basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 11

MATHEMATICS P2

PAST PAPER QUESTIONS ORGANISED BY TOPIC

October 2019 Edition

CONTENTS

All exam questions in this document have been extracted from Department of Education exam papers.
For any queries or suggestions for improvement regarding this resource please contact Mr C Borchers:
Email: ccborchers@gmail.com
Cell: 0615472363

PROGRESS TRACKER

Statistics	Attempts	DBE Nov 18 Q1		DBE Nov 18 Q2		DBE Nov 17 Q1		DBE Nov 17 Q2		DBE Nov 16 Q1		DBE Nov 16 Q2		DBE Nov 15 Q1		DBE Nov 15 Q2	
		[10]	%	[13]	%	[10]	%	[12]	%	[12]	%	[9]	%	[14]	%	[8]	%
	1st																
	2nd																
	3rd																
	Attempts	DBE Nov 14 Q1		DBE Nov 14 Q2		DBE Nov 13 Q1		DBE Nov 13 Q2		Exemplar 13 Q1		Exemplar 13 Q2					
		[10]	%	[10]	%	[15]	%	[6]	%	[14]	%	[9]	%				
	1st																
	2nd																
	3rd																

Analytic Geometry	Attempts	DBE Nov 18 Q3		DBE Nov 18 Q4		DBE Nov 17 Q3		DBE Nov 17 Q4		DBE Nov 16 Q3		DBE Nov 16 Q4		DBE Nov 15 Q3		DBE Nov 15 Q4	
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	2nd																
	3rd																
	Attempts	DBE Nov 14 Q3		DBE Nov 14 Q4		DBE Nov 13 Q3		DBE Nov 13 Q4		Exemplar 13 Q3		Exemplar 13 Q4					
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	1st																
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	3rd																

Trigonometry: Equations and Identities	Attempts	DBE Nov 18 Q5		DBE Nov 17 Q5		DBE Nov 16 Q5		DBE Nov 15 Q5		DBE Nov 14 Q5		DBE Nov 13 Q5		Exemplar 13 Q5	
		[28]	%	[26]	%	[31]	%	[24]	%	[33]	%	[26]	%	[26]	%
	1st														
	2nd														
	3rd														

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Trigonometry: Functions	Attempts	DBE Nov 18 Q6		DBE Nov 17 Q6		DBE Nov 16 Q6		DBE Nov 15 Q6		DBE Nov 14 Q6		DBE Nov 13 Q6		Exemplar 13 Q6	
		[11]	%	[16]	%	[9]	%	[21]	%	[11]	%	[10]	%	[9]	%
	1st														
	2nd														
	3rd														

Trigonometry: Sine, Cosine and & Rule	Attempts	DBE Nov 18 Q7		DBE Nov 17 Q6		DBE Nov 16 Q6		DBE Nov 15 Q6		DBE Nov 14 Q7.1		DBE Nov 13 Q6		Exemplar 13 Q6	
		[13]	%	[12]	%	[16]	%	[7]	%	[8]	%	[18]	%	[17]	%
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	2nd														
	3rd														

Measurement	Attempts	DBE Nov 18 Q8		DBE Nov 17 Q8		DBE Nov 16 Q8		DBE Nov 15 Q8		DBE Nov 14 Q7.2		DBE Nov 13 Q8		Exemplar 13 Q8	
		[7]	%	[6]	%	[6]	%	[8]	%	[6]	%	[5]	%	[6]	%
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	3rd														

Euclidean Geometry	Attempts	DBE Nov 18 Q9		DBE Nov 18 Q10		DBE Nov 18 Q11		DBE Nov 17 Q9		DBE Nov 17 Q10		DBE Nov 17 Q11		DBE Nov 16 Q9		DBE Nov 16 Q10	
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	2nd																
	3rd																
	Attempts	DBE Nov 16 Q10		DBE Nov 16 Q11		DBE Nov 16 Q12		DBE Nov 15 Q9		DBE Nov 15 Q10		DBE Nov 15 Q11		DBE Nov 14 Q8		DBE Nov 14 Q9	
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	1st																
	2nd																
	3rd																
	Attempts	DBE Nov 14 Q10		DBE Nov 13 Q9		DBE Nov 13 Q10		DBE Nov 13 Q11		DBE Nov 13 Q12		Exemplar 13 Q9		Exemplar 13 Q10		Exemplar 13 Q11	
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	3rd																

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QUESTION 1

A school held a sports day. One of the items on the programme was an obstacle race. Teams of 10 parents and learners participated in this race. The table below shows the time taken, in minutes, by each member of a particular team to complete the race.

4	12	13	16	17	18	20	22	22	25

1.1 How long, in minutes, did it take for the fastest member of this team to complete the race? (1)

1.2 Determine the mean time taken by this team. (2)

1.3 Calculate the standard deviation for the data. (1)

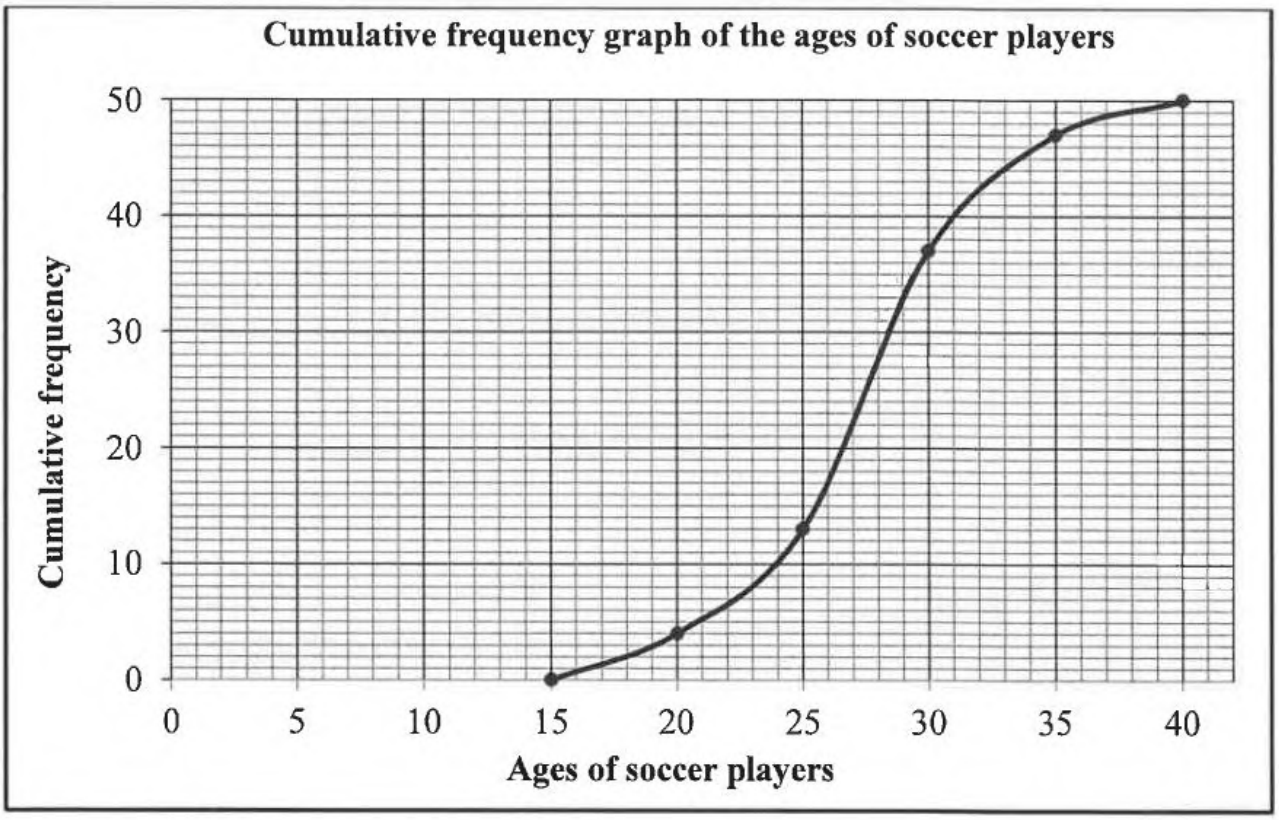
1.4 How many members of the team completed the obstacle race outside of two standard deviations of the mean? (3)

1.5 It took another team a total time of $x+5$ minutes to complete the race. Calculate the value of x if the overall mean of the two teams combined was 18 minutes. (3)

[10]

QUESTION 2

2.1 A survey was conducted of the ages of players at a soccer tournament. The results are shown in the cumulative frequency graph (ogive) below.



2.1.1 How many players took part in the soccer tournament? (1)

2.1.2 Determine the number of players between the ages of 24 and 31 years old. (2)

2.1.3 Complete the frequency column of the table below in the ANSWER BOOK.

CLASS INTERVAL	FREQUENCY	CUMULATIVE FREQUENCY
$15 \leq x < 20$		4
$20 \leq x < 25$		13
$25 \leq x < 30$		37
$30 \leq x < 35$		47
$35 \leq x < 40$		50

(3)

2.1.4 Use the grid provided in the ANSWER BOOK to draw a frequency polygon for the data. (4)

2.2 Two Grade 11 Mathematics classes have the same number of learners. The five-number summaries of the marks obtained by these classes for a test are shown below.

CLASS A (30 ; 48 ; 65 ; 82 ; 90)

CLASS B (50 ; 58 ; 65 ; 75 ; 90)

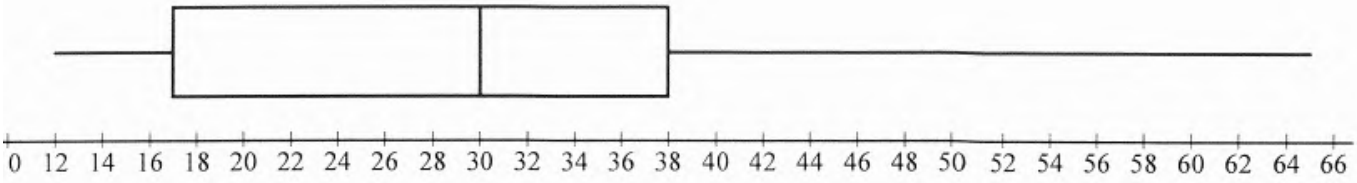
The parents of learners in CLASS A and CLASS B observe that both classes have the same median and the same maximum mark and therefore claim that there is no difference in the performance between these classes.

Do you agree with this claim? Use at least TWO different arguments to justify your answer. (3)

[13]

QUESTION 1

1.1 Mr Brown conducted a survey on the amount of airtime (in rands) EACH student had on his or her cellphone. He summarised the data in the box and whisker diagram below.



1.1.1 Write down the five-number summary of the data. (2)

1.1.2 Determine the interquartile range. (1)

1.1.3 Comment on the skewness of the data. (1)

1.2 A group of 13 students indicated how long it took (in hours) before their cellphone batteries required recharging. The information is given in the table below.

5	8	10	17	20	29	32	48	50	50	63	y	107

1.2.1 Calculate the value of y if the mean for this data set is 41. (2)

1.2.2 If $y = 94$, calculate the standard deviation of the data. (1)

1.2.3 The mean time before another group of 6 students needed to recharge the batteries of their cellphones was 18 hours. Combine these groups and calculate the overall mean time needed for these two groups to recharge the batteries of their cellphones. (3)

[10]

QUESTION 2

A student conducted a survey among his friends and relatives to determine the relationship between the age of a person and the number of marketing phone calls he or she received within one month. The information is given in the table below.

AGE OF PERSON IN SURVEY	FREQUENCY	CUMULATIVE FREQUENCY
$20 < x \leq 30$	7	7
$30 < x \leq 40$		27
$40 < x \leq 50$	25	
$50 < x \leq 60$		64
$60 < x \leq 70$		72
$70 < x \leq 80$	4	
$80 < x \leq 90$		80

2.1 Complete the frequency and cumulative frequency columns in the table given in the ANSWER BOOK. (4)

2.2 How many people participated in this survey? (1)

2.3 Write down the modal class. (1)

2.4 Draw an ogive (cumulative frequency graph) to represent the data on the grid given in the ANSWER BOOK. (3)

2.5 Determine the percentage of marketing calls received by people older than 54 years. (3)

[12]

QUESTION 1

The table below shows the number of cans of food collected by 9 classes during a charity drive.

5	8	15	20	25	27	31	36	75

1.1 Calculate the range of the data. (1)

1.2 Calculate the standard deviation of the data. (2)

1.3 Determine the median of the data. (1)

1.4 Determine the interquartile range of the data. (3)

1.5 Use the number line provided in the ANSWER BOOK to draw a box and whisker diagram for the data above. (3)

1.6 Describe the skewness of the data. (1)

1.7 Identify outliers, if any exist, for the above data. (1)

[12]

QUESTION 2

The table below shows the time (in minutes) that 200 learners spent on their cellphones during a school day.

TIME SPENT (IN MINUTES)	FREQUENCY
$95 < x \leq 105$	15
$105 < x \leq 115$	27
$115 < x \leq 125$	43
$125 < x \leq 135$	52
$135 < x \leq 145$	28
$145 < x \leq 155$	21
$155 < x \leq 165$	10
$165 < x \leq 175$	4

2.1 Complete the cumulative frequency column in the table provided in the ANSWER BOOK. (2)

2.2 Draw a cumulative frequency graph (ogive) of the data on the grid provided. (3)

2.3 Use the cumulative frequency graph to determine the value of the lower quartile. (2)

2.4 Determine, from the cumulative frequency graph, the number of learners who used their cellphones for more than 140 minutes. (2)

[9]

QUESTION 1

The table below shows the weight (to the nearest kilogram) of each of the 27 participants in a weight-loss programme.

56	68	69	71	71	72	82	84	85
88	89	90	92	93	94	96	97	99
102	103	127	128	134	135	137	144	156

1.1 Calculate the range of the data. (2)

1.2 Write down the mode of the data. (1)

1.3 Determine the median of the data. (1)

1.4 Determine the interquartile range of the data. (3)

1.5 Use the number line provided in the ANSWER BOOK to draw a box and whisker diagram for the data above. (2)

1.6 Determine the standard deviation of the data. (2)

1.7 The person weighing 127 kg states that she weighs more than one standard deviation above the mean. Do you agree with this person? Motivate your answer with calculations. (3)

[14]

QUESTION 2

The table below shows the weight (in grams) that each of the 27 participants in the weight-loss programme lost in total over the first 4 weeks.

WEIGHT LOSS OVER 4 WEEKS (IN GRAMS)	FREQUENCY
$1\,000 < x \leq 1\,500$	2
$1\,500 < x \leq 2\,000$	3
$2\,000 < x \leq 2\,500$	3
$2\,500 < x \leq 3\,000$	4
$3\,000 < x \leq 3\,500$	5
$3\,500 < x \leq 4\,000$	7
$4\,000 < x \leq 4\,500$	2
$4\,500 < x \leq 5\,000$	1

2.1 Estimate the average weight loss, in grams, of the participants over the first 4 weeks. (2)

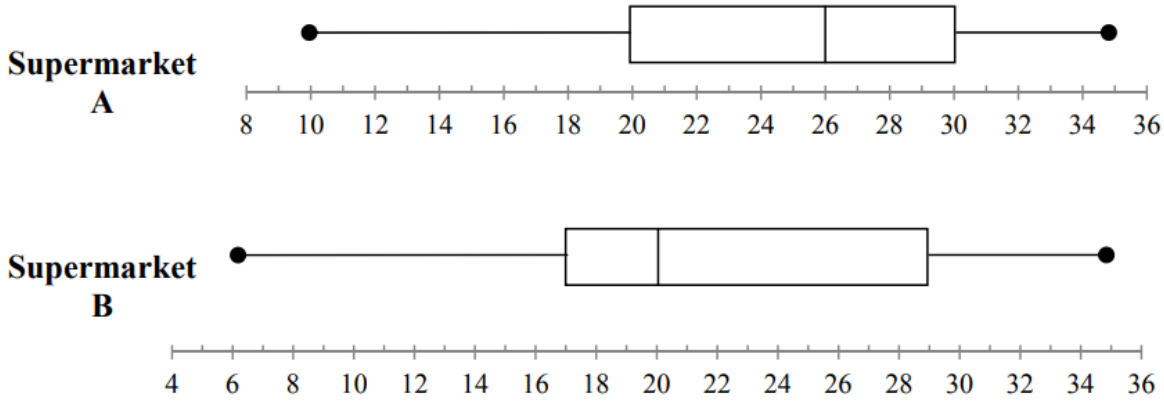
2.2 Draw an ogive (cumulative frequency graph) of the data on the grid provided. (4)

2.3 The weight-loss programme guarantees a loss of 800 g per week if a person follows the programme without cheating. Hence, determine how many of the participants had an average weight loss of 800 g or more per week over the first 4 weeks. (2)

[8]

QUESTION 1

1.1 The number of delivery trucks making daily deliveries to neighbouring supermarkets, Supermarket A and Supermarket B, in a two-week period are represented in the box-and-whisker diagrams below.



1.1.1 Calculate the interquartile range of the data for Supermarket A. (2)

1.1.2 Describe the skewness in the data of Supermarket A. (1)

1.1.3 Calculate the range of the data for Supermarket B. (2)

1.1.4 During the two-week period, which supermarket receives 25 or more deliveries per day on more days? Explain your answer. (2)

1.2 The number of delivery trucks that made deliveries to Supermarket A each day during the two-week period was recorded. The data is shown below.

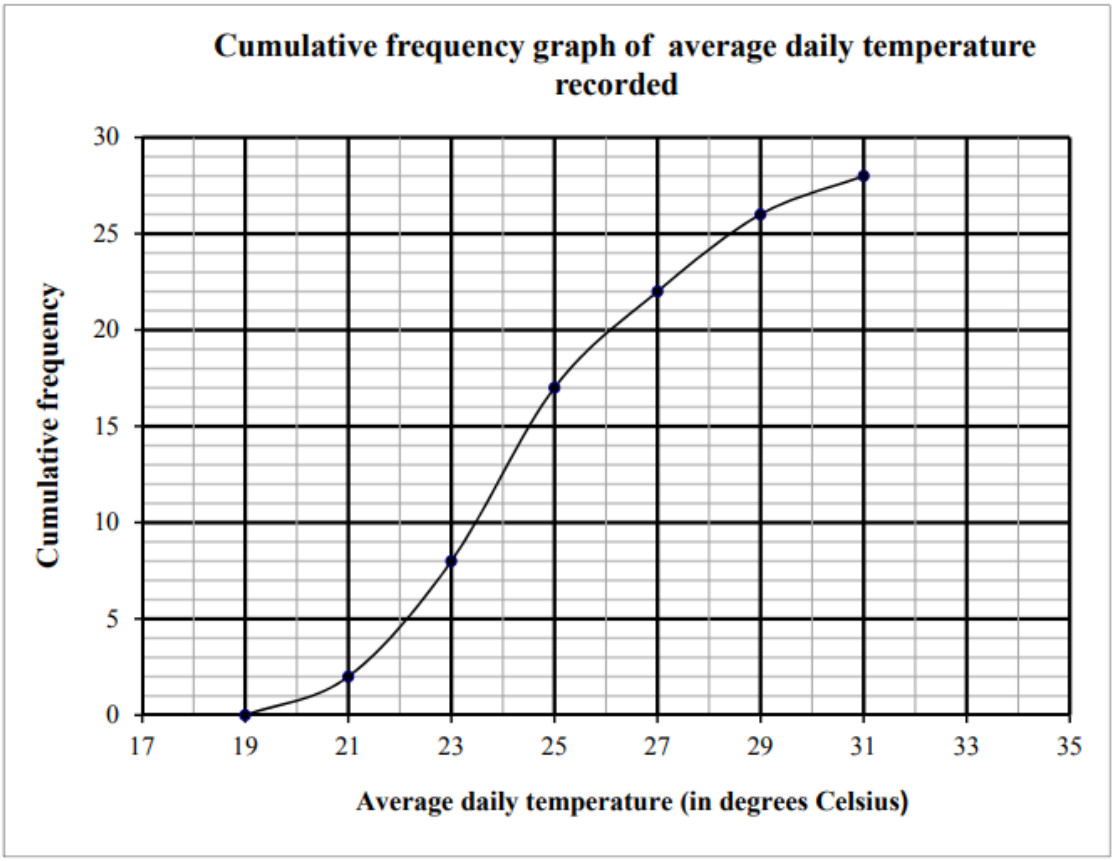
10	15	20	x	30	35	15	31	32	21	x	27	28	29

If the mean of the number of delivery trucks that made deliveries to supermarket A is 24,5 during these two weeks, calculate the value of x. (3)

[10]

QUESTION 2

The 2012 Summer Olympic Games was held in London. The average daily temperature, in degrees Celsius, was recorded for the duration of the Games. A cumulative frequency graph (ogive) of this data is shown below.



