin x}{\cos x} \\ & =-\tan x \\ & =\text { RHS } \end{aligned}$ | A $\square 1-2 s^{2} 7 x$ <br> $A \square 2 \sin x \cos x$ <br> A $\square$ simplification | (3) |
| 5.3.2 | $x=90^{\circ} ; 180^{\circ} ; 270^{\circ}$ | A $\backslash 90^{\circ}$ A $\backslash 180^{\circ}$ A $\quad 270^{\circ}$ | (3) |


| 5.3.3 | $\begin{aligned} & -\tan 2 x=\frac{1}{4} \\ & \tan 2 x=-\frac{1}{4} \\ & 2 x=165,96^{\circ}+k \cdot 180^{\circ} \\ & x=82,98^{\circ}+k \cdot 90^{\circ} \quad ; k \in Z \end{aligned}$ | A $\square \tan 2 x=-\frac{1}{4}$ <br> A $\square 165,96^{\circ}$ <br> CA $\square 82,98^{\circ}$ <br> $\mathrm{A} \square \mathrm{k} \in Z$ | (4) |
| :---: | :---: | :---: | :---: |
|  |  |  | [24] |

## QUESTION 6



## QUESTION 7



| 7.1 | $\begin{aligned} & Q S^{2}=d^{2}+d^{2}-2 d \cdot d \cdot \cos \theta \\ & Q S=2 d^{2}-2 d \cdot \cos \theta \\ & Q S=2 d^{2}(1-\cos ) \theta \\ & Q S=\sqrt{d(1-\cos ) \theta} \end{aligned}$ | A $\square$ subst into cos-rule <br> A $\square$ common factor | (2) |
| :---: | :---: | :---: | :---: |
| 7.2 |  | $\mathrm{A} \square \hat{\mathrm{Q} R}$ <br> $A \square$ subst into area rule $A \square$ simplify \& reduction | (2) |
|  |  |  | [5] |

## QUESTION 8

8.1


| 8.1.1 | $\hat{D}_{2}+\hat{\mathrm{D}}_{3}=90^{\circ}$ (tan $\perp$ radius) | $\mathrm{A} \square \mathrm{S} \mathrm{A} \square \mathrm{R}$ |  |
| :--- | :--- | :--- | :--- |
| $\therefore \hat{\mathrm{D}}_{2}=90^{\circ}-40^{\circ}=50^{\circ}$ | $\mathrm{A} \square$ answer | (3) |  |
| 8.1.2 | $\hat{\mathrm{D}}_{1}=\hat{\mathrm{C}}_{2}=65^{\circ}$ (angles opposite equal sides) | $\mathrm{A} \square \mathrm{S} / \mathrm{R}$ |  |
|  | $\hat{\mathrm{D}}_{1}+\hat{\mathrm{D}}_{2}=\mathrm{FBC}$ (ext angle of a cyclic quad) | $\mathrm{A} \square \mathrm{S} \mathrm{A} \square \mathrm{R}$ |  |
|  | $65^{\circ}+50^{\circ}=\mathrm{FB} \mathrm{C}$ |  |  |
| $\therefore \mathrm{FB} C=115^{\circ}$ | $\mathrm{CA} \square$ answer | (4) |  |


8.2


| 8.2.1 | $\hat{\mathrm{S}}=32^{\circ} \quad(\angle \mathrm{s}$ in the same segment) | $A \square S \quad A \square R$ | (2) |
| :---: | :---: | :---: | :---: |
| 8.2.2 | $\hat{\mathrm{O}}_{2}=64^{\circ}$ ( $\angle$ at centre $=2 \times \angle$ at circumference) | $\mathrm{A} \square \mathrm{S} \quad \mathrm{A} \square \mathrm{R}$ | (2) |
| 8.2.3 | $\begin{aligned} & \hline \hat{T}_{1}=90^{\circ} \quad \text { (line from centre to midpoint of chord) } \\ & \left.\hat{\mathrm{O}}_{1}+\hat{\mathrm{T}}_{1}+\hat{\mathrm{M}}_{2}=180^{\circ} \quad \text { (sum of angles of } \Delta\right) \\ & \hat{\mathrm{O}}_{1}+90+15=180 \\ & \hat{\mathrm{O}}_{1}=75^{\circ} \end{aligned}$ | A $\square \mathrm{S} / \mathrm{R}$ <br> A $\square$ Method <br> $A \square$ answer | (3) |
|  |  |  | [14] |

## QUESTION 9



| 9.1 | $\begin{aligned} & \hat{P}_{1}=\hat{Q}_{1}=x \quad \text { (tan chord theorem) } \\ & \hat{Q}_{1}=S=x \quad \text { (corresp } \angle \text { 's }(\mathrm{PQ} / / \mathrm{VS}) \\ & \hat{S}=x \end{aligned}$ | $A \square S A \square R$ <br> $A \square S A \square R$ | (4) |
| :---: | :---: | :---: | :---: |
| 9.2 | $\begin{aligned} & \hat{P}_{2}=\hat{Q}_{2} \quad \text { (tan chord theorem) } \\ & \hat{Q}_{2}=\hat{V}_{2} \quad \text { alt } \angle \text { 's }(\mathrm{PQ} / / \mathrm{VS}) \\ & \therefore \hat{P}_{2}=\hat{V}_{2} \\ & \therefore \text { PQTV is a cyclic quadrilateral (converse of angles in the same seg.) } \end{aligned}$ | $A \square S A \square R$ <br> $A \square S A \square R$ <br> A $\quad \mathrm{R}$ | (5) |
| 9.3 | $\begin{aligned} & \hat{Q}_{3}=\hat{P}_{1}(\angle \text { 's in the same segment }) \\ & \therefore \hat{Q}_{3}=\hat{P}_{2} \end{aligned}$ <br> $\therefore \mathrm{TQ}$ is a tangent (converse: tan chord theorem) | $A \square S \quad A \square R$ <br> $\mathrm{A} \square \mathrm{R}$ | (3) |
|  |  |  | [12] |



