education
MPUMALANGA PROVINCE
REPUBLIC OF SOUTH AFRICA

GRADE 12

MATHEMATICS

Date: 13 April 2021

Time: 2 hours **Marks: 100**

Instructions:

Read the following instructions carefully before answering the questions.

- **This question paper consists of 7 questions in Section A and one question in Section B**
- Answer ALL the questions in **SECTION A** and **SECTION B** is optional.
- Clearly show ALL calculations, diagrams, graphs, etc. that you have used in determining your answers.
- Answers only will not necessarily be awarded full marks.
- You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
- If necessary, round off answers to TWO decimal places, unless stated otherwise.

- Diagrams are NOT necessarily drawn to scale.
- An information sheet, with formulae, is included at the end of the question paper.
- THE diagram sheet that is included at the end of the paper must be handed in with your test, with construction lines added to the diagrams where necessary.
- Number the answers correctly according to the numbering system used in this question paper.
- Write legibly and present your work neatly.

SECTION A

QUESTION 1

Given the sequence –5; 4; 21; 46; ……

1.1 Determine the general term of the above sequence. (4)

1.2 Determine T_{15} (1)

1.3 Which term in the sequence will be equal to 364? (3)

[8]

QUESTION 2

2.1 $\sum_{i=2}^{m} 32(2)^{5-i} < 500$

2.1.1 Determine the value of m for which the above-mentioned statement is true, by using the correct sum formula. (4)

2.1.2 Determine the value for $S_{\infty} - S_4$ (3)

[7]

QUESTION 3

$2x;\ x+1;\ 6-x;\ . . .$ are the first three (3) terms of an arithmetic sequence.

3.1 Determine the value for x. (2)

3.2 If $x = 4$, how many terms in the sequence add up to -575. (4)

[6]

QUESTION 4

The sum of the first n terms of a series is given by : $S_n = \frac{n}{8}(14 - 4n)$

4.1 Determine the sum of the first 25 terms of this series. (1)

4.2 Determine the value of term 25. (3)

4.3 Determine the general term of the series (5)

[9]

Question 5

5.1 If $\cos 26° = q$, write the following in terms of p:

5.1.1 $\cos 334°$ (1)

5.1.2 $\sin 52°$ (3)

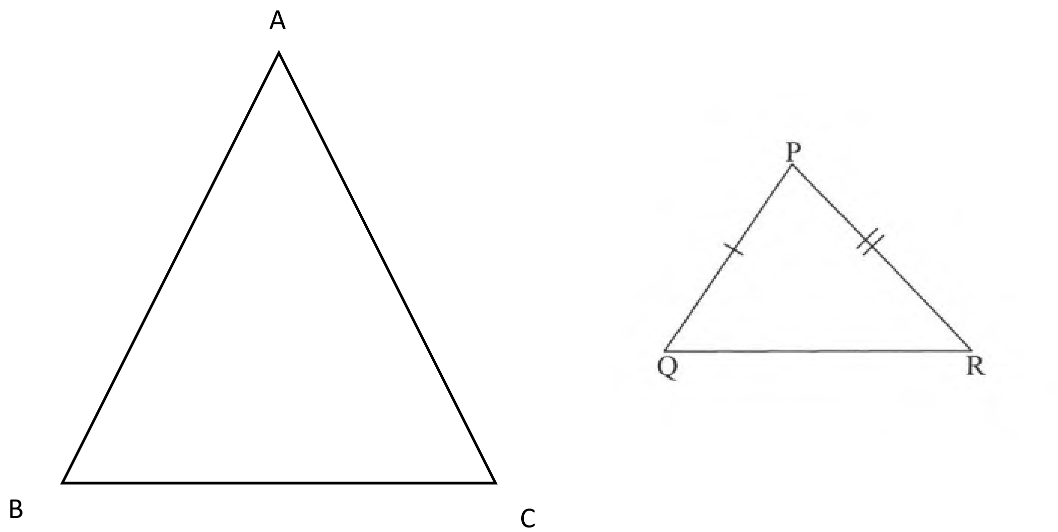
5.1.3 $\sin 86°$ (2)

[6]