2.1 Over how many days was the 2012 Summer Olympic Games held? (1)

2.2 Estimate the percentage of days that the average daily temperature was less than 24 °C. (2)

2.3 Complete the frequency table for the data on DIAGRAM SHEET 1. (3)

2.4 Hence, use the grid provided on DIAGRAM SHEET 1 to draw a frequency polygon of the data. (4)

[10]

QUESTION 1

The 100th Tour de France took place from 29 June 2013 to 21 July 2013. The race was made up of 21 stages of varying distances. The distance, in kilometres, covered in each stage is given in the table below:

Stage	Distance	Stage	Distance	Stage	Distance
1	213	8	195	15	247
2	156	9	168	16	168
3	145	10	197	17	32
4	25	11	33	18	172
5	228	12	218	19	204
6	176	13	173	20	125
7	205	14	191	21	133

[Source: www.letour.fr.le-tour/2013/us]

1.1 Calculate the mean distance. (3)

1.2 Calculate the standard deviation of the distances. (2)

1.3 Determine the number of stages that lie beyond ONE standard deviation of the mean. (2)

1.4 The distance covered in each stage has been rearranged in ascending order and is shown below. Determine the five-number summary of this data.

25 32 33 125 133 145 156
168 168 172 173 176 191 195
197 204 205 213 218 228 247 (4)

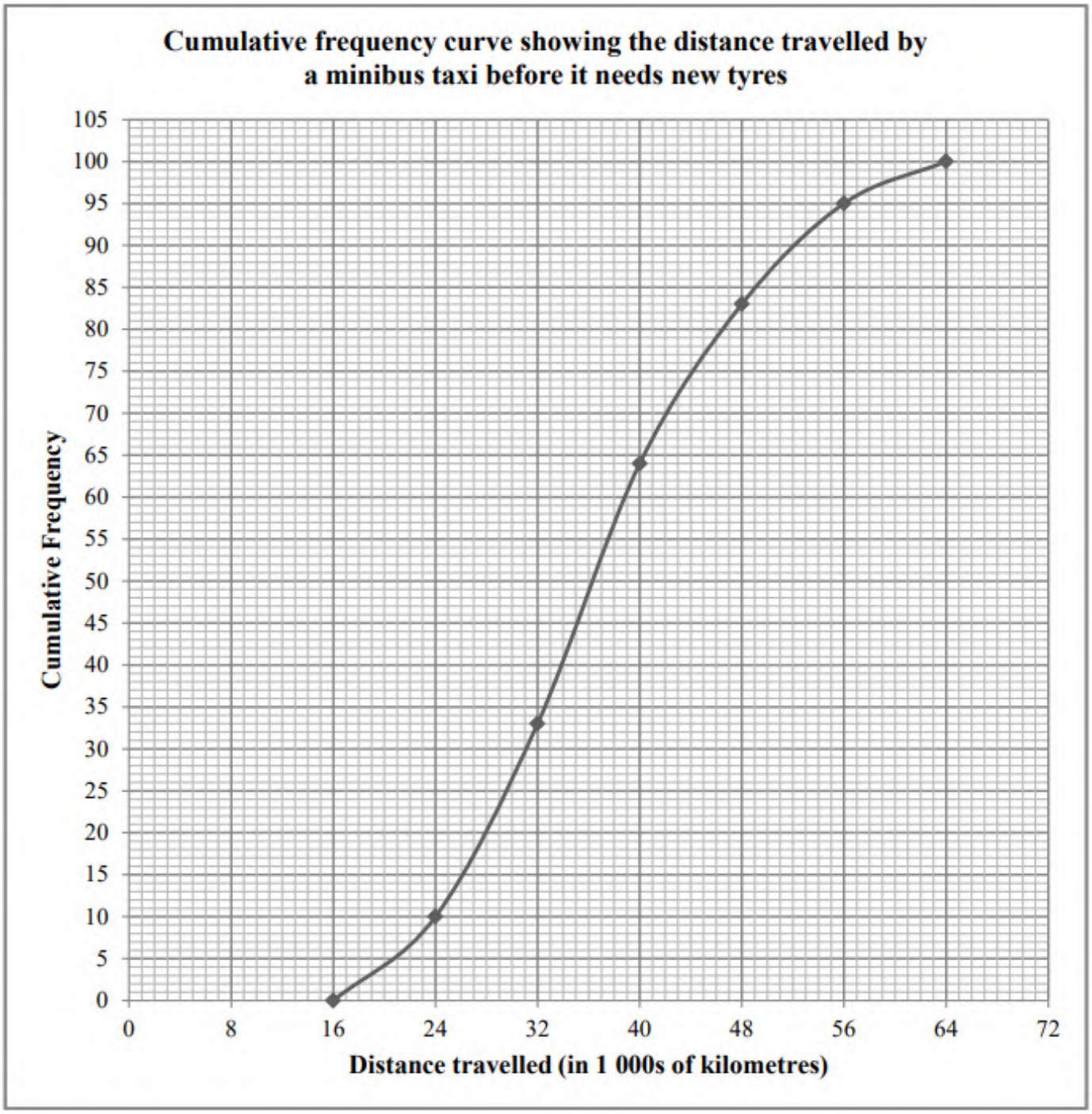
1.5 Use the scaled line provided in DIAGRAM SHEET 1 to draw a box and whisker diagram to represent the distance covered in each stage. (2)

1.6 Are there any outliers in the data set? Explain. (2)

[15]

QUESTION 2

A manufacturer recorded how far a minibus taxi travels before it needs new tyres. He recorded the distances, in 1 000s of kilometres, covered by a number of taxis that travelled the same route. This information is shown in the cumulative frequency graph (ogive) below.



2.1 How many times did they record the distance travelled by a minibus taxi before it needed new tyres? (1)

2.2 Write down the modal class of the data. (1)

2.3 Estimate the median distance travelled before new tyres are needed. (1)

2.4 Estimate the inter-quartile range for this data. (3)

[6]

QUESTION 1

The data below shows the number of people visiting a local clinic per day to be vaccinated against measles.

5	12	19	29
35	23	15	33
37	21	26	18
23	18	13	21
18	22	20	

1.1 Determine the mean of the given data. (2)

1.2 Calculate the standard deviation of the data. (2)

1.3 Determine the number of people vaccinated against measles that lies within ONE standard deviation of the mean. (2)

1.4 Determine the interquartile range for the data. (3)

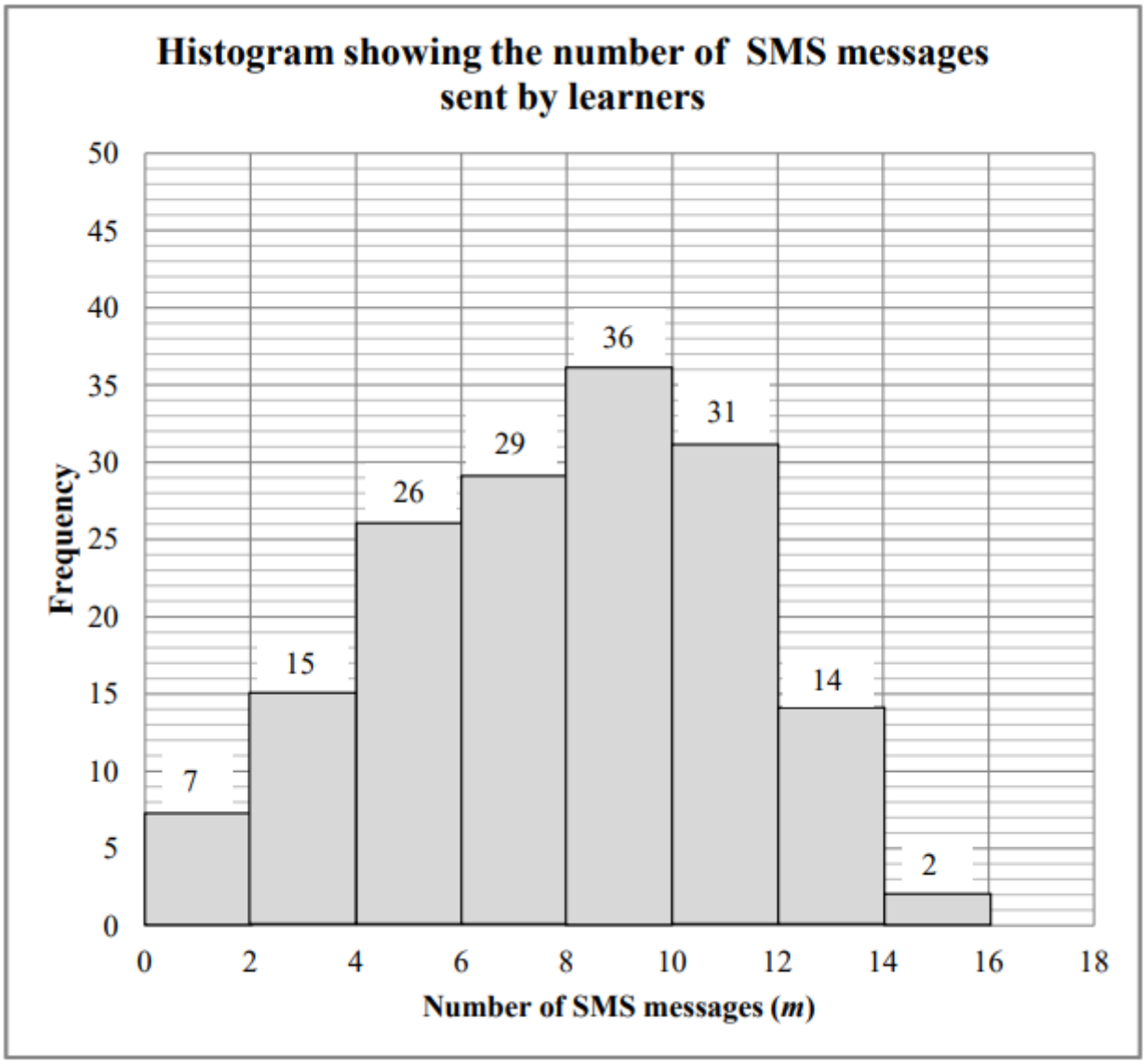
1.5 Draw a box and whisker diagram to represent the data. (3)

1.6 Identify any outliers in the data set. Substantiate your answer. (2)

[14]

QUESTION 2

A group of Grade 11 learners were interviewed about using a certain application to send SMS messages. The number of SMS messages, m, sent by each learner was summarised in the histogram below.



2.1 Complete the cumulative frequency table provided in DIAGRAM SHEET 1. (2)

2.2 Use the grid provided in DIAGRAM SHEET 2 to draw an ogive (cumulative frequency curve) to represent the data. (3)

2.3 Use the ogive to identify the median for the data. (1)

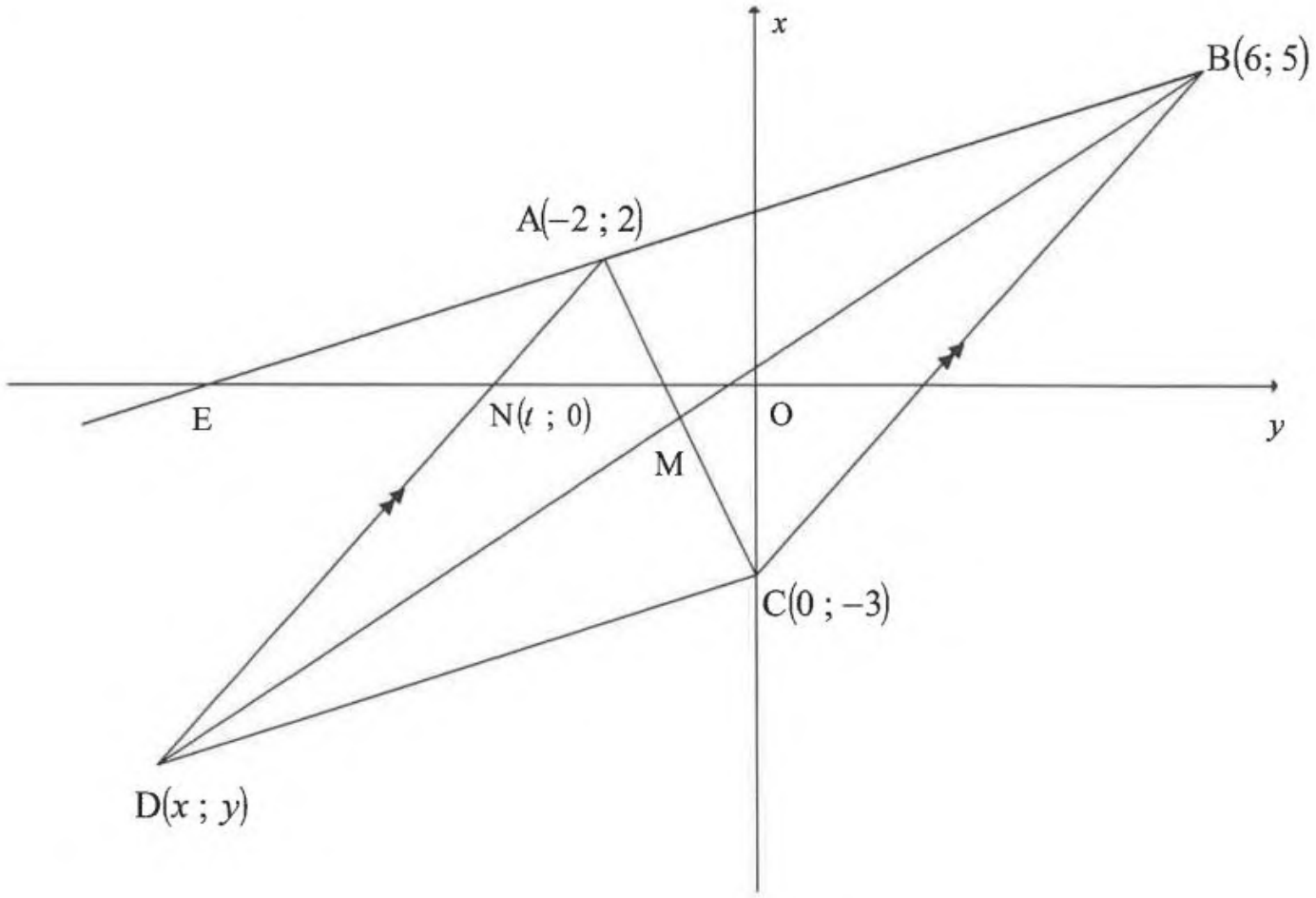
2.4 Estimate the percentage of the learners who sent more than 11 messages using this application. (2)

2.5 In which direction is the data skewed? (1)

[9]

QUESTION 3

In the diagram, A(–2 ; 2), B(6 ; 5), C(0 ; –3) and D(x ; y) are the vertices of a quadrilateral having AD || BC. BA produced has an x-intercept at E. BD and AC intersect at M. N(t ; 0) is a point on AD.



3.1 Calculate the gradient of BC. (2)

3.2 Determine the equation of AD. (3)

3.3 Determine the value of t. (2)

3.4 Calculate the length of AN. (2)

3.5 If DC is defined by $y=\frac{3}{8}x-3$, determine the coordinates of D. (4)

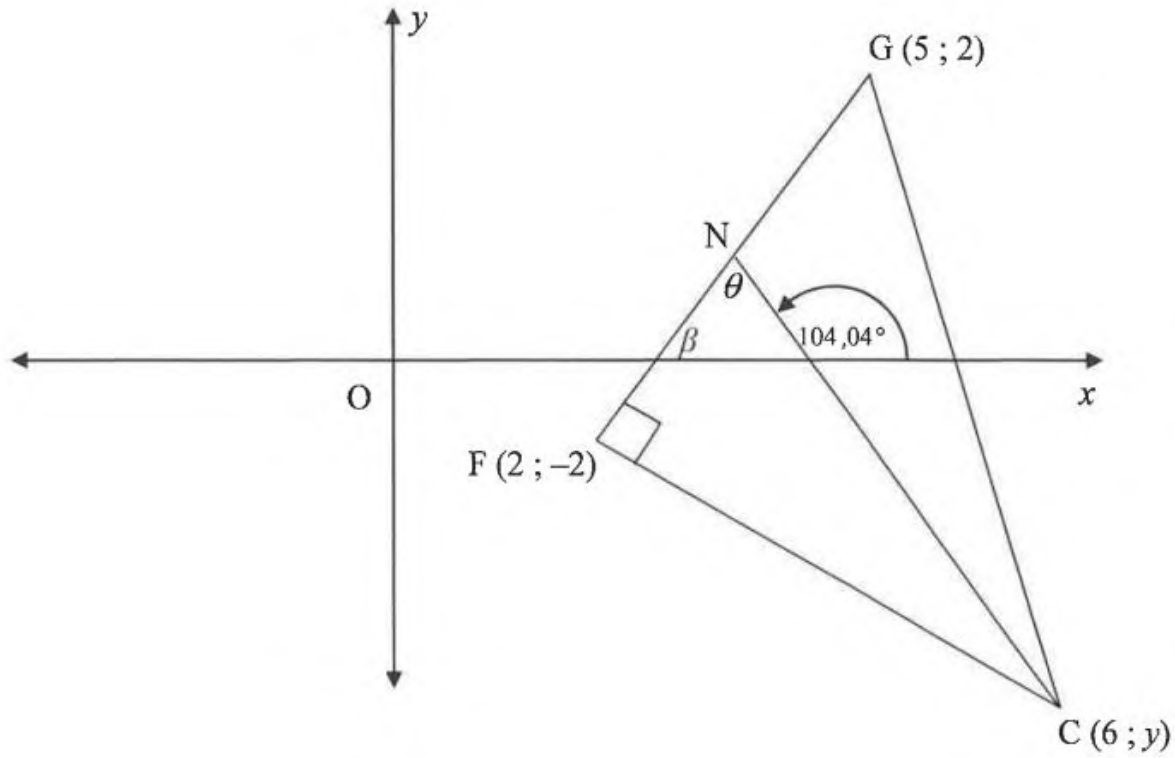
3.6 Prove that ABCD is a parallelogram. (3)

3.7 Calculate the coordinates of M. (3)

[19]

QUESTION 4

In the diagram, F(2 ; –2), G(5 ; 2) and C(6 ; y) are the vertices of ΔFGC. $FG \perp FC$.
N is a point on FG such that the inclination of NC is 104,04°.
The angle of inclination of FG is β and $F\hat{N}C = \theta$.



4.1 Calculate the gradient of FG. (2)

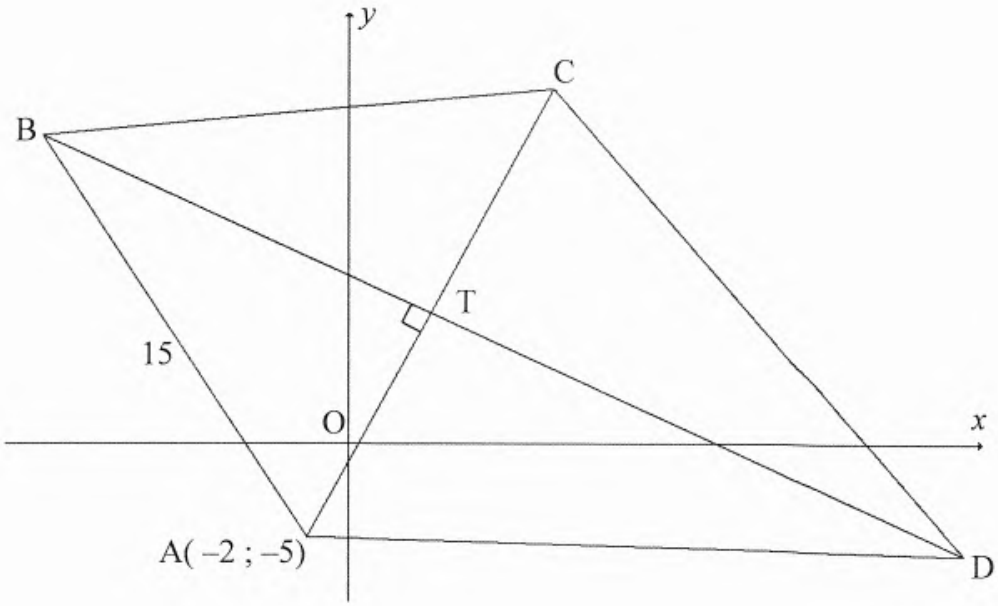
4.2 Calculate the value of y. (3)

4.3 Calculate the size of θ. (3)

4.4 Calculate the length of NC. (4)
[12]

QUESTION 3

A(–2 ; –5), B, C and D are the vertices of quadrilateral ABCD such that diagonal AC is perpendicular to diagonal BD at T.
The equation of BTD is given by $2y+x=18$ and AB = 15 units.