## QUESTION 10

10.1


| Construct AP = DE and AQ = DF and draw PQ | A $\square$ construction |  |
| :---: | :---: | :---: |
| In $\triangle$ APQ and $\triangle$ DEF: |  |  |
| $\mathrm{AP}=\mathrm{DE}$ |  |  |
| $\mathrm{AQ}=\mathrm{DF}$ |  |  |
| $\hat{\mathrm{A}}=\hat{\mathrm{D}}$ |  |  |
| $\therefore \triangle \mathrm{APQ} \equiv \triangle \mathrm{DEF}$ (SAS) | $\mathrm{A} \square \triangle \mathrm{APQ} \equiv \triangle \mathrm{DEF}$ |  |
| $\hat{\mathrm{P}}_{1}=\hat{\mathrm{E}}$, But $\hat{\mathrm{B}}=\hat{\mathrm{E}} \ldots$ (given) | $A \square$ SAS |  |
| $\hat{P}_{1}=\hat{B}$ |  |  |
| $\mathrm{PQ} \\| \mathrm{BC}$ (corresponding angles $=$ ) | $A \square S A \square R$ |  |
| $\therefore \frac{\mathrm{AB}}{\mathrm{AP}}=\frac{\mathrm{AC}}{\mathrm{AQ}}$ (proportionality theorem; $\mathrm{PQ} \\| \mathrm{BC}$ ) | $A \square S A \square R$ |  |
| $\therefore \frac{\mathrm{AB}}{\mathrm{DE}}=\frac{\mathrm{AC}}{\mathrm{DF}} \text { (construction) }$ |  |  |
|  |  | (7) |



| 10.2.1 | $\begin{aligned} & \hat{G} \quad \text { (tan chord theorem }) \\ & E \hat{F} H \quad(\text { alt } \angle \text { 's, DE\\|FH }) \\ & F \hat{E ̂} H=\hat{G}(\angle \text { 's opposite }=\text { sides }) \end{aligned}$ | $A \square S / R$ <br> A $\quad \mathrm{S} \square \mathrm{R}$ <br> A $\square \mathrm{S} / \mathrm{R}$ | (4) |
| :---: | :---: | :---: | :---: |
| 10.2.2(a) | In $\Delta$ 's DEF and DGE: <br> 1) $\hat{D}$ is common <br> 2) $D \hat{E} F=\hat{G}$ (proven) <br> 3) $D \hat{D E}=\hat{B E}($ sum of angles in triangle) <br> $\triangle \mathrm{DEF}\|\|\mid \triangle \mathrm{DGE}(\angle \angle \angle)$ | $\mathrm{A} \square \mathrm{S}$ <br> $\mathrm{A} \square \mathrm{S}$ $\mathrm{A} \square \mathrm{R}$ | (3) |
| 10.2.2(b) | Let $\mathrm{DEF}=x$ : <br> EFिD $=2 x$ (ext. angle of $\Delta$ ) $\begin{aligned} & \hat{\mathrm{D}}=2 x(\angle \text { s opposite }=\text { sides }) \\ & \therefore \begin{array}{c} 2 x+2 x+x=180^{\circ} \\ \quad x=36^{\circ} \end{array} \\ & \therefore \hat{\mathrm{D}}=2 x=72^{\circ} \end{aligned}$ | $A \square S A \square R$ <br> $A \square S$ <br> A $\square$ Equation <br> A $\square$ value of $x$ | (5) |



| 10.2.3 | $\begin{aligned} & \frac{\mathrm{DE}}{\mathrm{DG}}=\frac{\mathrm{DF}}{\mathrm{DE}}(\triangle \mathrm{DEF} \\|\| \| \Delta \mathrm{DGE}) \\ & \frac{\mathrm{FG}}{\mathrm{DG}}=\frac{\mathrm{DF}}{\mathrm{FG}} \quad(\mathrm{DE}=\mathrm{FG}) \\ & \frac{2}{k+2}=\frac{k}{2} \\ & k^{2}+2 k=4 \end{aligned}$ | $A \square S$ <br> A $\quad \mathrm{S}$ <br> $A \square$ substitution | (3) |
| :---: | :---: | :---: | :---: |
| 10.2.4 | $\frac{\mathrm{GH}}{\mathrm{GE}}=\frac{\mathrm{GF}}{\mathrm{GD}}=\frac{2}{k+2} \quad$ (Prop. Intercept theorem $\mathrm{DE} / / \mathrm{FH}$ ) | $A \square S \quad A \square R$ | (2) |
|  |  |  | [24] |

TOTAL: 150