Question 6

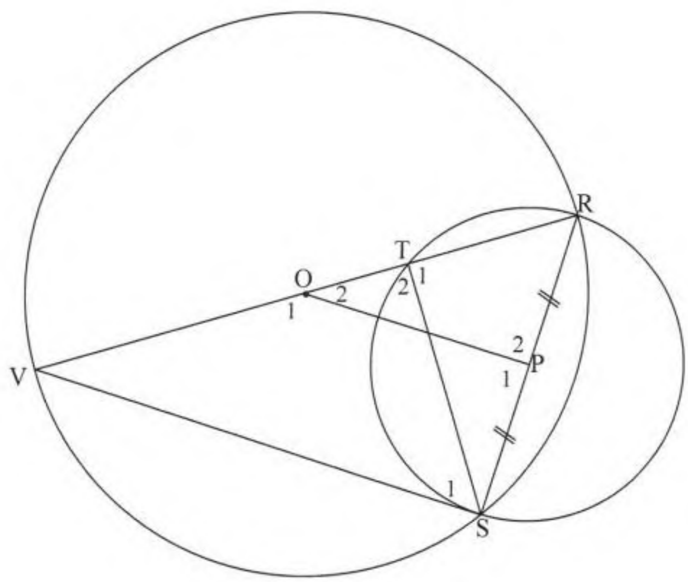
6.1 Given in the diagram below ΔABC and ΔPQR with

$\hat{A} = \hat{P}, \hat{B} = \hat{Q}$ and $\hat{C} = \hat{R}.$



Prove the theorem that states that if $\Delta ABC \,|||\, \Delta PQR$ then $\frac{AB}{PQ} = \frac{AC}{PR}$. (6)

6.2 Given in the diagram below, VR is the diameter of the circle with centre O. S is a point on the circumference. P is the midpoint of RS. The circle with RS as diameter intersects VR at T. ST, OP and SV are drawn.



6.2.1 Give a reason why $OP \perp RS$. (1)

6.2.2 Prove that $\Delta ROP \mid\mid\mid \Delta RVS$. (4)

6.2.3 Prove that $\Delta RVS \mid\mid\mid \Delta RST$. (3)

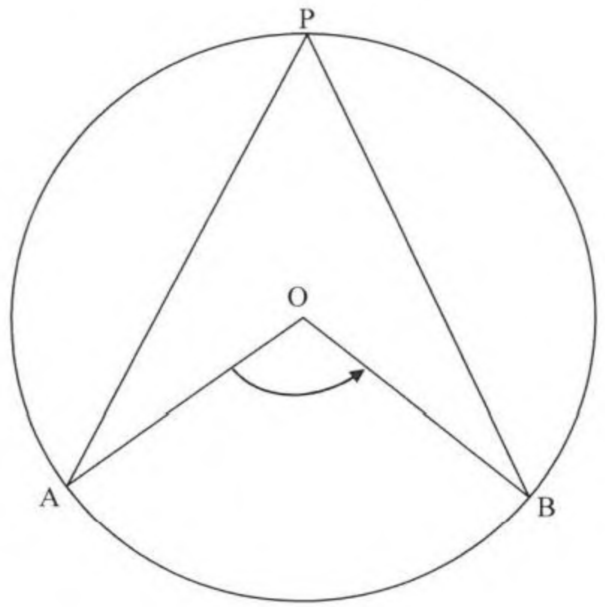
6.2.4 Prove that $ST^2 = VT.TR$ (5)

[19]

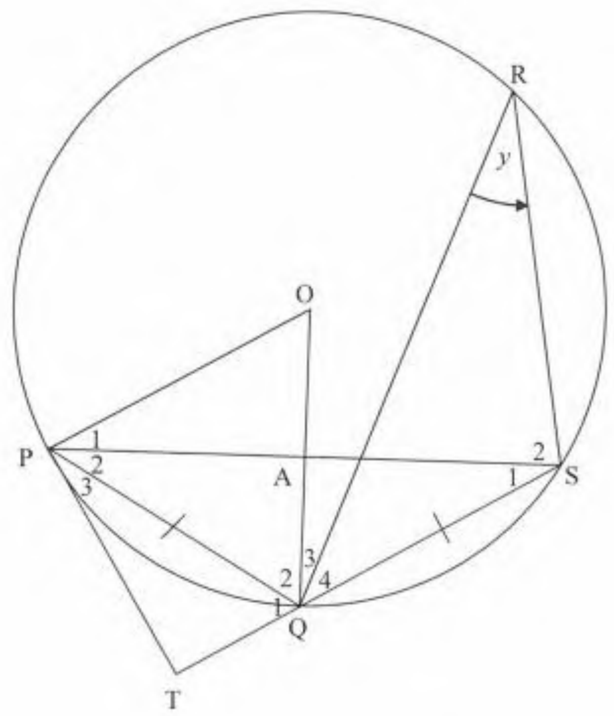
QUESTION 7

7.1 In the diagram below, O is the centre of the circle and P is a point on the circumference of the circle. Arc AB subtends $A\hat{O}B$ at the centre of the circle and $A\hat{P}B$ at the circumference of the circle.