3.1 Determine the gradient of line AC. (2)

3.2 Determine the equation of AC in the form $y=mx+c$. (2)

3.3 If the equation of AC is $y=2x-1$, calculate the coordinates of T. (3)

3.4 If ABCD is a kite with AB = BC:

3.4.1 Determine the coordinates of C (2)

3.4.2 Calculate the length of BT (4)

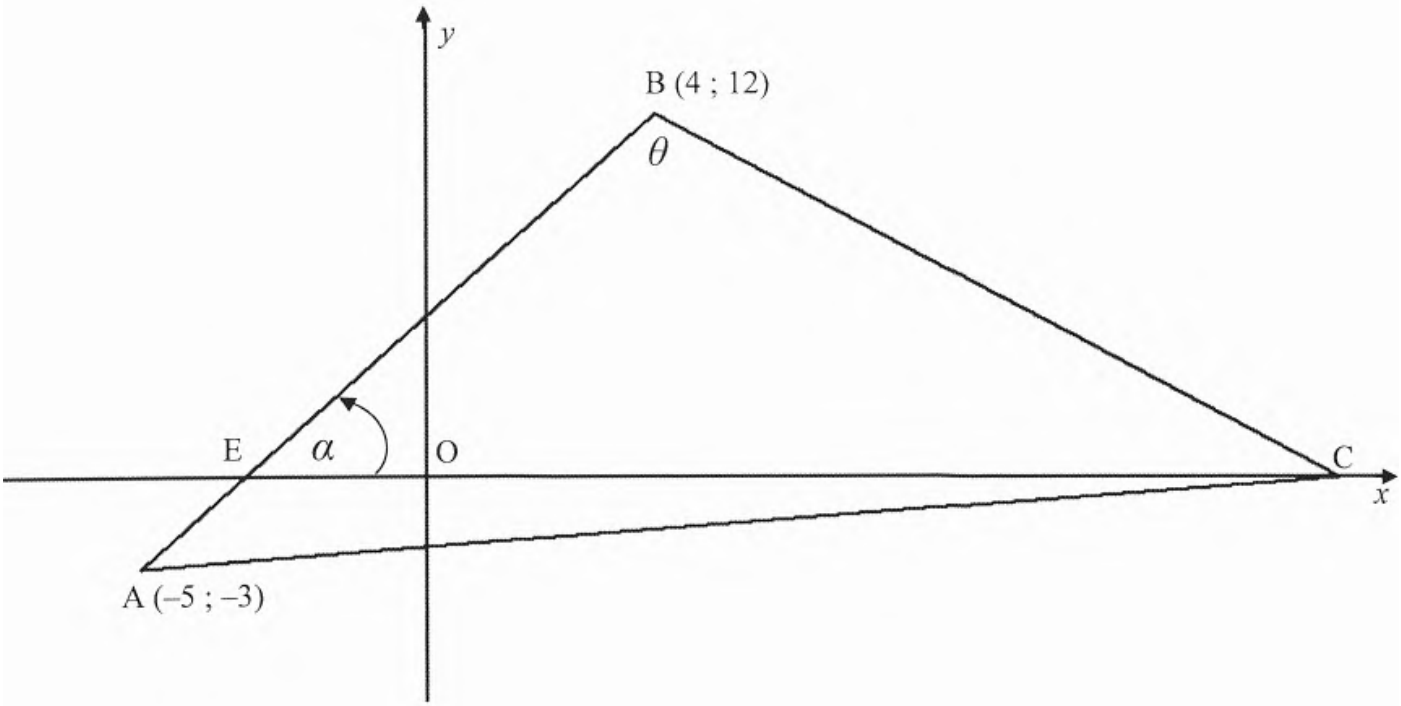
3.4.3 Write down the length of the radius of the circle passing through points B, C and T (2)

[15]

QUESTION 4

C, a point on the x-axis, $A(-5\,;\,-3)$ and $B(4\,;\,12)$ are the vertices of a triangle. AB intersects the x-axis at E.

$A\hat{B}C = \theta$ and $B\hat{E}C = \alpha$.



4.1 Calculate the gradient of AB. (2)

4.2 Determine the coordinates of point E. (3)

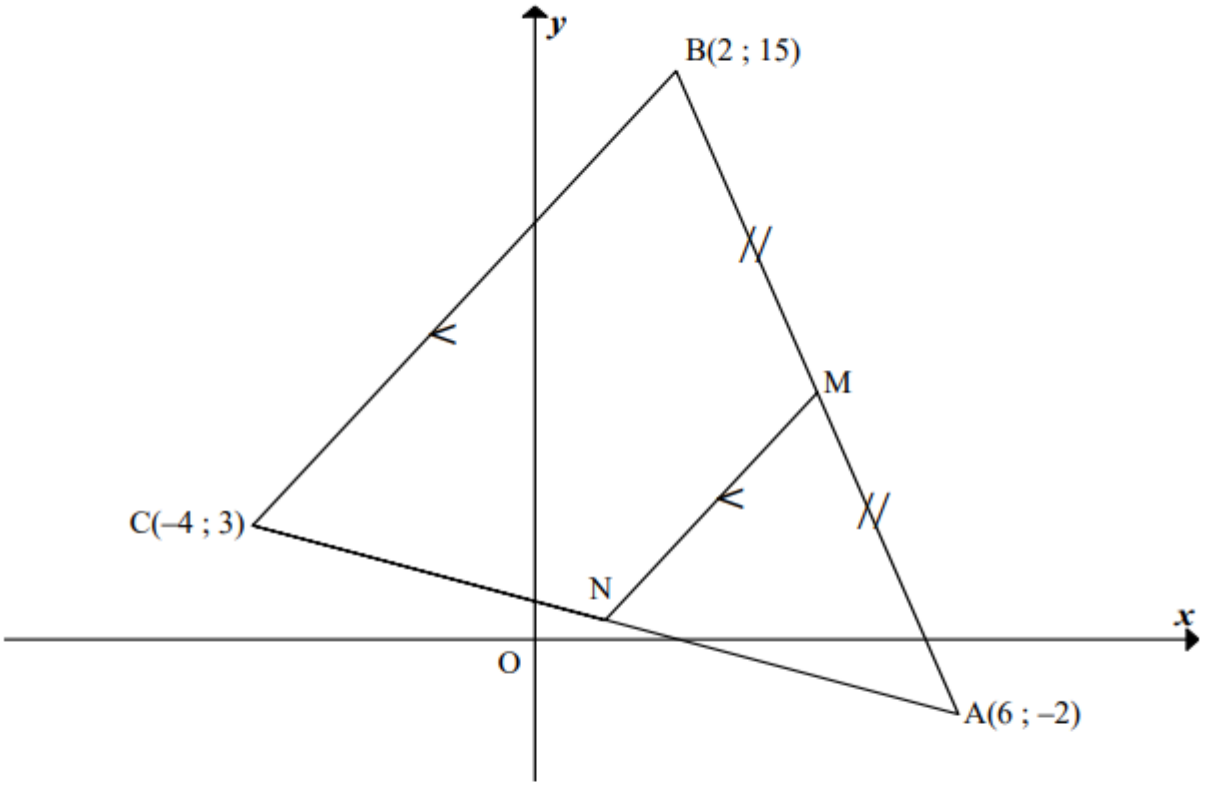
4.3 Determine the size of α. Round off to the nearest whole number. (2)

4.4 If $\theta = 76^\circ$, determine the equation of the line through A parallel to BC. (5)

[12]

QUESTION 3

In the diagram, A(6 ; – 2), B(2 ; 15) and C(– 4 ; 3) are the vertices of ΔABC.
M is the midpoint of AB. N is a point on CA such that MN || BC.



3.1 Determine the coordinates of M, the midpoint of AB. (2)

3.2 Determine the gradient of line MN. (3)

3.3 Hence, or otherwise, determine the equation of line MN, in the form $y = mx + c$. (2)

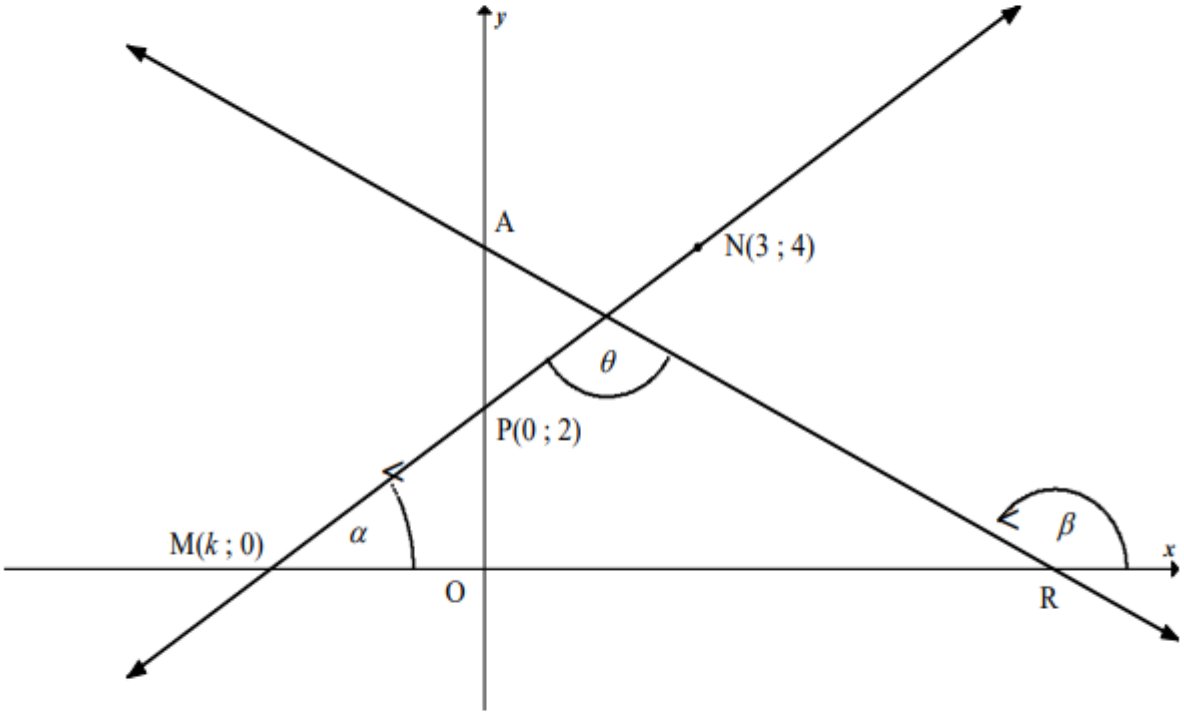
3.4 Calculate, with reasons, the coordinates of point N. (4)

3.5 If ABCD (in that order) is a parallelogram, determine the coordinates of point D. (4)
[15]

QUESTION 4

In the diagram, R and A are the x- and y-intercepts respectively of the straight line AR. The equation of AR is $y=-\frac{1}{2}x+4$. Another straight line cuts the y-axis at P(0 ; 2) and passes through the points M(k ; 0) and N(3 ; 4).

α and β are the angles of inclination of the lines MN and AR respectively.



4.1 Given that M, P and N are collinear points, calculate the value of k. (3)

4.2 Determine the size of θ, the obtuse angle between the two lines. (4)

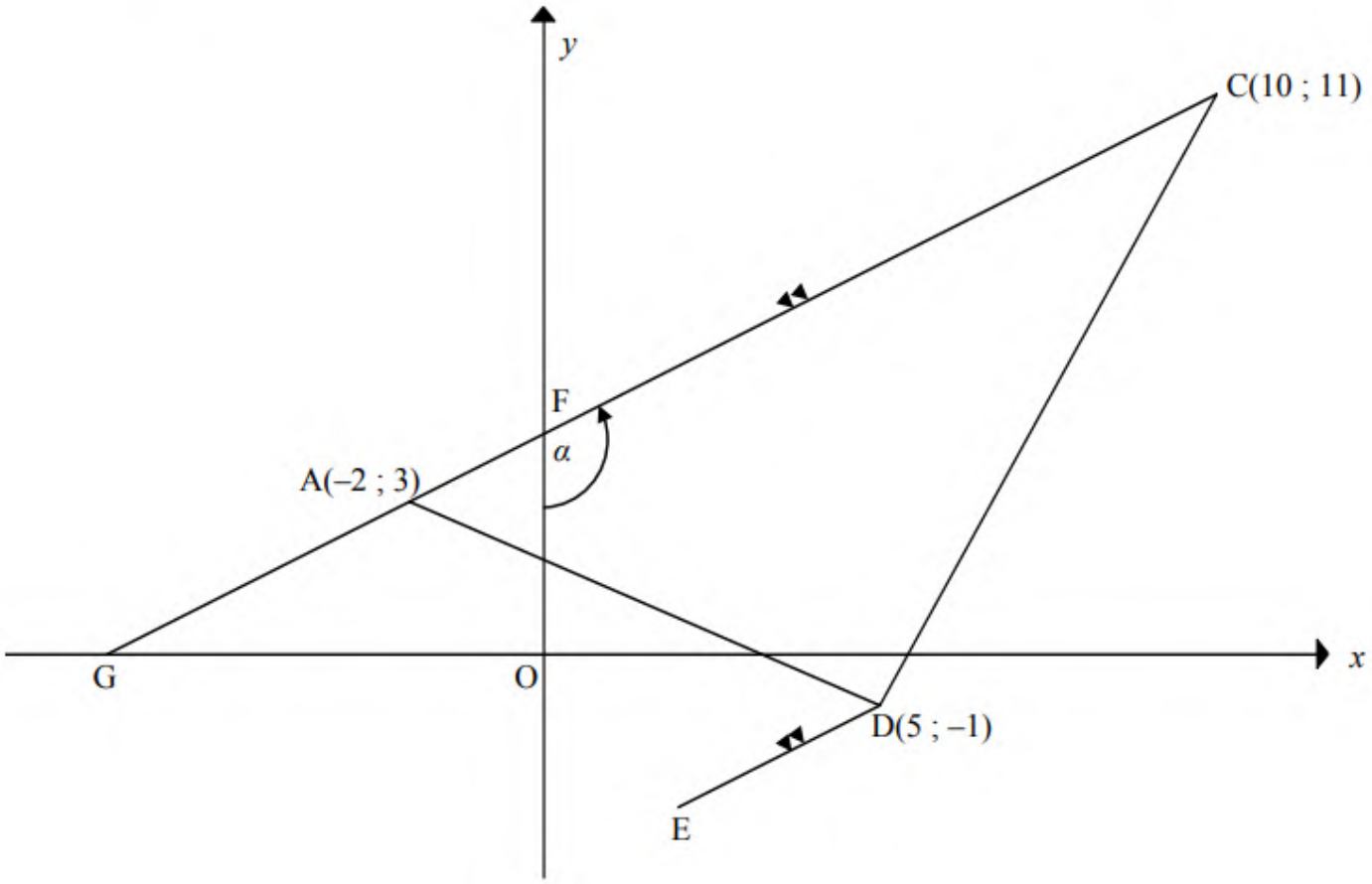
4.3 Calculate the length of MR. (3)

4.4 Calculate the area of ΔMNR. (3)

[13]

QUESTION 3

In the diagram, A(–2 ; 3), C(10 ; 11) and D(5 ; –1) are the vertices of ΔACD. CA intersects the y-axis in F and CA produced cuts the x-axis in G. The straight line DE is drawn parallel to CA. $C\hat{F}O = \alpha$.



3.1 Calculate the gradient of the line AC. (2)

3.2 Determine the equation of line DE in the form $y = mx + c$. (3)

3.3 Calculate the size of α. (3)

3.4 B is a point in the first quadrant such that ABDE, in that order, forms a rectangle. Calculate, giving reasons, the:

3.4.1 Coordinates of M, the midpoint of BE (3)

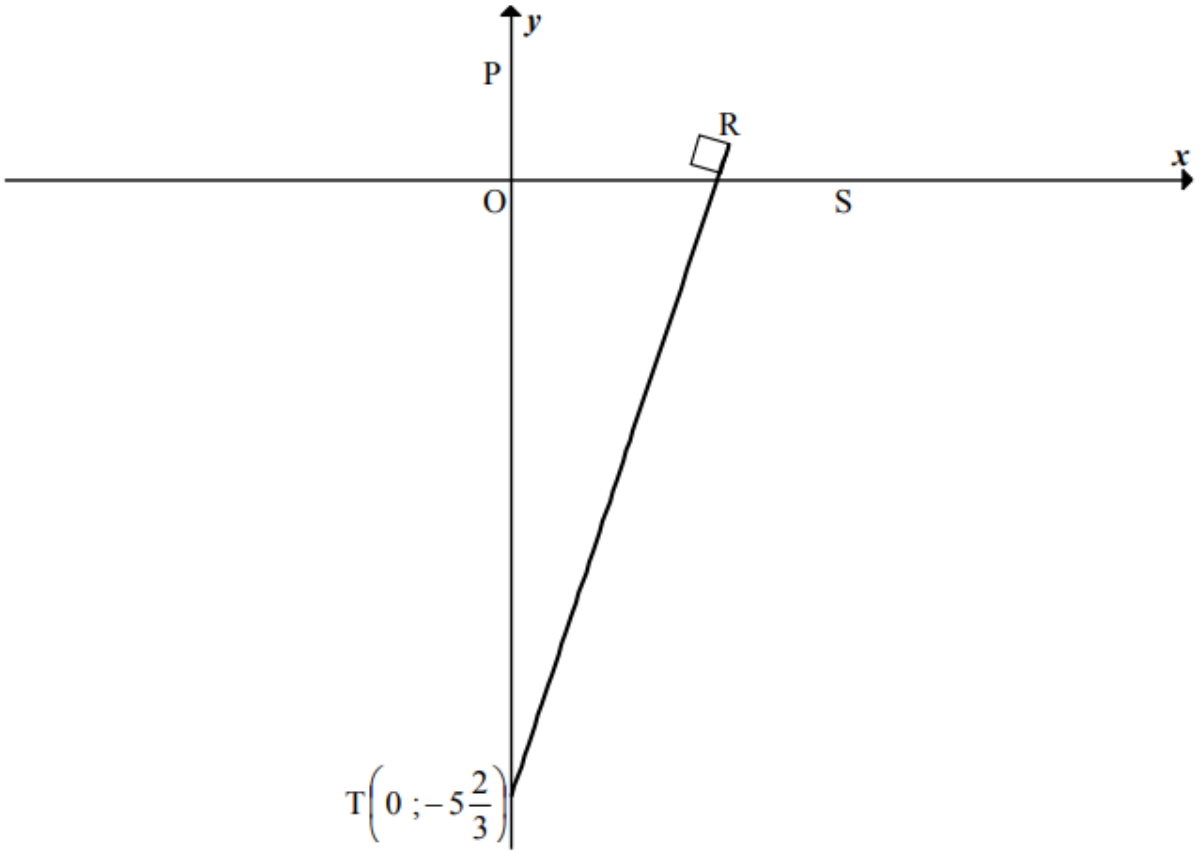
3.4.2 Length of diagonal BE (3)

[14]

QUESTION 4

In the diagram, the straight line SP is drawn having S and P as its x- and y-intercepts respectively. The equation of SP is $x + ay - a = 0$, $a > 0$. It is also given that OS = 3OP.

The straight line RT is drawn with R on SP and RT $\perp$ PS. RT cuts the y-axis in $T\left(0\ ;-5\frac{2}{3}\right)$.



4.1 Calculate the coordinates of P. (2)

4.2 Calculate the value of a. (2)

4.3 Determine the equation of RT in the form $y = mx + c$ if it is given that $a = 3$. (3)

4.4 Calculate the coordinates of R, the point where PS and TR meet. (4)

4.5 Calculate the area of ΔPRT if it is given that $R\left(2\ ;\ \frac{1}{3}\right)$. (3)

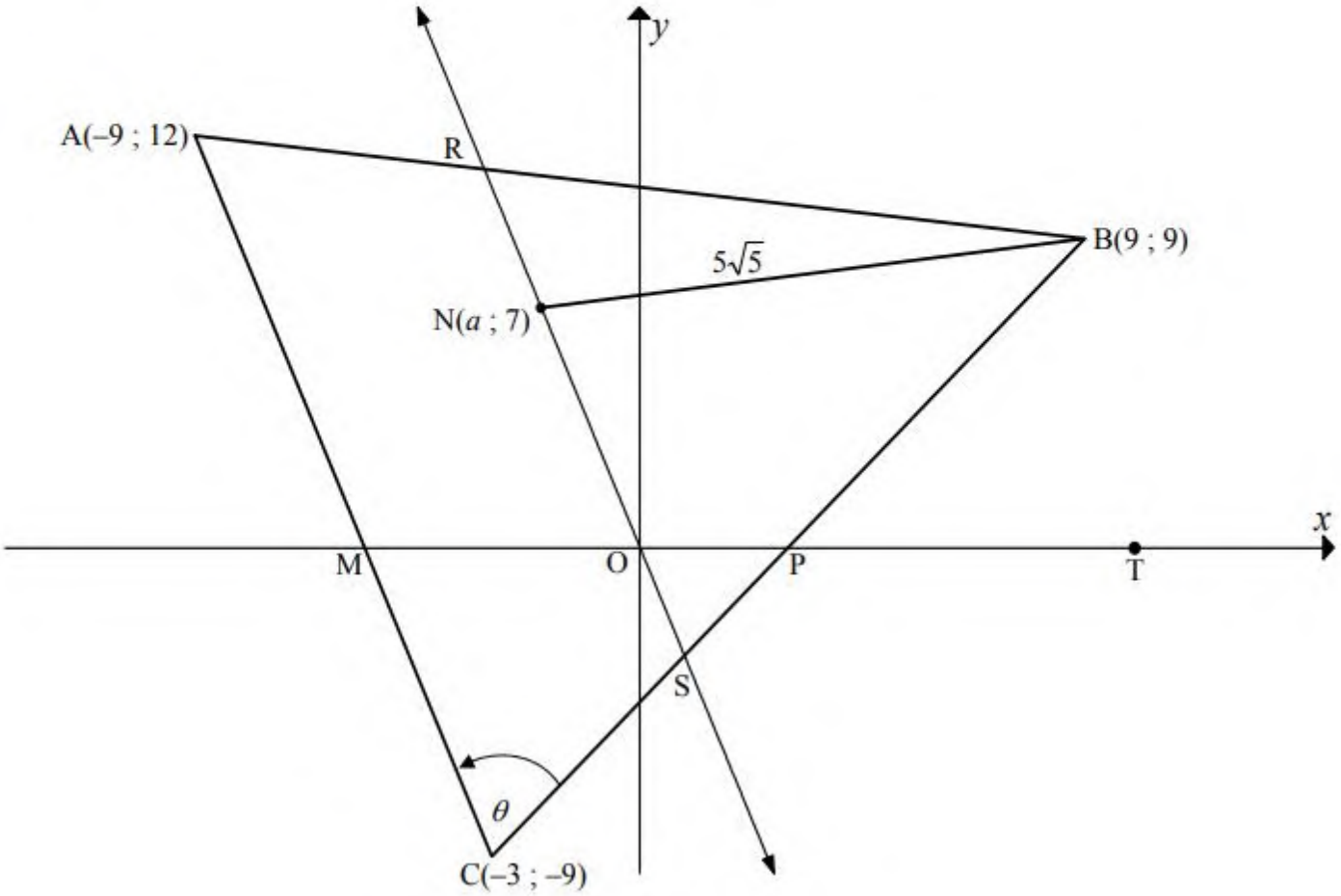
4.6 Calculate, giving reasons, the radius of a circle passing through the points P, R and T. (2)

[16]

QUESTION 3

In the diagram A(–9 ; 12), B(9 ; 9) and C(–3 ; –9) are the vertices of ΔABC. N(a ; 7) is a point such that BN = $5\sqrt{5}$. R is a point on AB and S is a point on BC such that RNS is parallel to AC and RNS passes through the origin. T lies on the x-axis to the right of point P.

$A\hat{C}B = \theta$, $A\hat{M}O = \alpha$ and $B\hat{P}T = \beta$.



3.1 Calculate the gradient of the line AC. (2)

3.2 Determine the equation of line RNS in the form $y = mx + c$. (2)

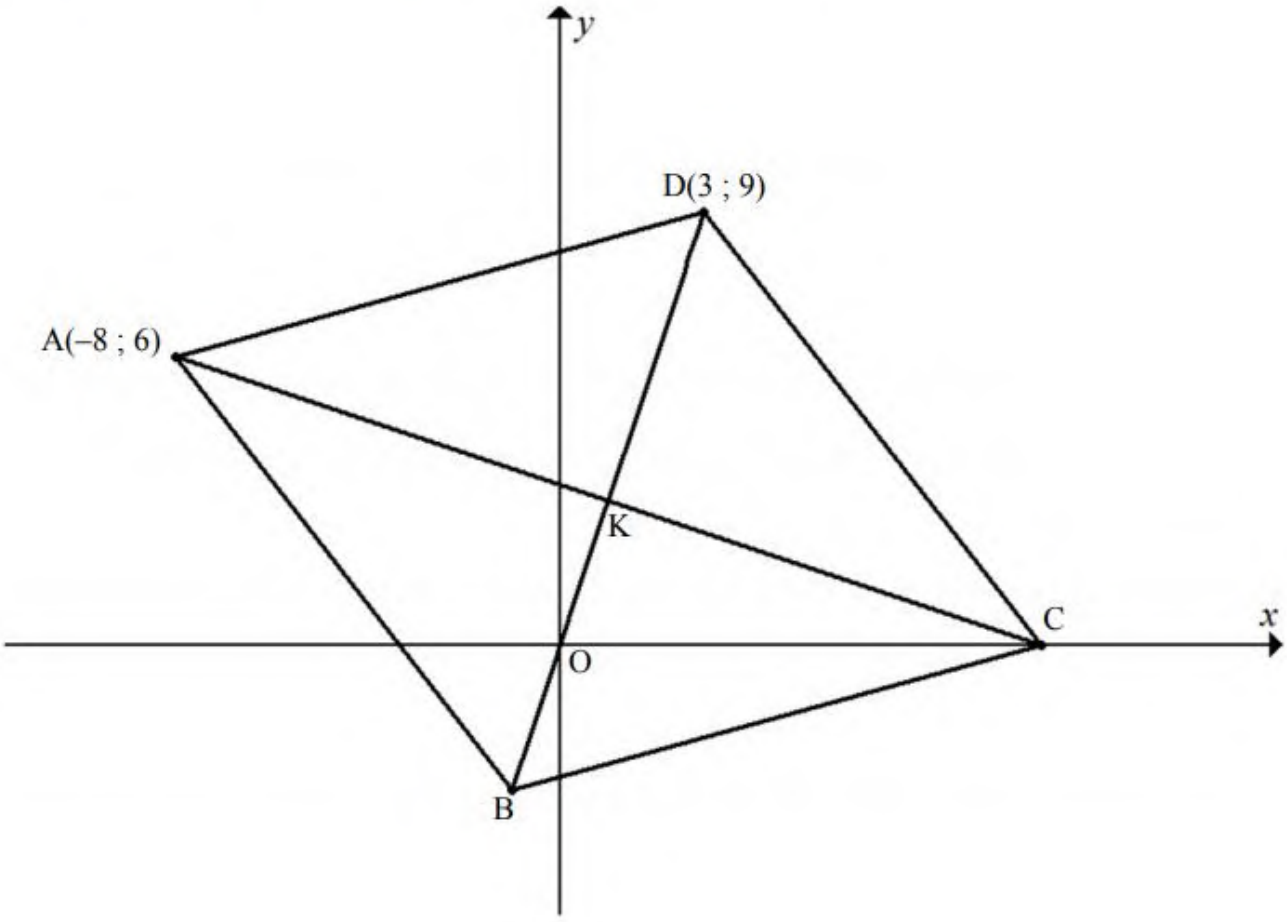
3.3 Calculate the value of a. (4)

3.4 Calculate the size of θ. (5)

[13]

QUESTION 4

In the diagram A(–8 ; 6), B, C and D(3 ; 9) are the vertices of a rhombus. The equation of BD is $3x - y = 0$. The diagonals of the rhombus intersect at point K.



4.1 Calculate the perimeter of ABCD. Leave your answer in simplest surd form. (3)

4.2 Determine the equation of diagonal AC in the form $y = mx + c$. (4)

4.3 Calculate the coordinates of K if the equation of AC is $x + 3y = 10$. (3)

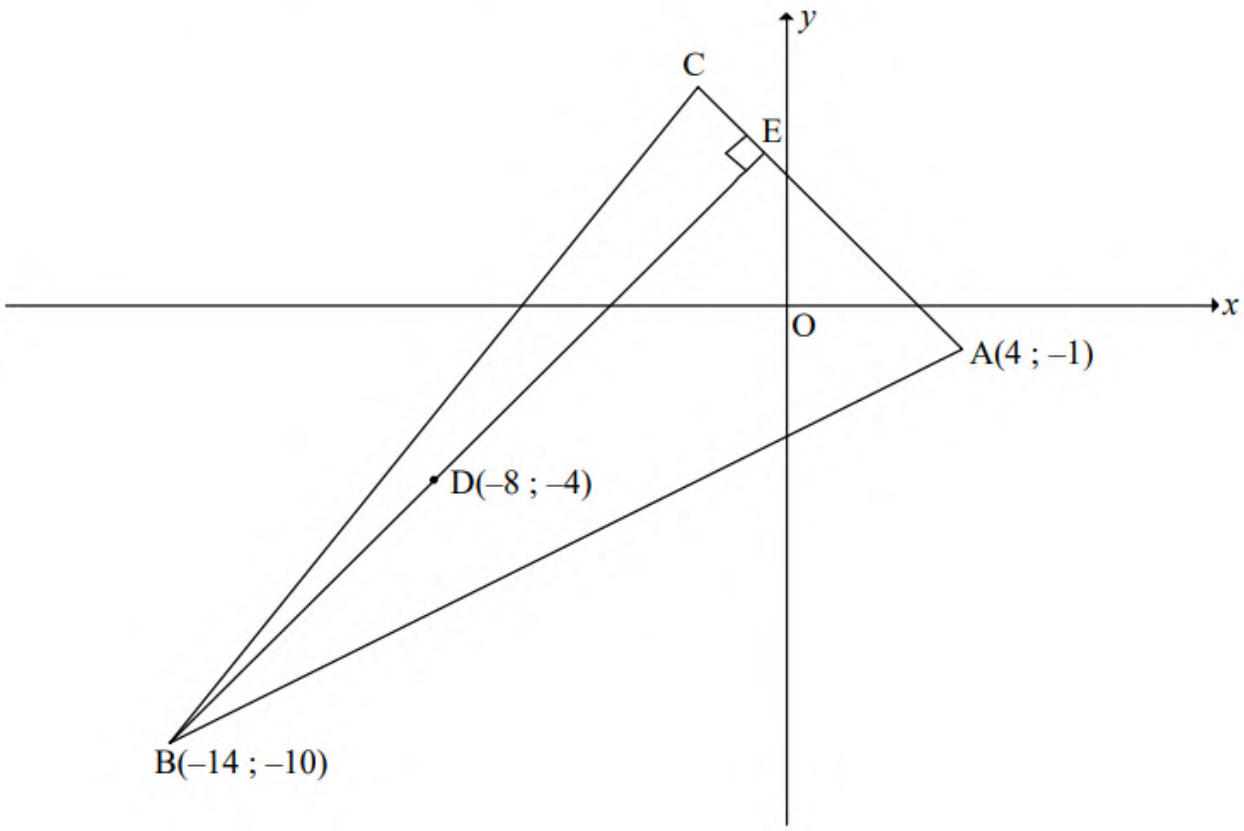
4.4 Calculate the coordinates of B. (2)

4.5 Determine, showing ALL your calculations, whether rhombus ABCD is a square or not. (5)

[17]

QUESTION 3

In the diagram below, A(4 ; –1), B(–14 ; –10) and C are the vertices of a triangle.
E is a point on AC such that BE ⊥ AC.
The point D(–8 ; –4) lies on BE.
The equation of the line BC is $4y - 5x - 30 = 0$.



3.1 Calculate the gradient of BD. (2)

3.2 Hence, write down the gradient of AC. (1)

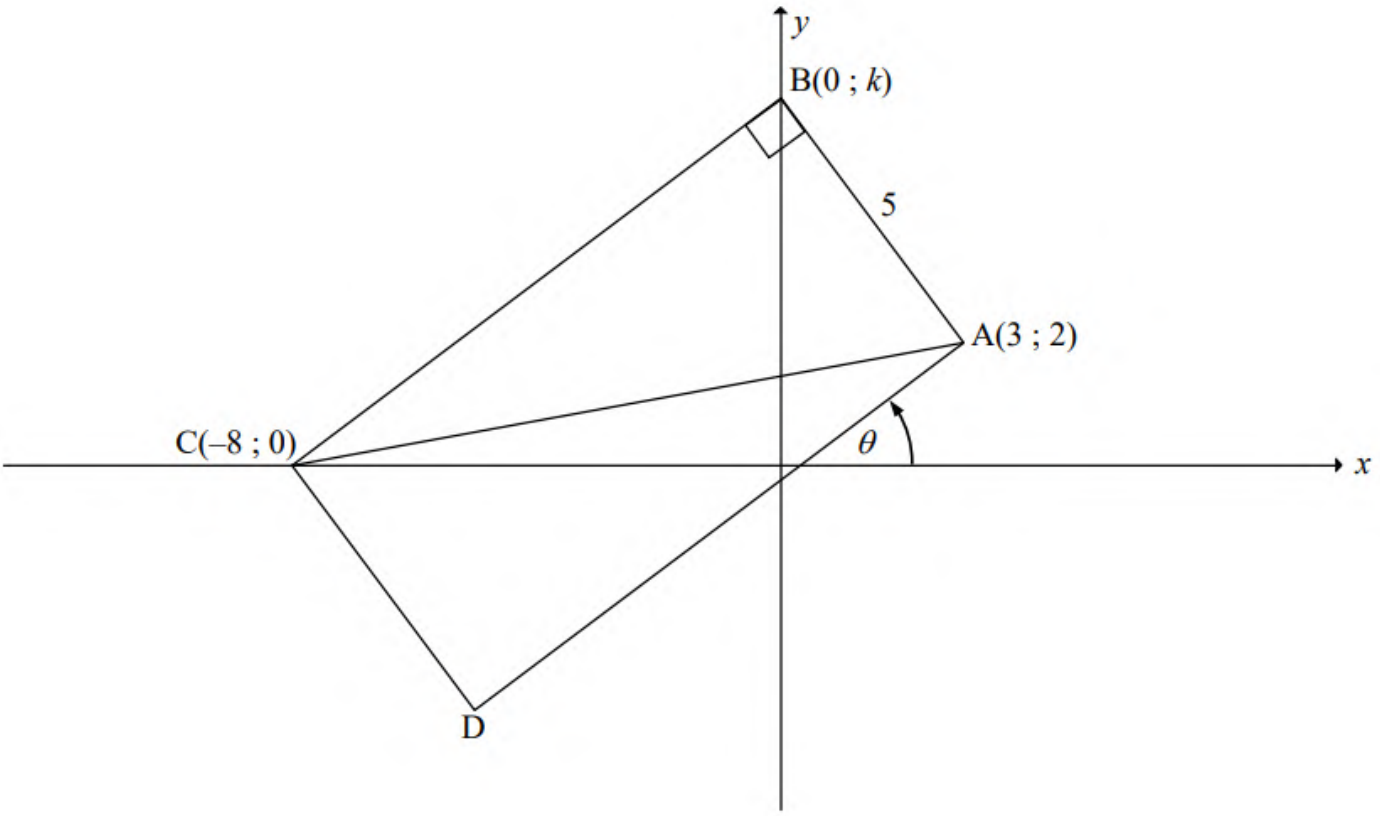
3.3 Determine the equation of AC in the form $y = mx + c$. (2)

3.4 The point G(p ; –5) lies on AB. Calculate the value of p. (3)

3.5 Calculate the coordinates of C. (4)
[12]

QUESTION 4

A(3 ; 2), B(0 ; k), C(–8 ; 0) and D are the vertices of a rectangle.
AB = 5 units.
The angle of inclination of AD is θ, as shown in the diagram.



4.1 Calculate the length of AC. (2)

4.2 Calculate the value of k. (4)

4.3 Determine the equation of BC in the form $y = mx + c$. (3)

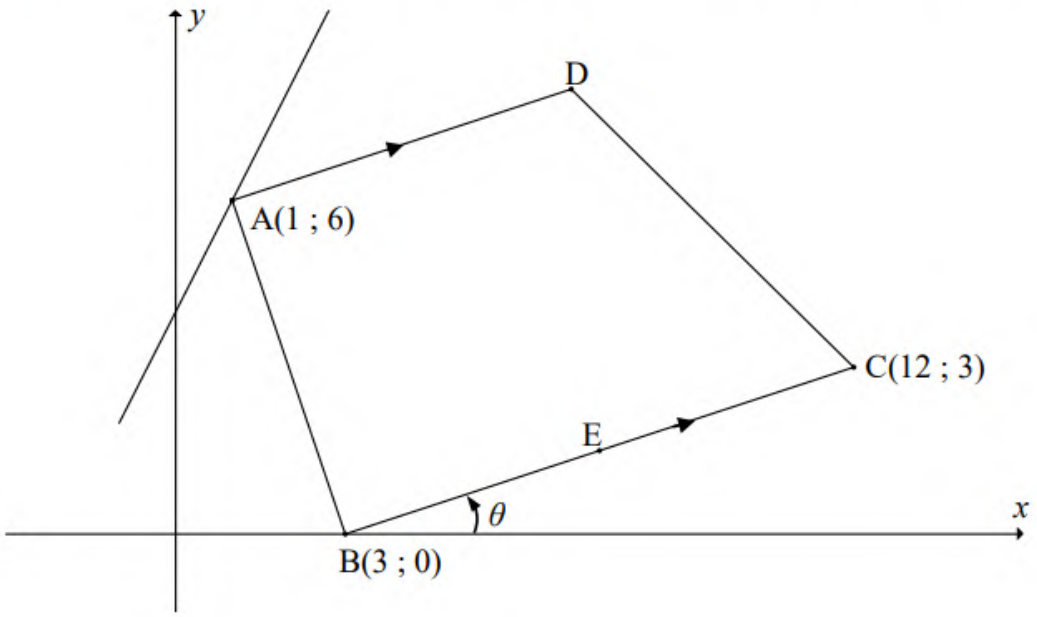
4.4 Calculate the size of θ. (3)

4.5 Calculate the area of ABCD. (3)

4.6 Calculate the size of $B\hat{A}C$. (2)
[17]

QUESTION 3

A(1 ; 6), B(3 ; 0), C(12 ; 3) and D are the vertices of a trapezium with AD || BC. E is the midpoint of BC. The angle of inclination of the straight line BC is θ, as shown in the diagram.