Use the diagram to prove the theorem that states that $A\hat{O}B = 2A\hat{P}B$ (5)



7.2 In the diagram, O is the centre of the circle and P, Q, S and R are points on the circle. PQ = QS and $Q\hat{R}S = y$. The tangent PT at P meets SQ produced at T. OQ intercepts PS at A.



7.2.1 Give a reason why $\hat{P}_2 = y$. (1)

7.2.2 Prove that PQ bisects $T\hat{P}S$. (4)

7.2.3 Determine $P\hat{O}Q$ in terms of y. (2)

7.2.4 Prove that PT is a tangent to the circle that passes through P, O and A. (2)

7.2.5 Prove that $O\hat{A}P = 90°$. (4)

[18]

Total Section A: 73 marks

SECTION B: OPTIONAL

QUESTION 8

8.1 Calculate the following without using calculator:

81.1 $\sin 236°.\cos 169° + \sin 371.\cos(-124°)$ (4)

8.1.2 $\dfrac{-\cos 10° + \sin^2 190°}{\cos(-145°).\cos 235°}$ (6)

8.2 Prove the following identities:

8.2.1 $\dfrac{\cos 2A + \sin A}{\cos^2 A} = \dfrac{2\sin A + 1}{1 + \sin A}$ (5)

8.2.2 $\dfrac{\sin(x + 45°)}{\cos(x - 45°)} = \dfrac{\sin 2x + 1}{(\sin x + \cos x)^2}$ (6)

8.3 Determine the general solution for:

8.3.3 $2\sin(3x - 15°) + 1 = 0$ (4)

8.3.4 Hence determine all possible values for x,

If $x \in [-270°; 90°]$ (2)

[27]

Total Section B: 27 marks

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $A = P(1-ni)$ $A = P(1-i)^n$ $A = P(1+i)^n$

$T_n = a + (n-1)d$ $S_n = \frac{n}{2}\left(2a + (n-1)d\right)$

$T_n = ar^{n-1}$ $S_n = \frac{a\left(r^n - 1\right)}{r-1}$; $r \neq 1$ $S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $P = \frac{x[1-(1+i)^{-n}]}{i}$

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ M$\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $y - y_1 = m(x - x_1)$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $m = \tan\theta$

$(x-a)^2 + (y-b)^2 = r^2$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $a^2 = b^2 + c^2 - 2bc.\cos A$ $area\ \Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum fx}{n}$ $\sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

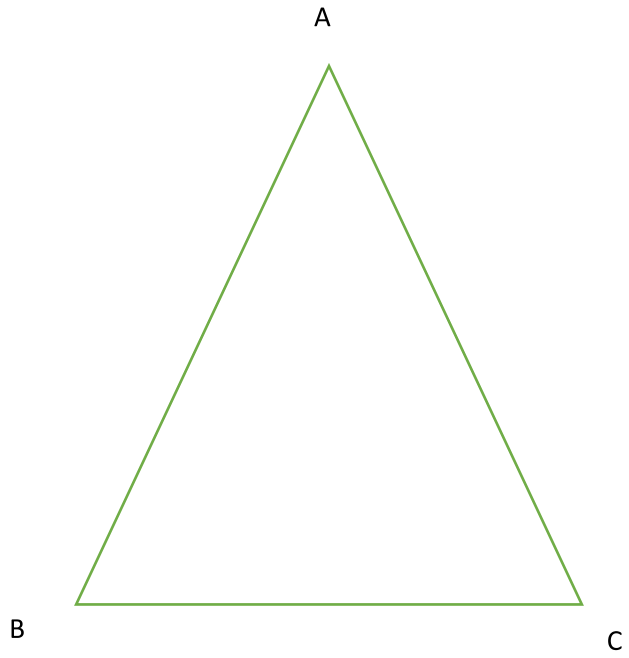
$P(A) = \frac{n(A)}{n(S)}$ $P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$

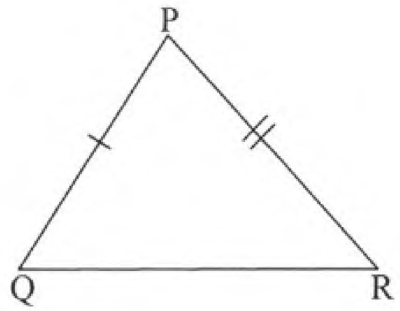
$\hat{y} = a + bx$ $b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$

Diagram Sheet

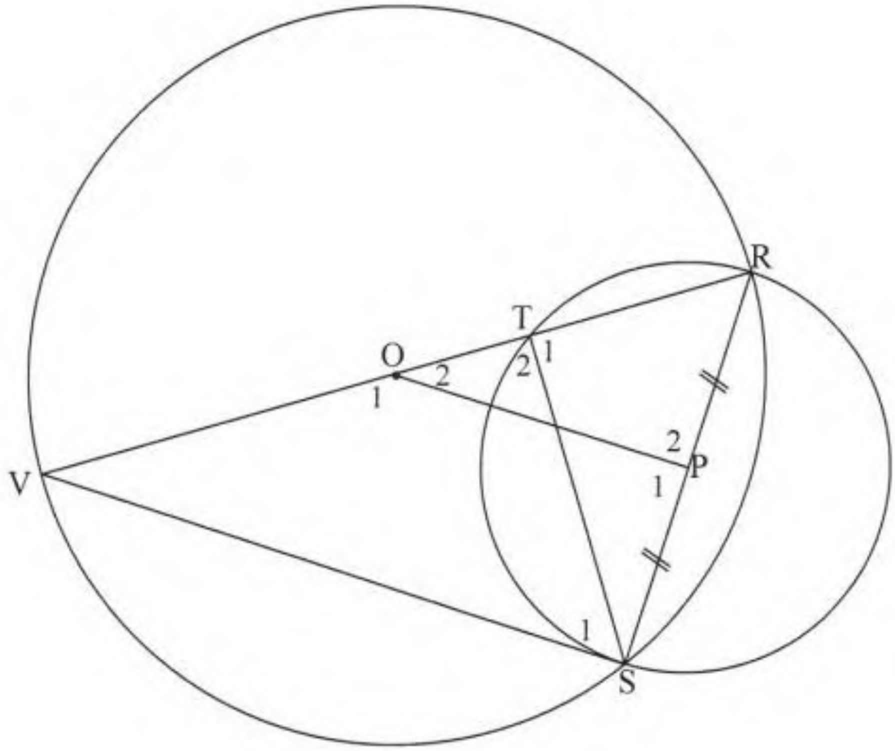
Question 6

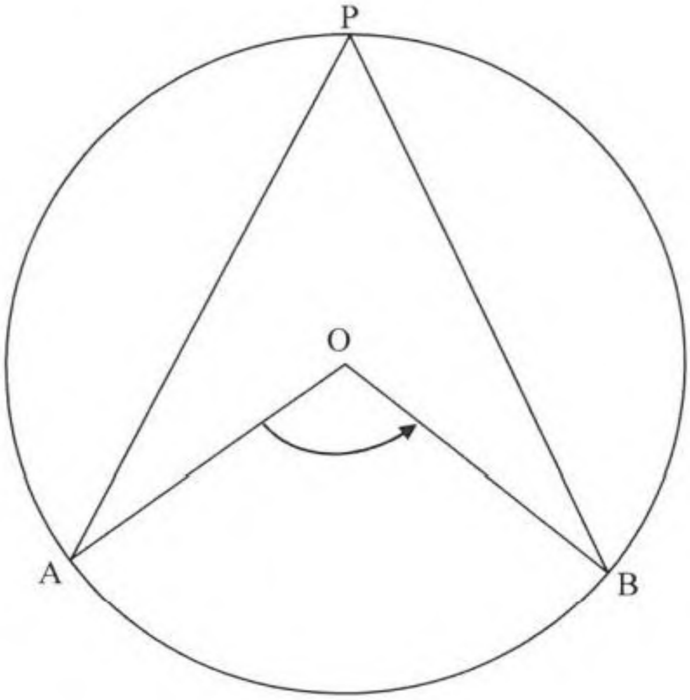
6.1





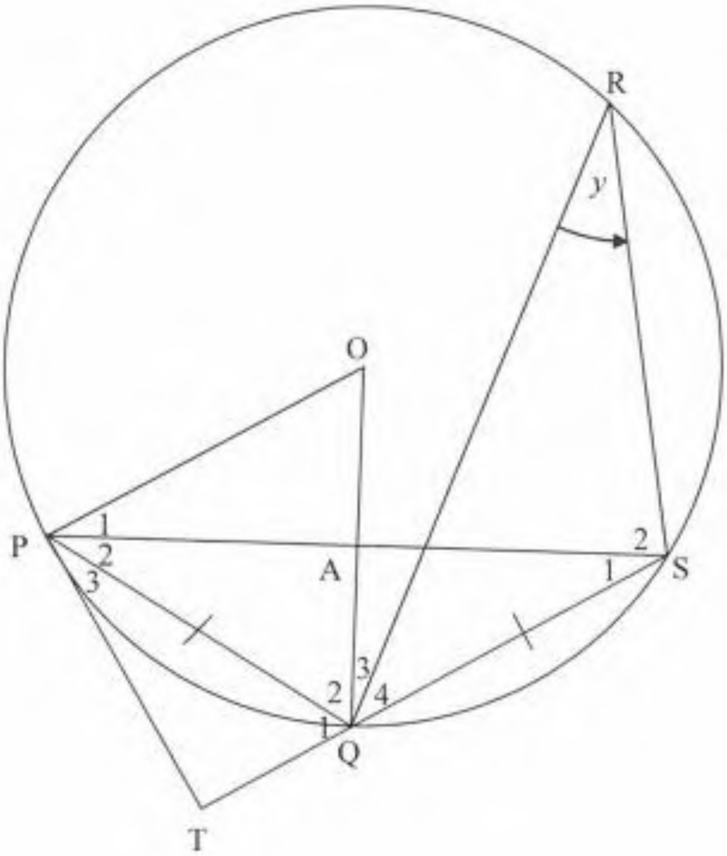
6.2





7.1

7.2



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NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS TEST

TERM 1

MARKING GUIDELINE

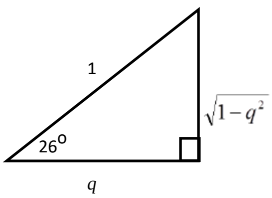
2021

Time: 1,5 hours **Marks: 73 Section A**

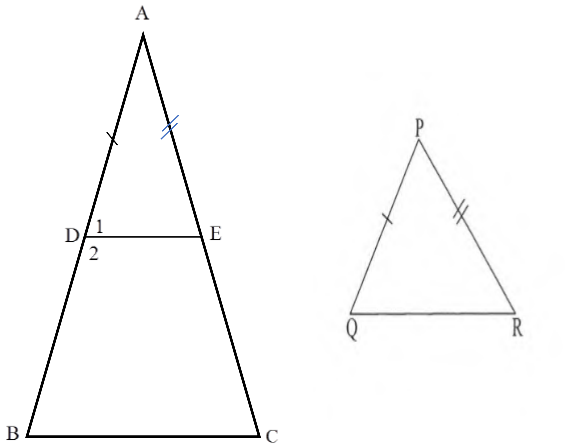
Time: 2 hours **Marks: 100 Section A and B**

QUESTION 1		