3.1 Calculate the coordinates of E. (2)

3.2 Determine the gradient of the line BC. (2)

3.3 Calculate the magnitude of θ. (2)

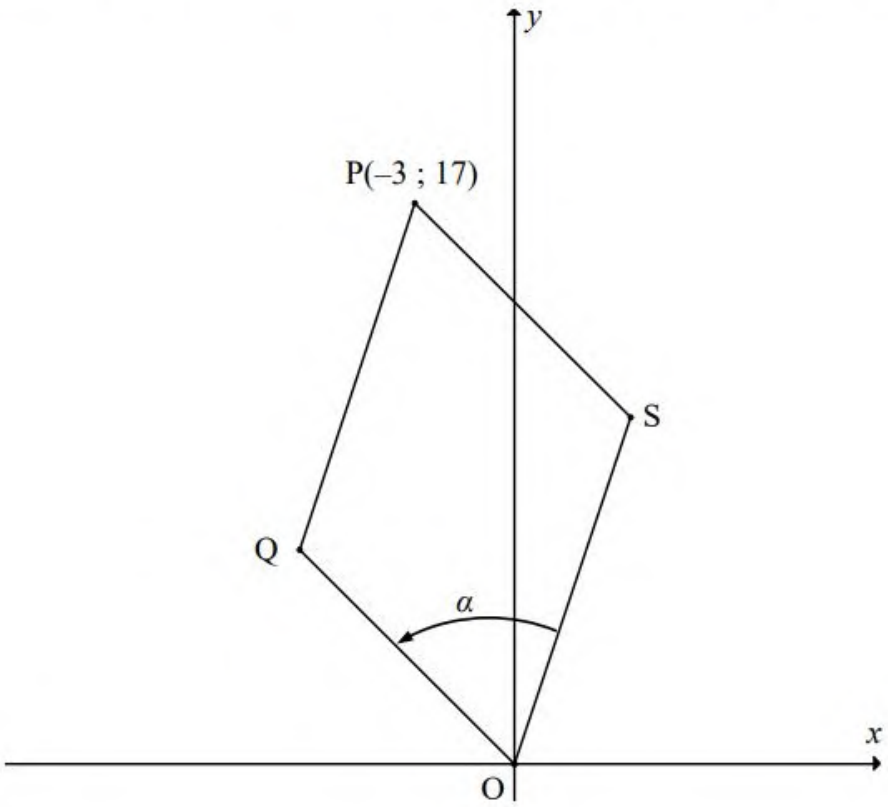
3.4 Prove that AD is perpendicular to AB. (3)

3.5 A straight line passing through vertex A does not pass through any of the sides of the trapezium. This line makes an angle of 45° with side AD of the trapezium. Determine the equation of this straight line. (5)

[14]

QUESTION 4

In the diagram below, P(–3 ; 17), Q, O and S are the vertices of a parallelogram. The sides OS and OQ are defined by the equations $y = 6x$ and $y = -x$ respectively. $\hat{QOS} = \alpha$.



4.1 Determine the equation of QP in the form $y = mx + c$. (3)

4.2 Hence, determine the coordinates of Q. (4)

4.3 Calculate the length of OQ. Leave your answer in simplified surd form. (2)

4.4 Calculate the size of α. (3)

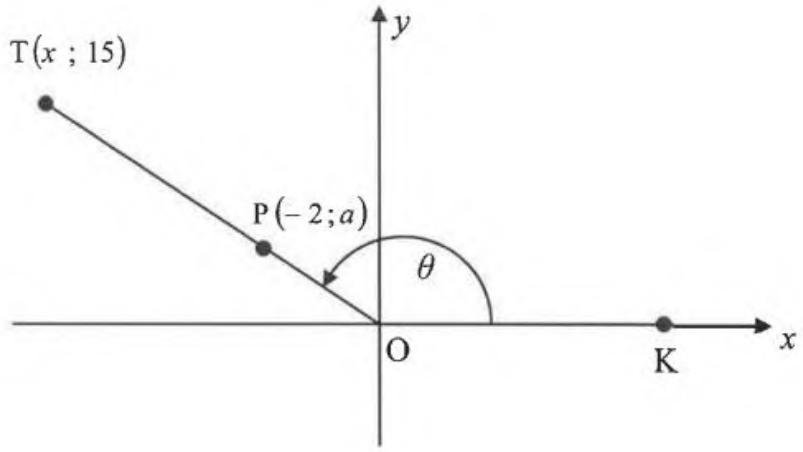
4.5 If $OS = \sqrt{148}$ units, calculate the length of QS. (3)

[15]

DBE NOV 18 Q5

QUESTION 5

5.1 In the diagram below, T$(x\,;15)$ is a point in the Cartesian plane such that OT = 17 units. P$(-2\,;a)$ lies on OT. K is a point on the positive x-axis and $\hat{TOK} = \theta$.



Determine, with the aid of the diagram, the following:

5.1.1 The value of x (2)

5.1.2 $\tan\theta$ (1)

5.1.3 $\cos(180° - \theta)$ (2)

5.1.4 $\sin^2\theta$ (2)

5.1.5 The value of a (3)

5.2 Simplify WITHOUT using a calculator:

$$\frac{\sin 120°.\cos 210°.\tan 315°.\cos 27°}{\sin 63°.\cos 540°}$$ (7)

5.3 Prove the identity:

$$\frac{1}{\cos\theta} - \frac{\cos\theta}{1+\sin\theta} = \tan\theta$$ (5)

5.4 Determine the general solution of $3\sin x = 2\tan x$ (6)

[28]

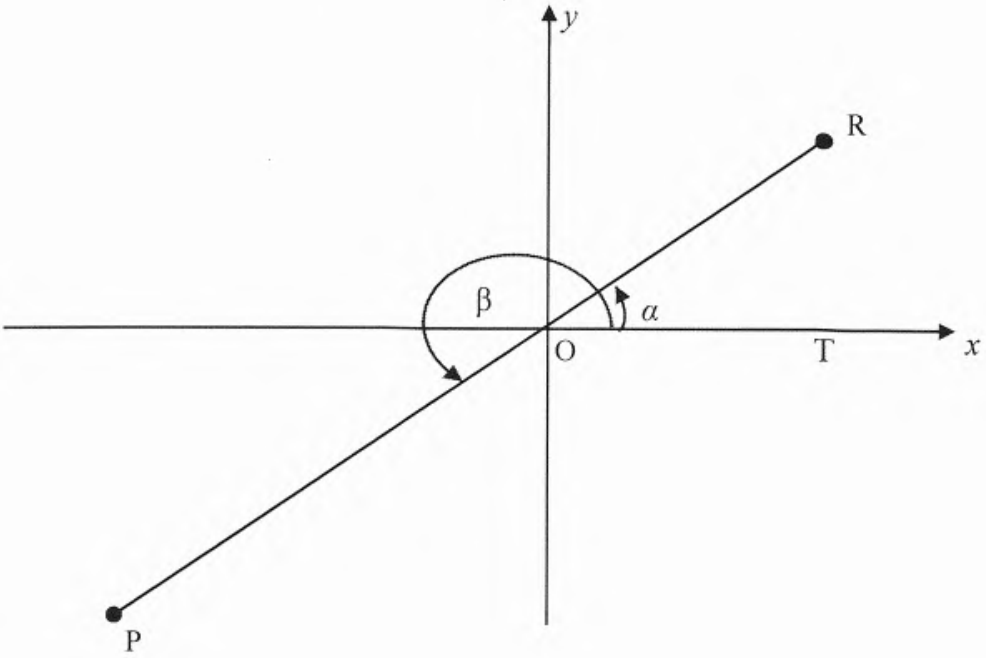
QUESTION 5

5.1 Simplify fully: $\sin(90^\circ - x).\cos(180^\circ + x) + \tan x.\cos x.\sin(x - 180^\circ)$ (6)

5.2 Prove, WITHOUT using a calculator, that

$$\frac{\sin 315^\circ.\tan 210^\circ.\sin 190^\circ}{\cos 100^\circ.\sin 120^\circ} = \frac{-\sqrt{2}}{3}$$ (6)

5.3 In the diagram below, R is a point in the first quadrant such that $\text{TOR} = \alpha$.
RO is extended to P such that $\text{OP} = 2\,\text{RO}$ and $\hat{\text{TOP}} = \beta$.
It is given that $\sin\alpha = \frac{3}{5}$.



WITHOUT using a calculator, determine:

5.3.1 The value of $\tan\alpha$ (3)

5.3.2 The value of $\sin\beta$ (3)

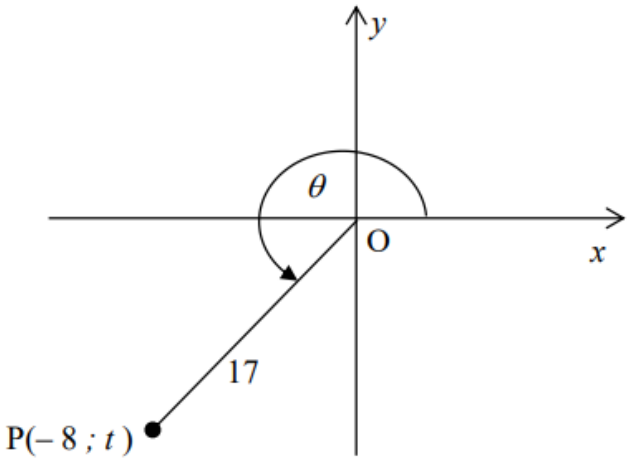
5.3.3 The coordinates of P (4)

5.4 Prove the identity: $\frac{\sin\theta - \tan\theta.\ \cos^2\theta}{\cos\theta - 1 + \sin^2\theta} = \tan\theta$ (4)

[26]

QUESTION 5

5.1 In the diagram below, P(–8 ; t) is a point in the Cartesian plane such that OP = 17 units and reflex $X\hat{O}P = \theta$.



5.1.1 Calculate the value of t. (2)

5.1.2 Determine the value of each of the following WITHOUT using a calculator:

(a) $\cos(-\theta)$ (2)

(b) $1-\sin\theta$ (2)

5.2 If $\sin 17° = a$, WITHOUT using a calculator, express the following in terms of a:

5.2.1 $\tan 17°$ (3)

5.2.2 $\sin 107°$ (2)

5.2.3 $\cos^2 253° + \sin^2 557°$ (4)

5.3 Simplify fully, WITHOUT the use of a calculator:

$$\frac{\cos(-225°).\sin 135° + \sin 330°}{\tan 225°}$$ (6)

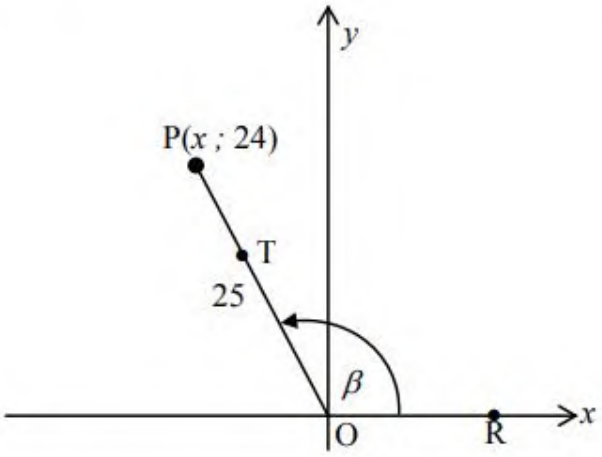
5.4 Prove the identity: $\dfrac{1}{(\cos x+1)(\cos x-1)} = \dfrac{-1}{\tan^2 x.\cos^2 x}$ (4)

5.5 Determine the general solution for $2\sin x.\cos x = \cos x$. (6)

[31]

QUESTION 5

5.1 In the diagram below, P(x ; 24) is a point such that OP = 25 and $\hat{ROP} = \beta$, where β is an obtuse angle.



5.1.1 Calculate the value of x. (2)

5.1.2 Determine the value of each of the following WITHOUT using a calculator:

(a) $\sin\beta$ (1)

(b) $\cos(180° - \beta)$ (2)

(c) $\tan(-\beta)$ (2)

5.1.3 T is a point on OP such that OT = 15. Determine the coordinates of T WITHOUT using a calculator. (4)

5.2 Determine the value of the following expression:

$$\frac{2\sin x.\cos x\,(1+\tan^2 x)}{\tan x}$$ (4)

5.3 Consider: $\dfrac{1-\cos^2 A}{4\cos(90°+A)}$

5.3.1 Simplify the expression to a single trigonometric term. (3)

5.3.2 Hence, determine the general solution of $\dfrac{1-\cos^2 2x}{4\cos(90°+2x)} = 0{,}21$. (6)

[24]

QUESTION 5

5.1 If $\cos 23° = p$, express, **without the use of a calculator**, the following in terms of p:

5.1.1 $\cos 203°$ (2)

5.1.2 $\sin 293°$ (3)

5.2 Simplify the following expression to a single trigonometric term:

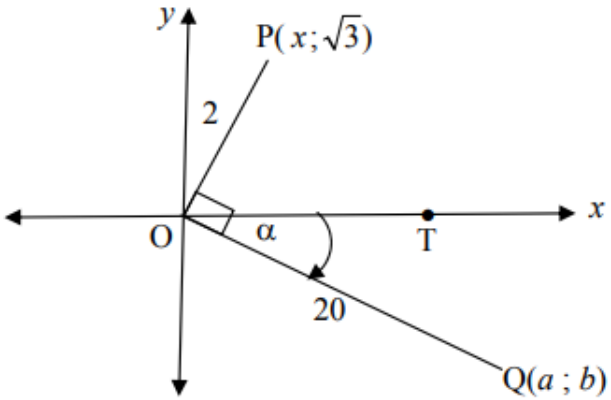
$$\frac{\sin(360° - x).\tan(-x)}{\cos(180° + x).(\sin^2 A + \cos^2 A)}$$ (6)

5.3 5.3.1 Prove the identity: $\frac{\cos x}{1+\sin x} + \frac{1+\sin x}{\cos x} = \frac{2}{\cos x}$ (5)

5.3.2 For which values of x in the interval $0° \leq x \leq 360°$ will the identity in QUESTION 5.3.1 be undefined? (2)

5.4 Determine the general solution of: $\sin 2x = 4\cos 2x$ (5)

5.5 In the diagram below $P(x;\sqrt{3})$ is a point on the Cartesian plane such that $OP = 2$. $Q(a\ ;\ b)$ is a point such that $T\hat{O}Q = \alpha$ and $OQ = 20$. $P\hat{O}Q = 90°$.



5.5.1 Calculate the value of x. (2)

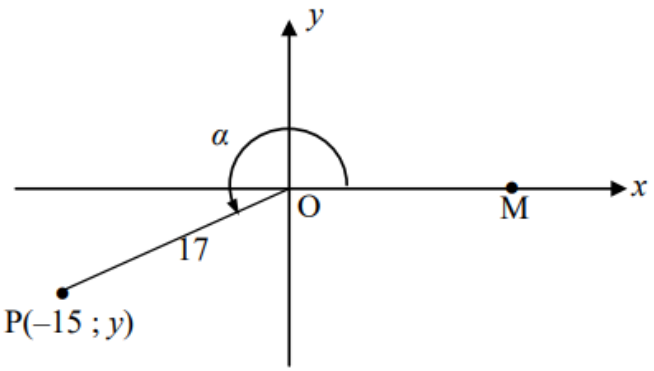
5.5.2 Hence, calculate the size of α. (3)

5.5.3 Determine the coordinates of Q. (5)

[33]

QUESTION 5

5.1 In the diagram, P(–15 ; y) is a point in the Cartesian plane.
OP = 17 units and reflex $M\hat{O}P = \alpha$.



Determine the value of the following **without using a calculator**:

5.1.1 y (2)

5.1.2 $\sin(90° + \alpha)$ (2)

5.1.3 $\tan\beta$, if $\alpha + \beta = 540°$ (3)

5.2 Simplify the following expression to a single trigonometric ratio:

$$\frac{\sin(180° - x) - 2\cos(90° - x)\cos x}{2\cos^2(360° + x) - \cos(-x)}$$ (6)

5.3 5.3.1 Prove that $\frac{1 - \tan x}{1 + \tan x} = \frac{\cos x - \sin x}{\cos x + \sin x}$ (3)

5.3.2 For which value(s) of x in the interval $0° \le x \le 180°$ is the identity in QUESTION 5.3.1 undefined? (2)

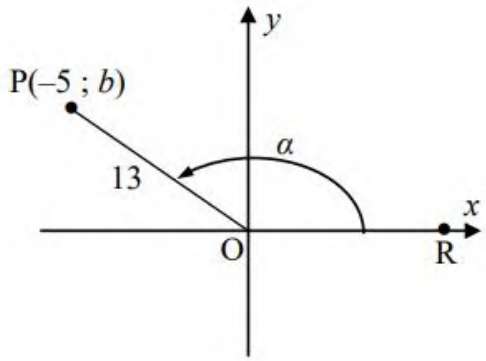
5.4 Determine the general solution of the following equation:

$2\tan x = 5\sin x$ (8)

[26]

QUESTION 5

5.1 In the figure below, the point P(–5 ; b) is plotted on the Cartesian plane. OP = 13 units and $R\hat{O}P = \alpha$.



Without using a calculator, determine the value of the following:

5.1.1 $\cos\alpha$ (1)

5.1.2 $\tan(180° - \alpha)$ (3)

5.2 Consider: $\dfrac{\sin(\theta - 360°)\sin(90° - \theta)\tan(-\theta)}{\cos(90° + \theta)}$

5.2.1 Simplify $\dfrac{\sin(\theta - 360°)\sin(90° - \theta)\tan(-\theta)}{\cos(90° + \theta)}$ to a single trigonometric ratio. (5)

5.2.2 Hence, or otherwise, **without using a calculator,** solve for θ if $0° \le \theta \le 360°$:

$$\frac{\sin(\theta - 360°)\sin(90° - \theta)\tan(-\theta)}{\cos(90° + \theta)} = 0{,}5$$ (3)

5.3 5.3.1 Prove that $\dfrac{8}{\sin^2 A} - \dfrac{4}{1 + \cos A} = \dfrac{4}{1 - \cos A}$. (5)

5.3.2 For which value(s) of A in the interval $0° \le A \le 360°$ is the identity in QUESTION 5.3.1 undefined? (3)

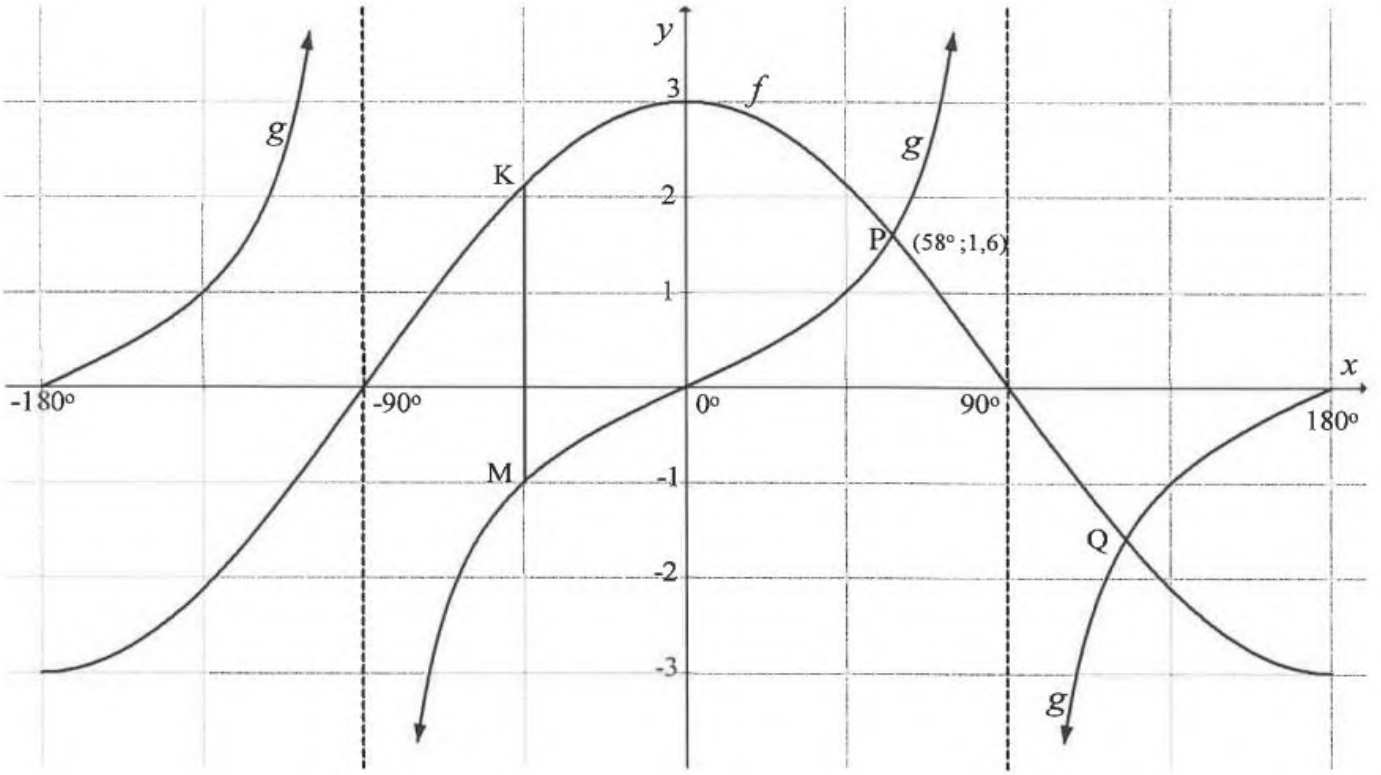
5.4 Determine the general solution of $8\cos^2 x - 2\cos x - 1 = 0$. (6)

[26]

DBE NOV 18 Q6

QUESTION 6

The graphs of the functions $f(x) = a\cos b\theta$ and $g(x) = c\tan\theta$ for $x \in [-180° ; 180°]$ are sketched below. The graphs intersect at P(58° ; 1,6) and Q.