1.1	$-5; \quad 4; \quad 21; \quad 46;.....$ $9 \quad 17 \quad 25$ $8 \quad 8$ $2a=8$ $\therefore a=4$ $3a+b=9$ $3(4)+b=9$ $\therefore b=-3$ $a+b+c=-5$ $4-3+c=-5$ $\therefore c=-6$ $T_n=4n^2-3n-6$	✓ $a=4$ ✓ $b=-3$ ✓ $c=-6$ ✓ T_n (4)
1.2	$T_{(15)}=4(15)^2-3(15)-6$ $=849$	✓answer (1)
1.3	$364=4n^2-3n-6$ $4n^2-3n-370=0$ $(4n+37)(n-10)=0$ $n=\frac{-37}{4}$ or $n=10$ $\therefore n=10$	✓ $360=4n^2-3n-6$ ✓ $n=\frac{-37}{4}$, NA ✓ $\therefore n=10$ (3)
		[8]
QUESTION 2		
2.1.1	$\sum_{i=2}^{m} 32(2)^{5-i}<500$ $256+128+64+\ldots<500$ $S_n=\frac{a(1-r^m)}{1-r}$ $\frac{256\left(1-\frac{1}{2}^m\right)}{1-\frac{1}{2}}<500$ $\frac{512\left(1-\frac{1}{2}^m\right)}{1}<500$	✓ $\frac{256\left(1-\frac{1}{2}^m\right)}{1-\frac{1}{2}}<500$ ✓correct use of logs ✓ $\therefore m>5.4$ ✓ $m=6$ (4)