6.1 Write down the range of f. (2)

6.2 If M(–45° ; –1) lies on g, determine the value of c. (1)

6.3 Write down the values of a and b. (2)

6.4 Determine the coordinates of Q. (2)

6.5 K lies on f such that KM is parallel to the y-axis.
Calculate the length of KM . (2)

6.6 If the system of axes is shifted 45° to the left and the graphs remain fixed, write down the equation that is now represented by graph f. (2)
[11]

QUESTION 6

6.1 Determine the general solution for $\sin(x-30^\circ) = \cos 2x$ (5)

6.2 Consider the functions $f(x) = \sin(x-30^\circ)$ and $g(x) = \cos 2x$

6.2.1 Write down the period of g. (1)

6.2.2 State the range of f. (2)

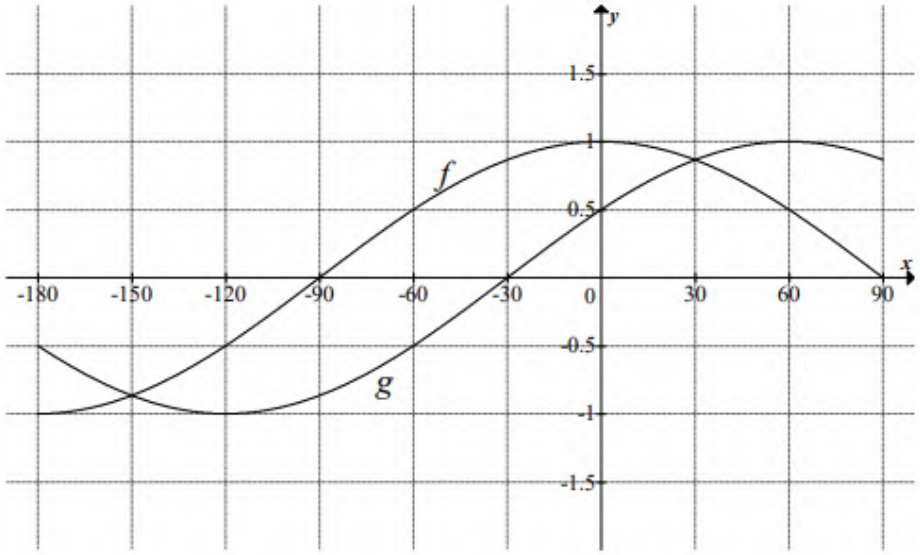
6.2.3 On the grid provided in the ANSWER BOOK, draw the graphs of f and g for $x \in [-90^\circ; 180^\circ]$.
Clearly show ALL intercepts with the axes, turning points and end points. (5)

6.2.4 Write down the x-coordinates of the points of intersection of f and g in the interval $x \in [-90^\circ; 180^\circ]$. (3)

[16]

QUESTION 6

In the diagram the graphs of $f(x)=\cos x$ and $g(x)=\sin(x+b)$ are drawn for the interval $-180° \le x \le 90°$.



6.1 Write down the value of b. (1)

6.2 Write down the period of g. (1)

6.3 Write down the value(s) of x in the interval $-180° \le x \le 90°$ for which $f(x)-g(x)=0$. (2)

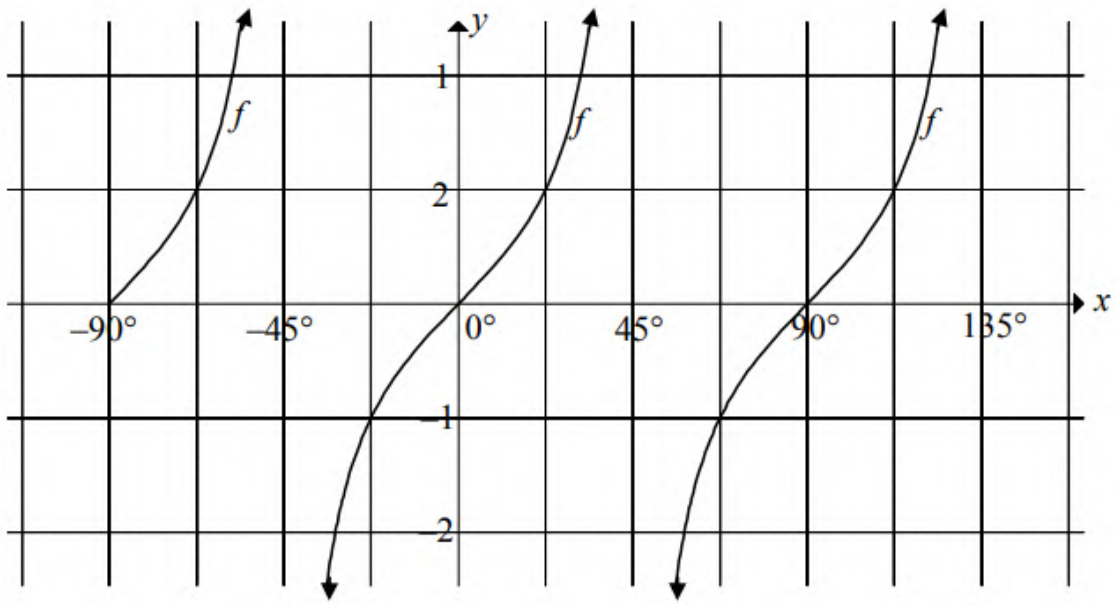
6.4 For which values of x in the interval $-180° \le x \le 90°$ is $\sin(90°-x) > g(x)$? (3)

6.5 The graph of h is obtained by shifting f 3 units upwards. Determine the range of h. (2)

[9]

QUESTION 6

6.1 In the diagram, the graph of $f(x) = \tan bx$ is drawn for the interval $-90° \le x \le 135°$.

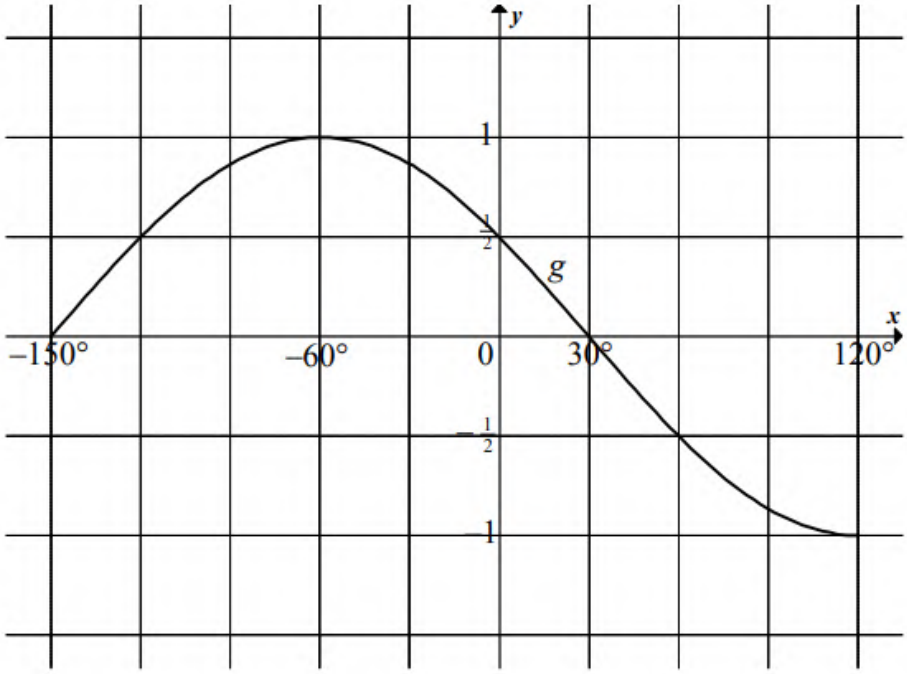


6.1.1 Determine the value of b. (1)

6.1.2 Determine the values of x in the interval $0° \le x \le 135°$ for which $f(x) \le -1$. (2)

6.1.3 Graph h is defined as $h(x) = \tan b(x + 55°)$. Write down the equations of the asymptotes of h in the interval $-90° \le x \le 135°$. (2)

6.2 In the diagram, the graph of $g(x) = \cos(x + 60°)$ is drawn for the interval $-150° \le x \le 120°$.



6.2.1 On the same system of axes, draw the graph of $k(x) = -\sin x$ for the interval $-150° \le x \le 120°$. Show ALL the intercepts with the axes as well as the coordinates of the turning points and end points of the graph. (4)

6.2.2 Determine the minimum value of $h(x) = \cos(x + 60°) - 3$. (2)

6.2.3 Solve the equation $\cos(x + 60°) + \sin x = 0$ for the interval $-150° \le x \le 120°$. (6)

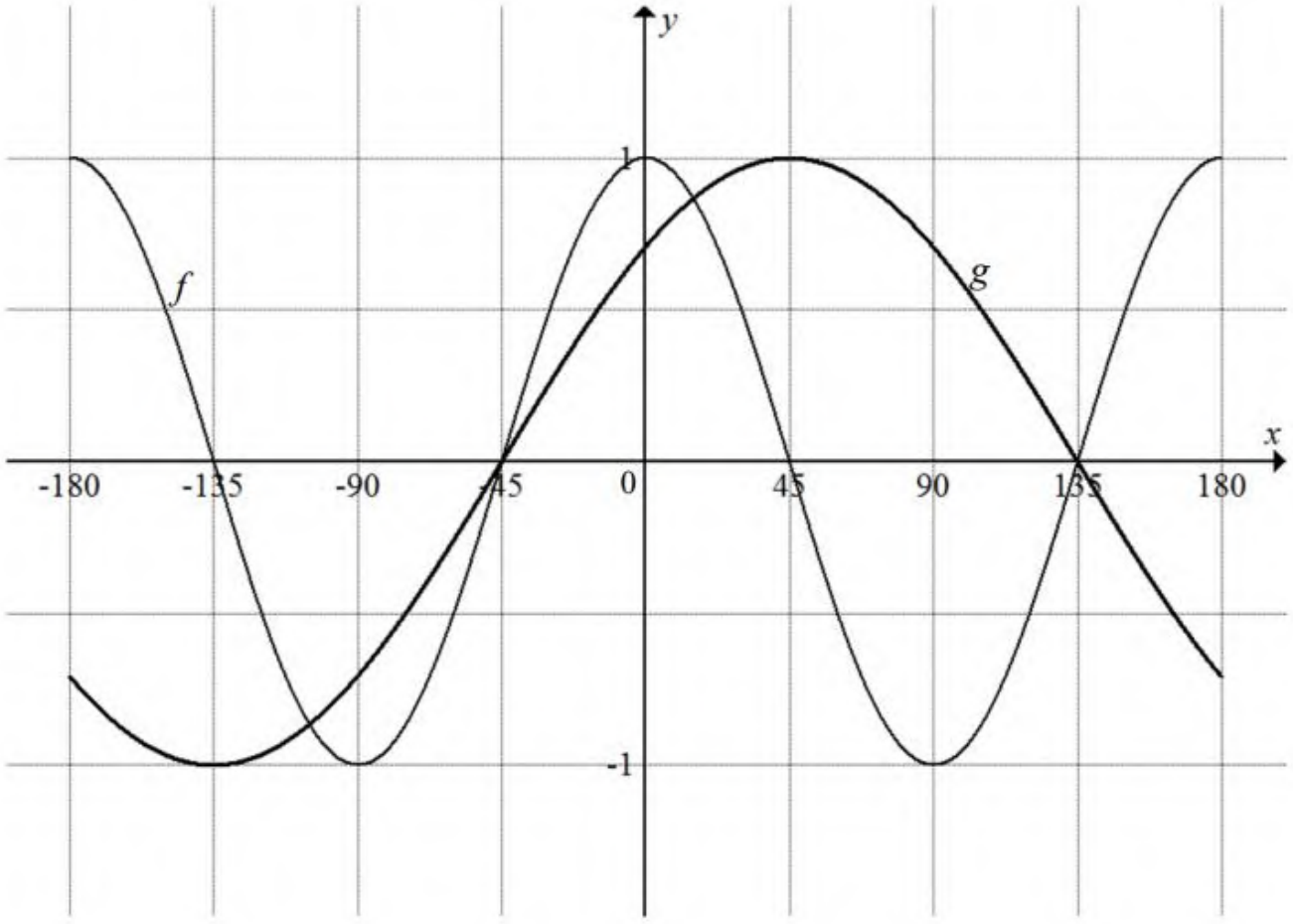
6.2.4 Determine the values of x for the interval $-150° \le x \le 120°$, for which $\cos(x + 60°) + \sin x > 0$. (2)

6.2.5 The function g can also be defined as $y = -\sin(x - \theta)$, where θ is an acute angle. Determine the value of θ. (2)

[21]

QUESTION 6

In the diagram below the graphs of $f(x) = a \cos bx$ and $g(x) = \sin(x + p)$ are drawn for $x \in [-180° ; 180°]$.



6.1 Write down the values of a, b and p. (3)

6.2 For which values of x in the given interval does the graph of f increase as the graph of g increases? (2)

6.3 Write down the period of $f(2x)$. (2)

6.4 Determine the minimum value of h if $h(x) = 3f(x) - 1$. (2)

6.5 Describe how the graph g must be transformed to form the graph k, where $k(x) = -\cos x$. (2)

[11]

QUESTION 6

6.1 Use the system of axes provided on DIAGRAM SHEET 1 to draw the graphs of $f(x) = \cos 2x$ and $g(x) = -\sin x + 1$ for the interval $-180° \leq x \leq 180°$. Show clearly ALL intercepts with the axes, turning points and end points. (6)

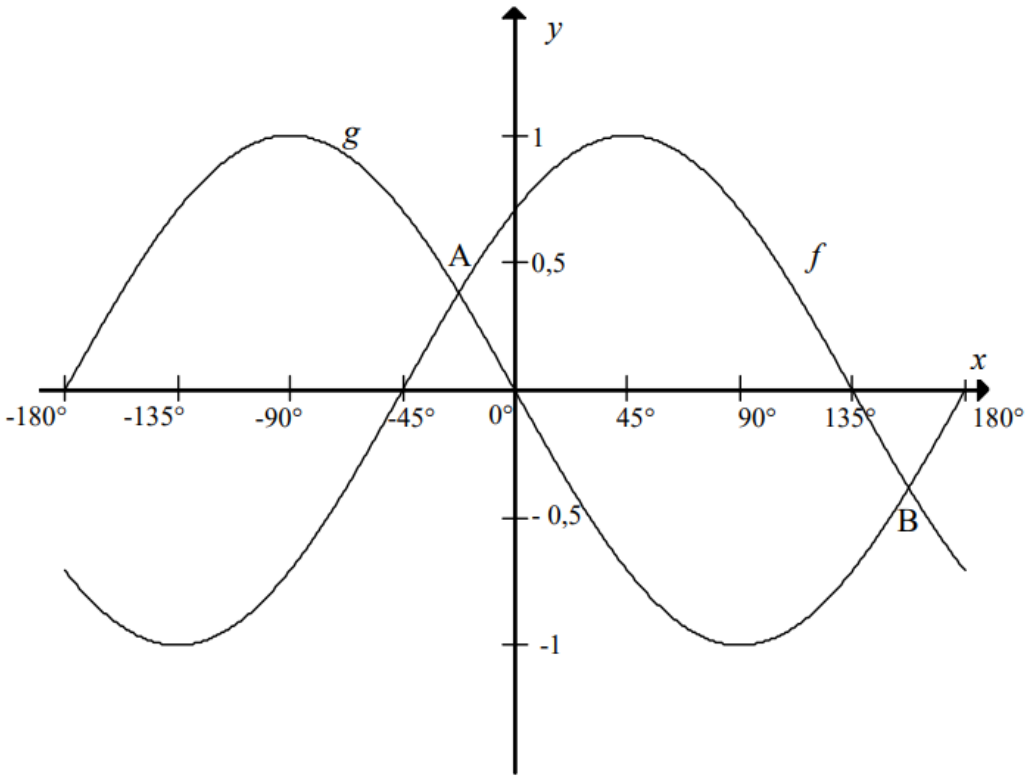
6.2 Write down the period of f. (1)

6.3 For which value(s) of x in the interval $-180° \leq x \leq 180°$ will $g(x) - f(x)$ be a maximum? (1)

6.4 The graph f is shifted $45°$ to the right to obtain a new graph h.
Write down the equation of h in its simplest form. (2)
[10]

QUESTION 6

In the diagram below, the graphs of $f(x) = \cos(x + p)$ and $g(x) = q \sin x$ are shown for the interval $-180° \leq x \leq 180°$.



6.1 Determine the values of p and q. (2)

6.2 The graphs intersect at A(–22,5° ; 0,38) and B. Determine the coordinates of B. (2)

6.3 Determine the value(s) of x in the interval $-180° \leq x \leq 180°$ for which $f(x) - g(x) < 0$. (2)

6.4 The graph f is shifted $30°$ to the left to obtain a new graph h.

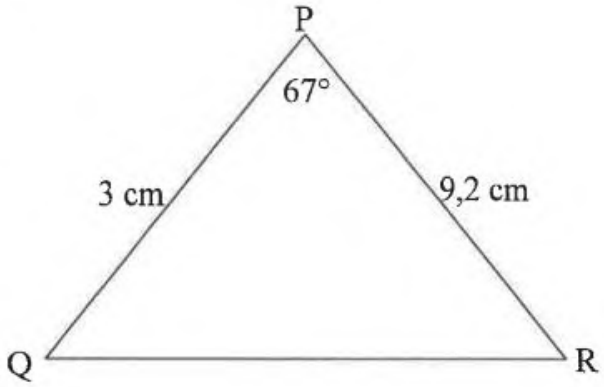
6.4.1 Write down the equation of h in its simplest form. (2)

6.4.2 Write down the value of x for which h has a minimum in the interval $-180° \leq x \leq 180°$. (1)

[9]

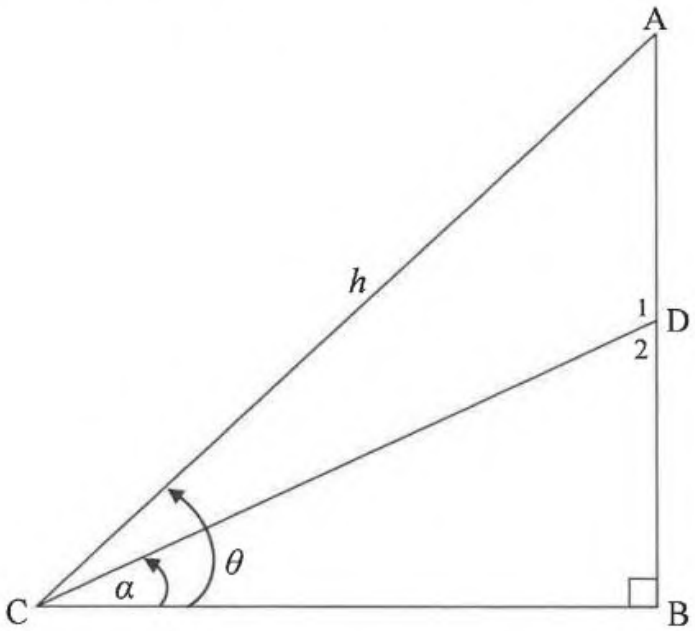
QUESTION 7

7.1 In the diagram, $\hat{P}=67°$, PQ = 3 cm and PR = 9,2 cm.
Determine the length of QR.



(3)

7.2 In the diagram below, $D\hat{C}B = \alpha$, AC = h units and $A\hat{C}B = \theta$.



7.2.1 Determine size of $A\hat{C}D$ in terms of θ and α. (1)

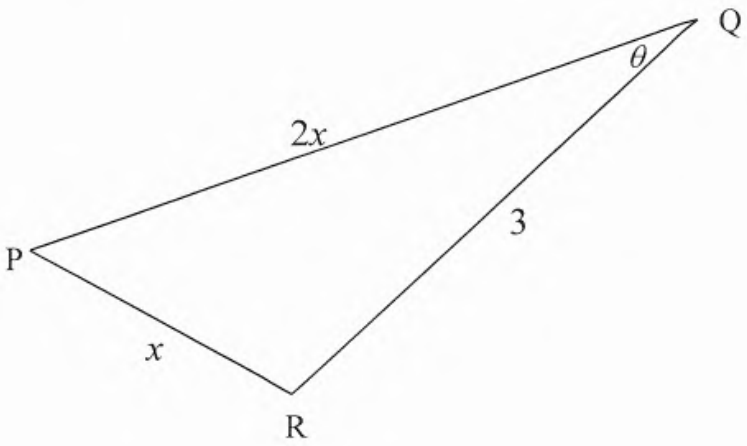
7.2.2 Prove that $AD = \dfrac{h\sin(\theta-\alpha)}{\cos\alpha}$ (4)

7.2.3 Determine the length of AD if $h = 17$ units, $\theta = 58°$ and $\alpha = 23°$. (2)

7.2.4 Calculate the area of ΔADC. (3)
[13]

QUESTION 7

In ΔPQR, $QR = 3$ units, $PR = x$ units, $PQ = 2x$ units and $P\hat{Q}R = \theta$.



7.1 Show that $\cos\theta = \dfrac{x^2 + 3}{4x}$ (3)

7.2 If $x = 2{,}4$ units:

7.2.1 Calculate θ (3)

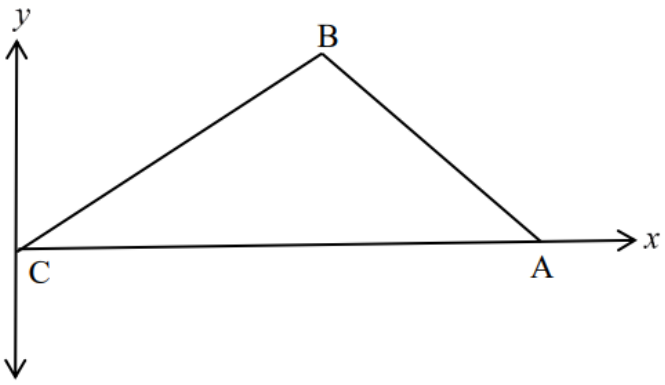
7.2.2 Calculate the area of ΔPQR (2)

7.3 Calculate the values of x for which the triangle exists. (4)

[12]

QUESTION 7

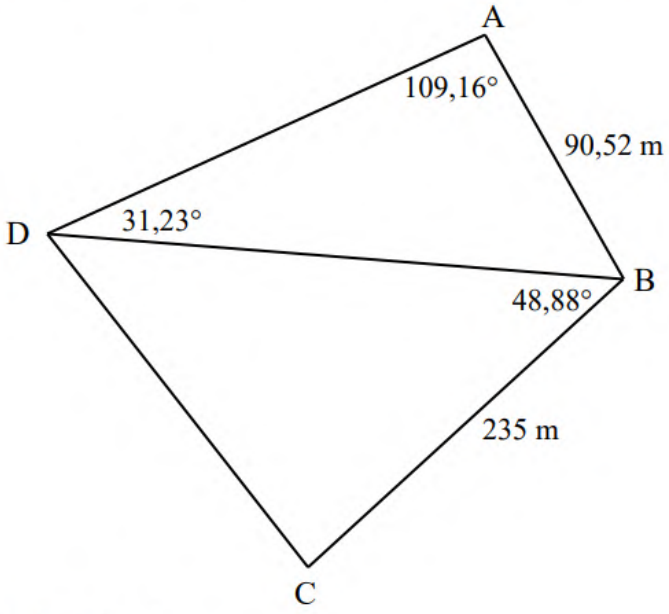
7.1 In the figure below, acute-angled ΔABC is drawn having C at the origin.



7.1.1 Prove that $c^2 = a^2 + b^2 - 2ab\cos C$. (6)

7.1.2 Hence, deduce that $1 + \cos C = \dfrac{(a+b+c)(a+b-c)}{2ab}$ (4)

7.2 Quadrilateral ABCD is drawn with $BC = 235\text{ m}$ and $AB = 90{,}52\text{ m}$. It is also given that $A\hat{D}B = 31{,}23°$; $D\hat{A}B = 109{,}16°$ and $C\hat{B}D = 48{,}88°$.



Determine the length of:

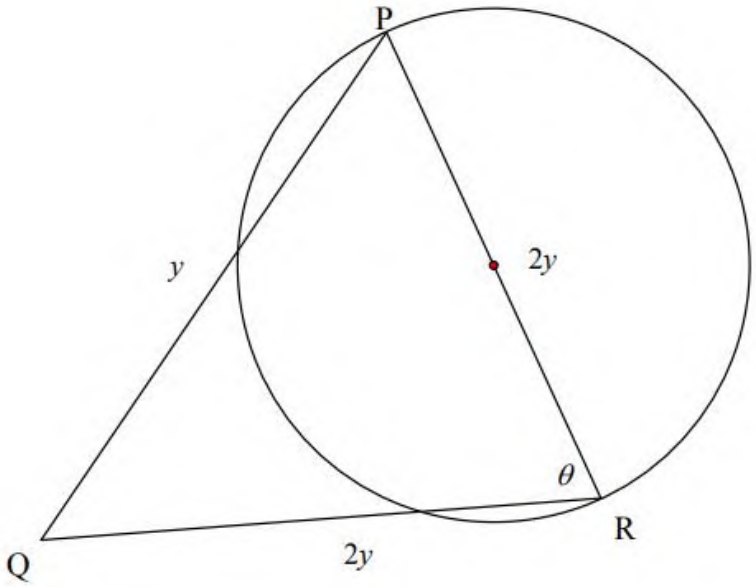
7.2.1 BD (3)

7.2.2 CD (3)

[16]

QUESTION 7

In the diagram, PR is the diameter of the circle. Triangle PQR is drawn with vertex Q outside the circle. $\hat{R} = \theta$, $PR = QR = 2y$ and $PQ = y$.



7.1 Determine the value of $\cos\theta$. (4)

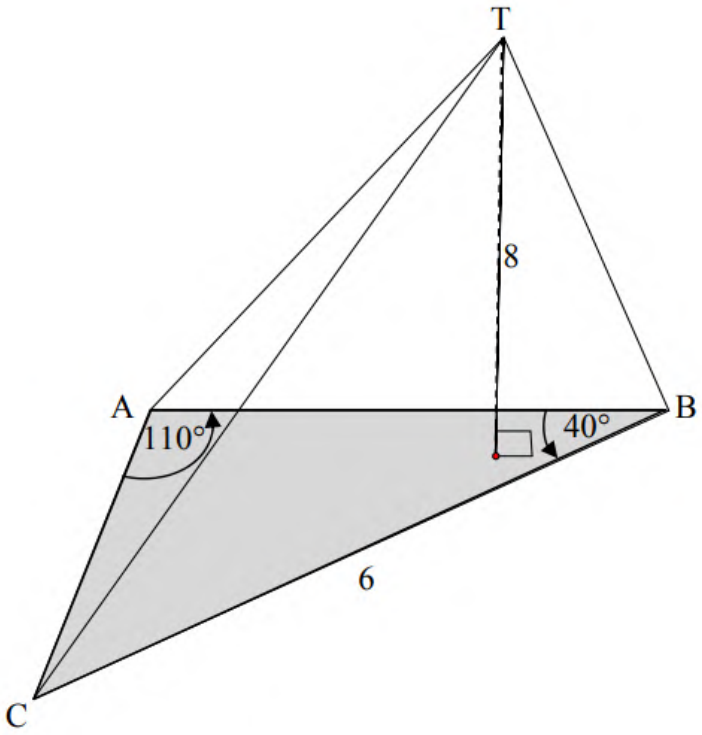
7.2 If QR cuts the circumference of the circle at T, determine PT in terms of y and θ. (3)

[7]

QUESTION 7

Surface area = $\pi r^2 + \pi r S$ where S is the slant height.
Volume = $\frac{1}{3}$ area of base × perpendicular height
Volume = $\frac{1}{3}\pi r^2 h$

7.1 In the diagram, the base of the pyramid is an obtuse-angled ΔABC with $\hat{A} = 110°$, $\hat{B} = 40°$ and BC = 6 metres. The perpendicular height of the pyramid is 8 metres.



7.1.1 Calculate the length of AB. (3)

7.1.2 Calculate the area of the base, that is ΔABC. (2)

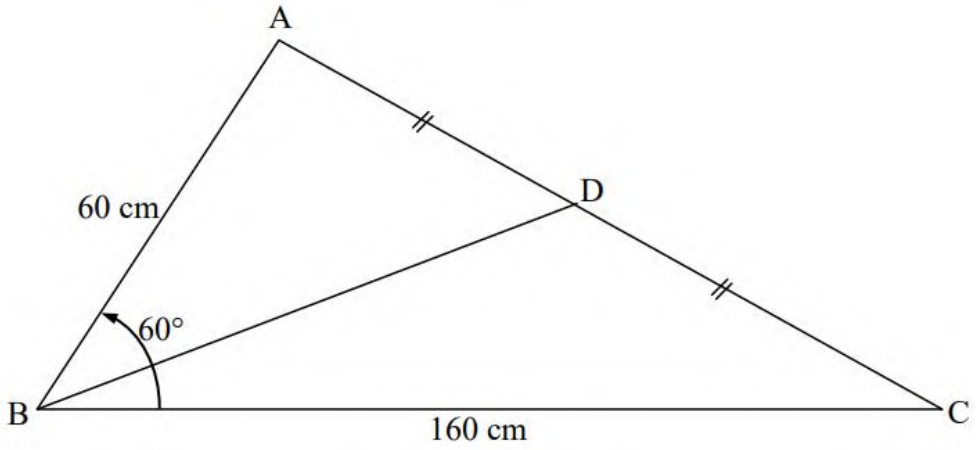
7.1.3 Calculate the volume of the pyramid. (3)

[8]

QUESTION 7

7.1 Prove that in any acute-angled ΔABC, $c^2 = a^2 + b^2 - 2ab\cos C$. (6)

7.2 In ΔABC, AB = 60 cm, BC = 160 cm and $A\hat{B}C = 60°$.
BD is the bisector of AC with D a point on AC.

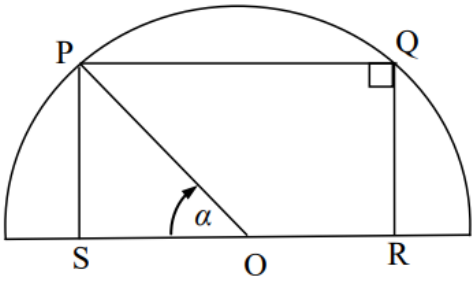


7.2.1 Calculate the length of AC. (3)

7.2.2 Determine the value of sin A. Leave the answer in its simplest surd form. (3)

7.2.3 Calculate the area of ΔABD. Give your answer correct to ONE decimal place. (3)

7.3 In the diagram, O is the centre of a semi-circle.
PQRS is a rectangle drawn inside the semi-circle such that O lies on RS.
$P\hat{O}S = \alpha$.

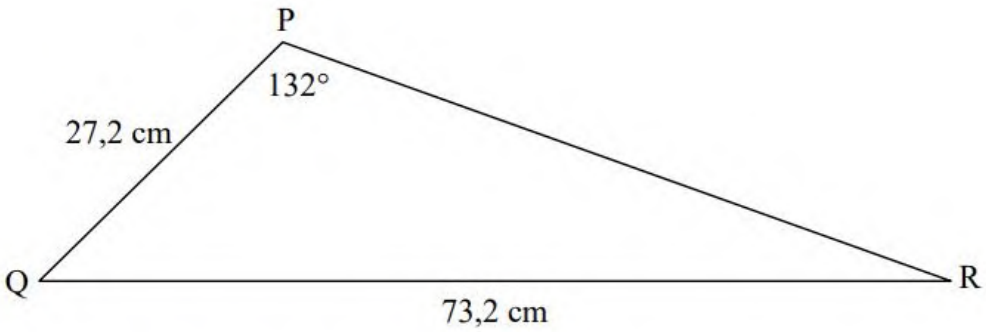


Calculate the size of α for which PQRS will be a square. (3)
[18]

QUESTION 7

7.1 Prove that in any acute-angled ΔABC, $\frac{\sin A}{a} = \frac{\sin C}{c}$. (5)

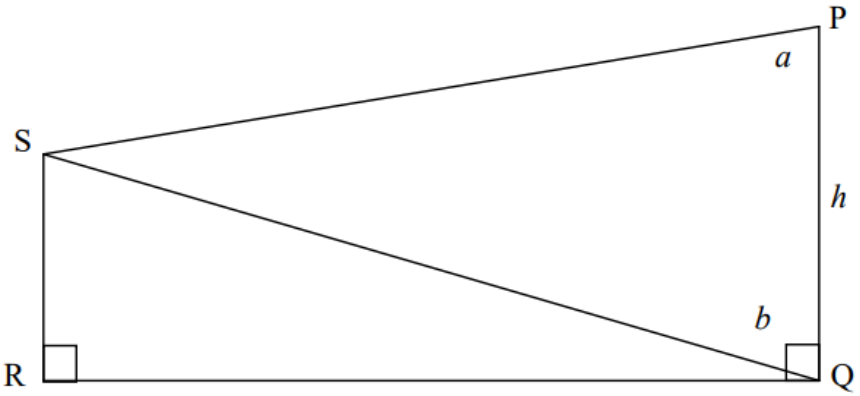
7.2 In ΔPQR, $\hat{P} = 132°$, PQ = 27,2 cm and QR = 73,2 cm.



7.2.1 Calculate the size of $\hat{R}$. (3)

7.2.2 Calculate the area of ΔPQR. (3)

7.3 In the figure below, $S\hat{P}Q = a$, $P\hat{Q}S = b$ and PQ = h. PQ and SR are perpendicular to RQ.



7.3.1 Determine the distance SQ in terms of a, b and h. (3)

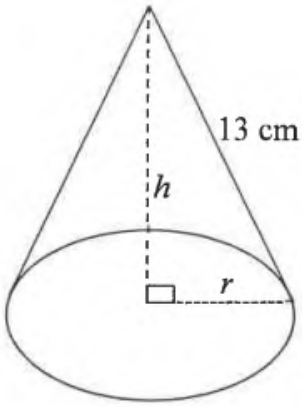
7.3.2 Hence show that $RS = \frac{h \sin a \cos b}{\sin(a+b)}$. (3)

[17]

DBE NOV 18 Q8

QUESTION 8

The diagram below shows a cone with a perpendicular height of h cm, a radius of r cm and a slant height of 13 cm.



Volume of cone $= \frac{1}{3}\pi r^2 h$

Total surface area of the cone
$= \pi r^2 + \pi r s$

8.1 Show that the volume of the cone is given by $V = \frac{169\pi h - \pi h^3}{3}$ (4)

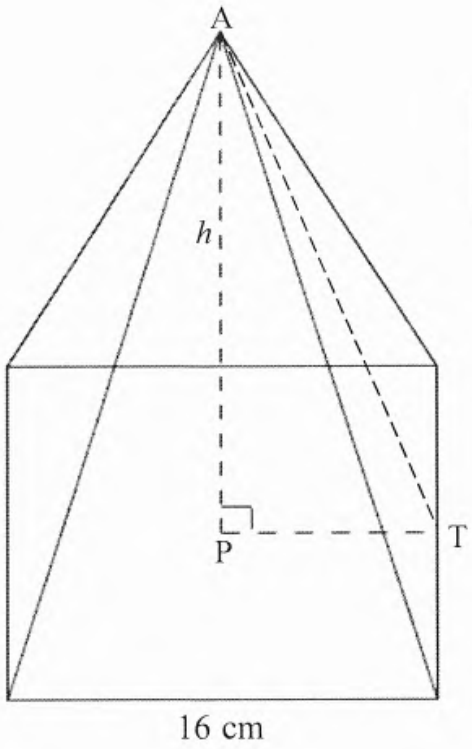
8.2 If $h = 12$ cm, determine the total surface area of the cone. (3)

[7]

QUESTION 8

A pyramid with a square base with a side length of 16 cm is sketched below. P lies on the square base directly below A.

The volume of the pyramid is 640 cm^3.



Volume of pyramid $= \frac{1}{3}Ah$

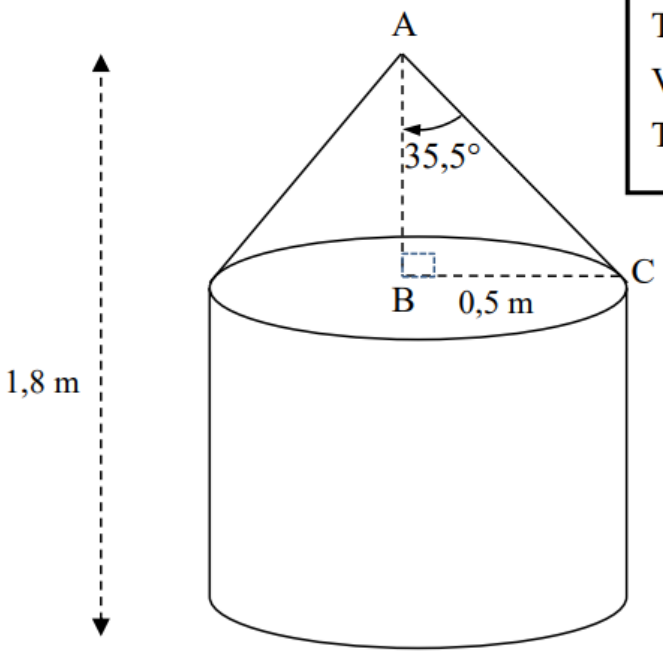
8.1 Show that the perpendicular height of the pyramid, AP, is 7,5 cm. (2)

8.2 Hence, determine the total surface area of the pyramid. (4)

[6]

QUESTION 8

The diagram below shows a water tank which is made up of a cylinder and cone having equal radii. The height of the tank is 1,8 m and the radius is 0,5 m. The angle between the perpendicular height, AB, and the slant height, AC, of the conical section is 35,5°.



Volume of cone $= \frac{1}{3}\pi r^2 h$

Total surface area of cone $= \pi r^2 + \pi r s$

Volume of cylinder $= \pi r^2 h$

Total surface area of cylinder $= 2\pi r^2 + 2\pi r h$

8.1 Calculate the perpendicular height, AB, of the cone. (2)

8.2 When the tank is full, an electric pump switches on and pumps the water from the tank into an irrigation system at a rate of 0,52 m^2/h. The pump automatically switches off when the tank is $\frac{1}{4}$ full.

Calculate how long, in hours, the pump feeds water into the irrigation system. (4)

[6]

QUESTION 8

A cylindrical aerosol can has a lid in the shape of a hemisphere that fits exactly on the top of the can. The height of the can is 16 cm and the radius of the base of the can is 2,9 cm.