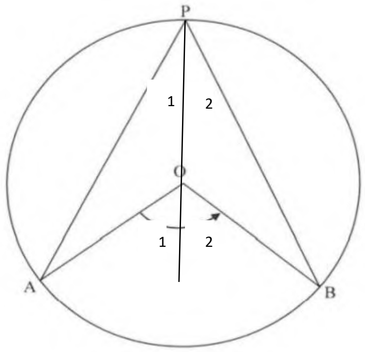
	$1-\frac{1}{2}^{m} < \frac{125}{128}$ $\frac{1}{2}^{m} > \frac{3}{128}$ $m > \log_{\frac{1}{2}} \frac{3}{128}$ $m > 5.4$ $\therefore m = 6$	
2.1.2	$S_\infty - S_4 = \frac{a}{1-r} - \frac{a(1-r^n)}{1-r}$ $= \frac{256}{1-\frac{1}{2}} - \frac{256\left(1-\frac{1}{2}^{4}\right)}{1-\frac{1}{2}}$ $= 512 - 480$ $= 32$	✓ $S_\infty - S_4$ ✓substitution ✓answer (3)
		[7]
	QUESTION 3	
3.1	$2x;\ x+1;\ 6-x;\ \ldots$ $x+1-2x = 6-x-(x+1)$ $-+1 = -2x+5$ $x = 4$	✓ $T_2 - T_1 = T_3 - T_2$ ✓answer (2)
3.2	$8;\ 5;\ 2;\ \ldots$ $-575 = \frac{n}{2}[2(8)+(n-1)(-3)]$ $-1150 = n(19-3n)$ $3n^2 - 19n - 1150 = 0$ $(3n+50)(-23) = 0$ $n = 23 \ \ or \ \ n \neq -\frac{50}{3}$	✓ substitution of S_n ✓substitution of a and d ✓standard form ✓ $n = 23$ only (4)
		[6]

QUESTION 4		
4.1	$S_{25}=\frac{25}{8}[14-4(25)]$ $=-268\frac{3}{4}$	✓answer (1)
4.2	$T_{25}=S_{25}-S_{24}$ $T_{25}=-268\frac{3}{4}-\left(\frac{24}{8}[14-4(24)]=22\frac{3}{4}\right)$	✓method ✓substitution ✓answer (3)
4.3	$S_1=T_1=\frac{1}{8}[14-4(1)]=\frac{5}{4}$ $T_2=\frac{2}{8}[14-4(2)]-\frac{5}{4}=\frac{1}{4}$ $T_3=\frac{3}{8}[14-4(3)]-\frac{3}{2}=-\frac{3}{4}$ $5;\ 1;\ -3;\ \rightarrow T_n=9-4n$ $\frac{5}{4};\frac{1}{4};-\frac{3}{4};\ \rightarrow\ T_n=\frac{9-4n}{4}$	✓$S_1=T_1=\frac{5}{4}$ ✓$T_2=\frac{1}{4}$ ✓$T_3=-\frac{3}{4}$ ✓$T_n=9-4n$ ✓$T_n=\frac{9-4n}{4}$ (5)
		[9]
QUESTION 5		
5.1.1	$\cos 334°=\cos(360°-26°)$ $=\cos 26°$ $=q$	✓answer (1)
5.1.2	 $\sin 52°=\sin 2(26°)$ $=2\sin 26°\cos 26°$ $=2.\sqrt{1-q^2}(q)$ $=2q\sqrt{1-q^2}$	✓diagram ✓identity ✓answer (3)
5.1.3	$\sin 86°=\sin(60°+26°)$ $=\sin 60°\cos 26°+\cos 60°\sin 26°$ $=\frac{\sqrt{3}}{2}.q+\frac{1}{2}\sqrt{1-q^2}$	✓identity ✓answer (2)

		[6]

QUESTION 6		
6.1	 Constructions: Draw PQ on AB such that PQ =AD Draw PR on AE such that PR=AE In ΔADE *and* ΔPQR 1. AD = PQ [construction] 2. AE = PR [construction] 3. $\hat{A} = \hat{P}$ [given] $\Delta ADE \equiv \Delta PQR$ [$S\angle S$] $\hat{D}_1 = \hat{Q}$ [from congruency] But $\hat{D}_1 = \hat{B}$ $\therefore \hat{D}_1 = \hat{B}$ $\therefore$ DE//BC [*corrsponding* $\angle S$ *equal*] $\frac{AB}{AD} = \frac{AC}{AE}$ [line || side of Δ] But AD = PQ and AE = PR [construction] $\frac{AB}{PQ} = \frac{AC}{PR}$	✓Construction ✓S/R ✓S/R ✓$\therefore \hat{D}_1 = \hat{B}$ ✓S/R ✓S/R (6)

6.2.1	Line from centre to midpoint of chord	✓Reason (1)
6.2.2	In ΔROP and ΔRVS 1. $\hat{R} = \hat{R}$ [common] 2. $\hat{S}_1 = 90°$ [angle in semi-circle] $\hat{P}_2 = 90°$ (proven) $\hat{S}_1 = \hat{P}_2$ 3. $\hat{V} = \hat{O}_2$ (angles in Δ) $\Delta ROP \mathbin{\vert\vert\vert} \Delta RVS$ [∠;∠;∠]	✓S/R ✓R ✓S ✓R (4)
6.2.3	In ΔRVS and ΔRST 1. $\hat{R} = \hat{R}$ (common) 2. $\hat{T}_1 = \hat{S}_1 = 90°$ (angle in semi circle) 3. $T\hat{S}R = \hat{V}$ (∠s in Δ) $\Delta RVS \mathbin{\vert\vert\vert} \Delta RST$ (∠;∠;∠)	✓S ✓R ✓R (3)
6.2.4	In ΔSTV and ΔRST $R\hat{T}S = V\hat{T}S = 90°$ (Angles on straight line) $\hat{R} = 90° - T\hat{S}R$ $= T\hat{S}V$ $T\hat{S}R = \hat{V}$ (angles in Δ) $\Delta RST \mathbin{\vert\vert\vert} \Delta STV$ ([A,A,A) $\frac{RT}{ST} = \frac{TS}{VT}$ (from similarity) $ST^2 = VT.TR$	✓S ✓R ✓S ✓R ✓S (5)
		[19]

QUESTION 7		
7.1	 Construction: Draw PO extended OP = OA (radii) $\hat{P}_1 = \hat{A}$ (angles opp. equal sides) But $\hat{O}_1 = \hat{P}_1 + \hat{A}$ (ext. angle of triangle) $\hat{O}_1 = 2\hat{P}_1$ Similarly $\hat{O}_2 = 2\hat{P}_2$ $A\hat{O}B = 2A\hat{P}B$	✓Construction ✓S/R ✓S/R ✓S ✓S (5)
7.2.1	Angles in the same segment	✓answer (1)
7.2.2	$\hat{P}_2 = \hat{S}_1 = y$ (angles opp equal sides) $\hat{S}_1 = \hat{P}_3 = y$ (tan cord theorem) $\hat{P}_2 = \hat{P}_3$ PQ bisects $T\hat{P}S$	✓S ✓R ✓S ✓R (4)
7.2.3	$P\hat{O}Q = 2\hat{S}_1 = 2y$ (∠at centre = 2 ∠ at circumference)	✓S ✓R (2)
7.2.4	$T\hat{P}A = \hat{P}_2 + \hat{P}_3$ (proven) $T\hat{P}A = P\hat{Q}O$ (proven) PT is a tangent (converse theorem tan cord)	✓S ✓R (2)
7.2.5	$O\hat{P}Q + O\hat{Q}P = 180°\text{-}2y$ (angles of triangle) $O\hat{Q}P = 90°\text{-}y$ (angles opp equal sides) $90° - y + y + Q\hat{A}P = 180°$ $Q\hat{A}P = 90°$	✓S ✓R ✓S/R ✓S (4)
		[18]

OPTIONAL:

QUESTION 8

8.1.1	$\sin 236°.\cos 169° + \sin 371.\cos(-124°)$ $= -\sin 56°(-\cos 11°) + \sin 11°.(-\cos 56°)$ $= \sin(56° - 11°)$ $= \sin 45°$ $= \frac{1}{\sqrt{2}}$	✓ $-\sin 56°(-\cos 11°)$ ✓ $\sin 11°.(-\cos 56°$ ✓ $\sin 45°$ ✓ $\frac{1}{\sqrt{2}}$ (4)
8.1.2	$\frac{-\cos^2 10° + \sin^2 190°}{\cos(-145°).\cos 235°}$ $= \frac{-\cos^2 10° + \sin^2(180° + 10°)}{\cos(180 - 35°).\cos(270° - 35)}$ $= \frac{-\cos^2 10° + \sin^2 10°}{-\cos 35°.(-\sin 35°)}$ $= \frac{-(\cos^2 10° - \sin^2 10°)}{\cos 35° \sin 35°}$ $= \frac{-\cos 2 \times 10°}{\cos 35° \sin 35°}$ $= \frac{-\cos 20°}{\cos 35° \sin 35°}$ $= \frac{-2\cos 20°}{2\cos 35° \sin 35°}$ $= \frac{-2\cos 20°}{\sin 2 \times 35°}$ $= \frac{-2\sin 70°}{\sin 70°}$ $= -2$	✓ $+\sin^2 10°$ ✓ $-\cos 35°$ ✓ $-\sin 35°$ ✓ $-\cos 20°$ ✓ $-2\sin 70°$ ✓ $\sin 70°$ (6)
8.2.1	$\frac{\cos 2A + \sin A}{\cos^2 A} = \frac{2\sin A + 1}{1 + \sin A}$ LHS: $\frac{\cos 2A + \sin A}{\cos^2 A} = \frac{1 - 2\sin^2 A + \sin A}{1 - \sin^2 A}$ $= \frac{1 + \sin A - 2\sin^2 A}{1 - \sin^2 A}$	✓ $1 - 2\sin^2 A$ ✓ $1 - \sin^2 A$

	$=\frac{(1+2\sin A)(1-\sin A)}{(1-\sin A)(1+\sin A)}$ $=\frac{1+2\sin A}{(1+\sin A)}$ $\therefore LHS=RHS$	✓factorise numerator ✓factorise denominator (4)
8.2.2	$\frac{\sin(x+45°)}{\cos(x-45°)}=\frac{\sin 2x+1}{(\sin x+\cos x)^2}$ LHS: $\frac{\sin(x+45°)}{\cos(x-45°)}=\frac{\sin x\cos 45°+\cos c\sin 45°}{\cos x\cos 45°+\sin x\sin 45°}$ $=\frac{\sin x.\frac{\sqrt{2}}{2}+\cos x\frac{\sqrt{2}}{2}}{\cos x\frac{\sqrt{2}}{2}+\sin x\frac{\sqrt{2}}{2}}$ $=1$ **RHS:** $\frac{\sin 2x+1}{(\sin x+\cos x)^2}=\frac{\sin 2x+1}{\sin^2 x+\sin x\cos x+\cos^2 x}$ $=\frac{\sin 2x+1}{1+2\sin x\cos x}$ $=\frac{\sin 2x+1}{1+\sin 2x}$ $=1$ $\therefore LHS=RHS$	✓ $\sin x\cos 45°+\cos c\sin 45°$ ✓ $\cos x\cos 45°+\sin x\sin 45°$ ✓Substituting $\frac{\sqrt{2}}{2}$ ✓1 ✓simplifying denominator ✓square identity ✓1 (7)
8.3.1	$2\sin(3x-15°)+1=0$ $\sin(3x-15°)=-\frac{1}{2}$ Ref angle: $x=30°$ **<u>3rd</u>** $3x-15°=180°+30°+k.360°;\quad k\in Z$ $3x=225°+k.360°;\quad k\in Z$ $x=75°+k.120°;\quad k\in Z$ **4th** $3x-15°=360°-30°+k.360°;\quad k\in Z$ $3x=345°+k.360°;\quad k\in Z$	✓ $\sin(3x-15°)=-\frac{1}{2}$ ✓ $x=75°+k.120°$ ✓ $k\in Z$ ✓ $x=115°+k.120°$ (4)

	$x = 115° + k.120°; \quad k \in Z$	
8.3.2	$x \in \{-245°; -165°; -125°; -45°; -5°, 75°\}$	✓ three correct ✓six correct (2)