Volume of sphere = $\frac{4}{3}\pi r^3$

Surface area of sphere = $4\pi r^2$

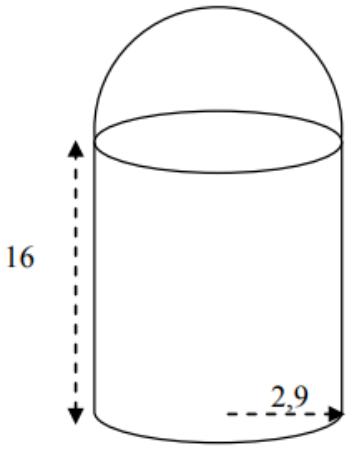


FIGURE 1

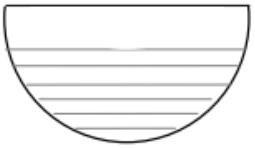


FIGURE 2

8.1 Calculate the surface area of the can with the lid in place, as shown in FIGURE 1. (5)

8.2 If the lid is 80% filled with a liquid, as shown in FIGURE 2, calculate the volume of the liquid in the lid. (3)

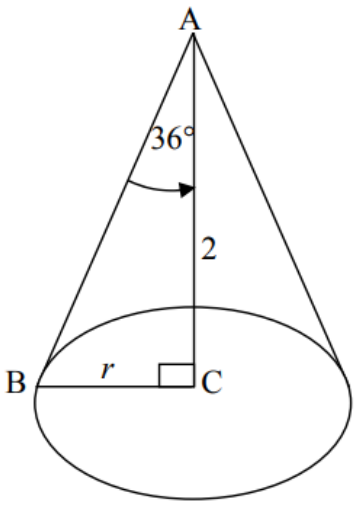
[8]

QUESTION 7

Surface area = $\pi r^2 + \pi r S$ where S is the slant height.
Volume = $\frac{1}{3}$ area of base × perpendicular height
Volume = $\frac{1}{3}\pi r^2 h$

7.2 The perpendicular height, AC, of the cone below is 2 metres and the radius is r. AB is the slant height.

$B\hat{A}C = 36°$

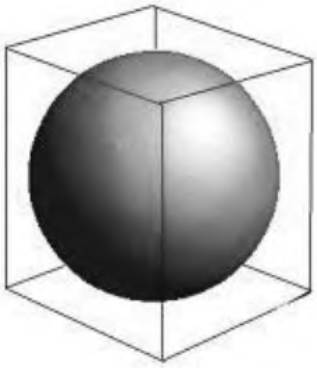


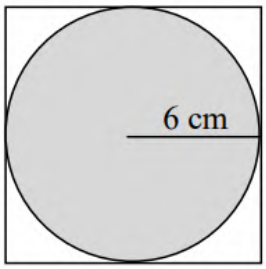
Calculate the total surface area of the cone. (6)

[6]

QUESTION 8

A spherical glass ball is tightly packed in a box. The box is in the shape of a cube, as shown in the picture on the LEFT. The radius of the ball is 6 cm. The diagram on the RIGHT shows the cross-section of the glass ball placed in the box.

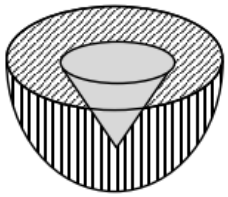




What volume of the box remains after the glass ball is placed in it? **[5]**

QUESTION 8

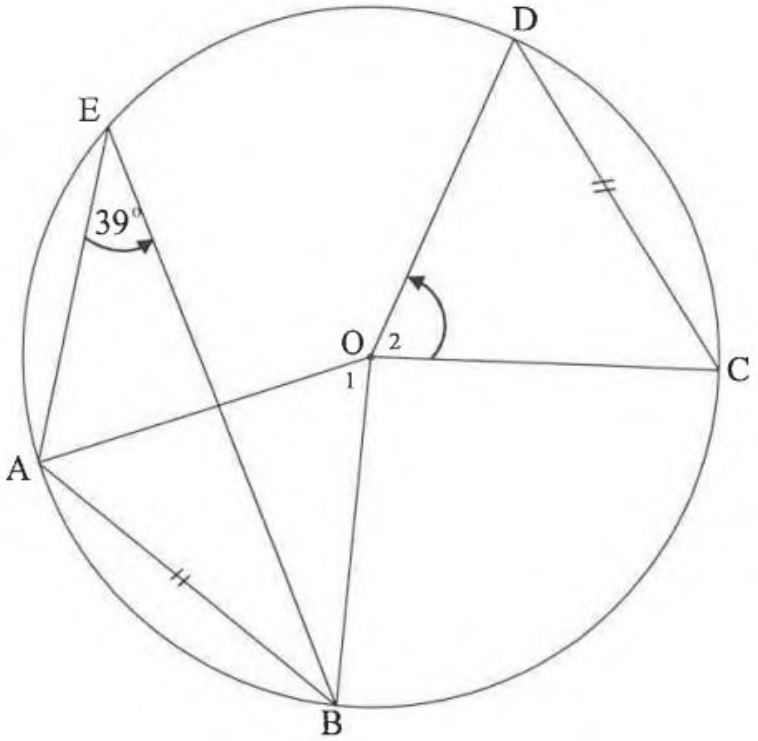
A solid metallic hemisphere has a radius of 3 cm. It is made of metal A. To reduce its weight a conical hole is drilled into the hemisphere (as shown in the diagram) and it is completely filled with a lighter metal B. The conical hole has a radius of 1,5 cm and a depth of $\frac{8}{9}$ cm.



Calculate the ratio of the volume of metal A to the volume of metal B. **[6]**

QUESTION 9

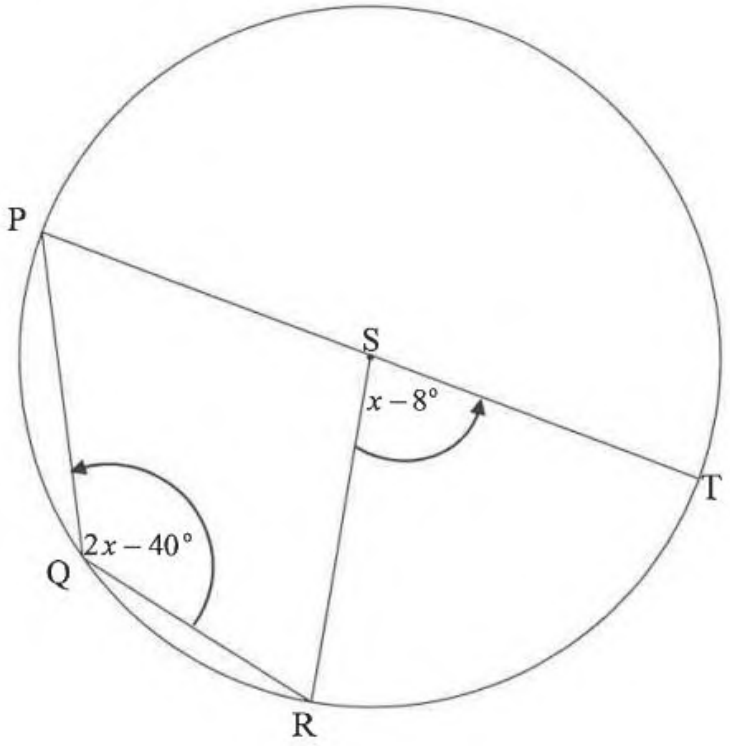
9.1 In the figure, O is the centre of the circle. A, B, C, D and E lie on the circle such that chord AB and chord DC are equal in length and $A\hat{E}B = 39°$.



9.1.1 Determine the size of $\hat{O}_1$. (2)

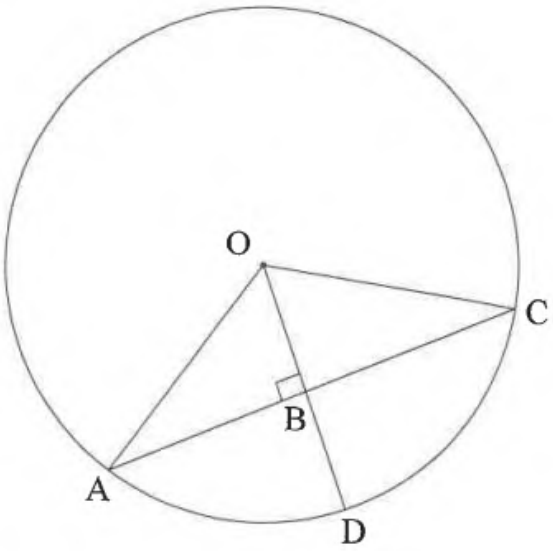
9.1.2 Determine the size of $\hat{O}_2$. (2)

9.2 In the diagram, S is the centre of circle PQRT. PT is a diameter.
$R\hat{S}T = x - 8°$ and $P\hat{Q}R = 2x - 40°$.



Determine the value of x. (4)

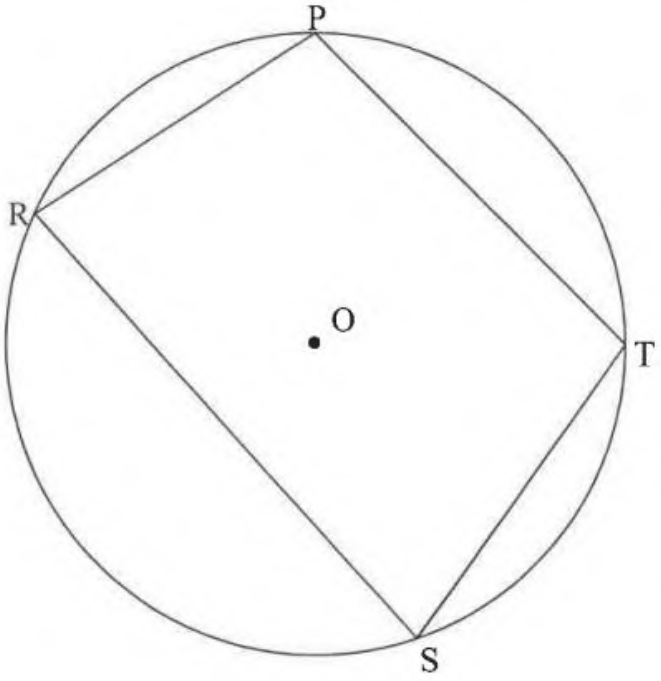
9.3 In the diagram, O is the centre of the circle. Chord AC is perpendicular to radius OD at B. $OB = 2x$ units and $AC = 8x$ units.



Show that the length of BD is $2x\left(\sqrt{5}-1\right)$ units. (5)
[13]

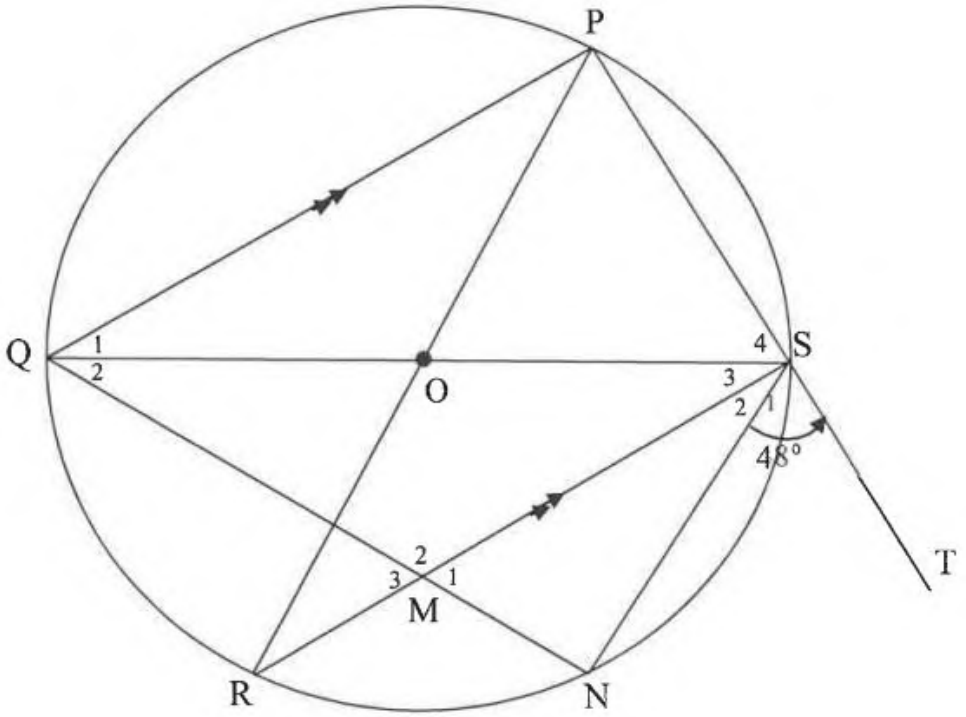
QUESTION 10

10.1 In the diagram below, O is the centre of the circle and PTSR is a cyclic quadrilateral.



10.2 In the figure, QS and PR are diameters of the circle with centre O such that PQ // SR. RS is produced to T. N is a point on the circle such that $\hat{Q}_1 = \hat{Q}_2$. SN is drawn.

RS intersects QN at M. $\hat{S}_1 = 48^\circ$



10.2.1 Determine, with reasons, the size of:

(a) $\hat{Q}_1$ (3)

(b) $\hat{R}$ (2)

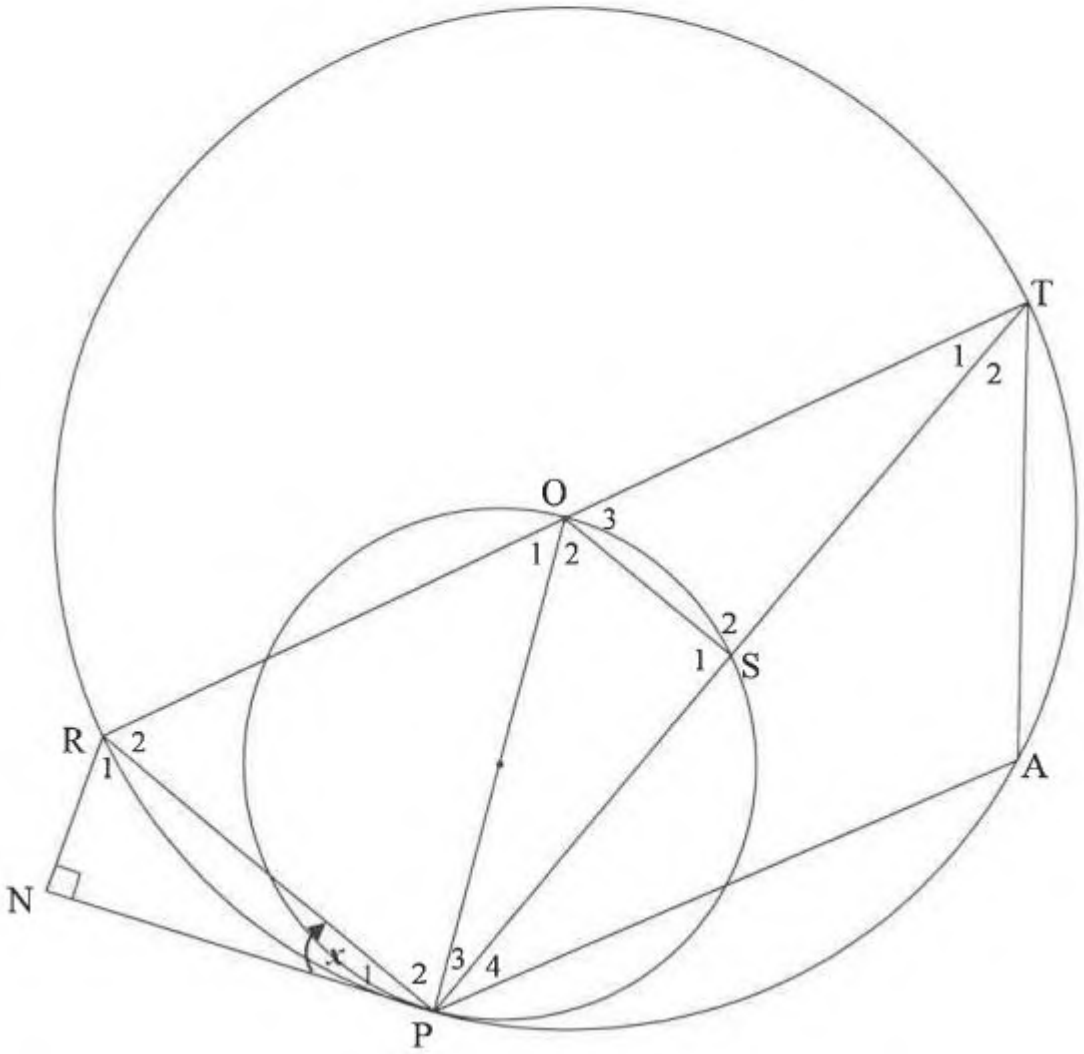
(c) $\hat{M}_1$ (2)

10.2.2 Prove that ST is a tangent to the circle passing through M, N and S. (2)
[14]

QUESTION 11

O is the centre of the larger circle RTAP. OP is the diameter of the smaller circle PSO. NP is a tangent to both circles at P. RN ⊥ NP.

Let $\hat{P}_1 = x$.



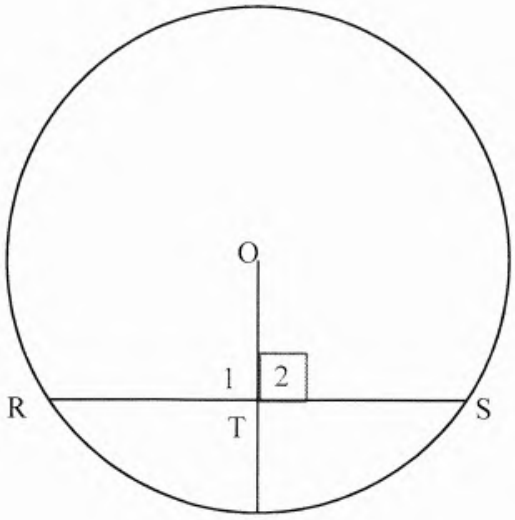
11.1 Prove that PR bisects $O\hat{R}N$. (5)

11.2 Prove that $R\hat{O}S = P\hat{A}T$. (5)

[10]

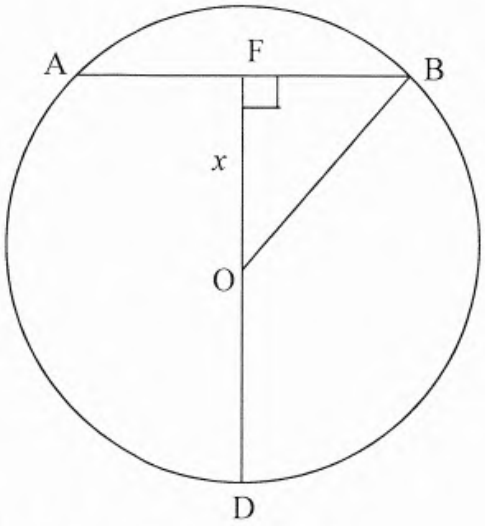
QUESTION 9

9.1 In the diagram below, O is the centre of the circle and point T lies on chord RS. Prove the theorem which states that if $OT \perp RTS$ then $RT = TS$.



(5)

9.2 In the diagram, O is the centre of circle ABD. F is a point on chord AB such that $DOF \perp AB$. $AB = FD = 8$ cm and $OF = x$ cm.

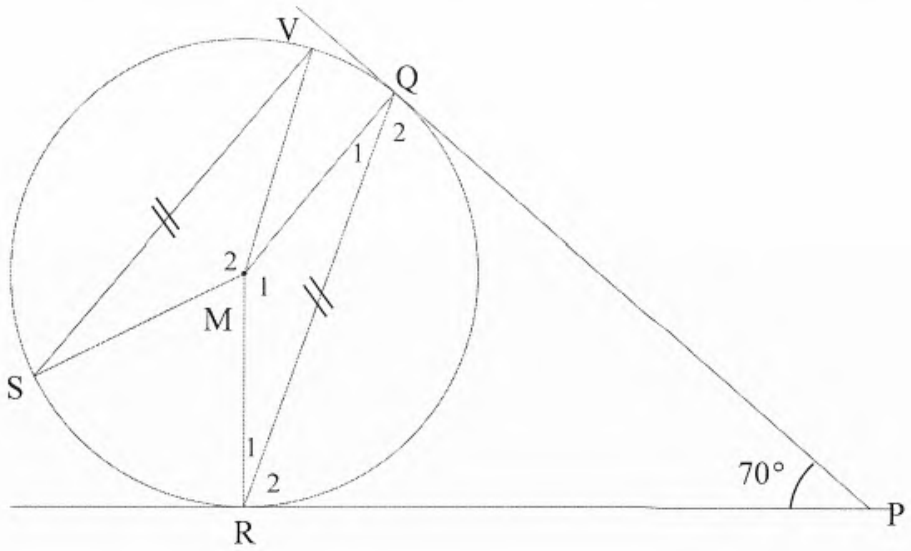


Determine the length of the radius of the circle. (5)

[10]

QUESTION 10

M is the centre of the circle SVQR having equal chords SV and QR.
RP and QP are tangents to the circle at R and Q respectively such that $R\hat{P}Q = 70°$.



10.1 Calculate the size of $\hat{R}_2$. (4)

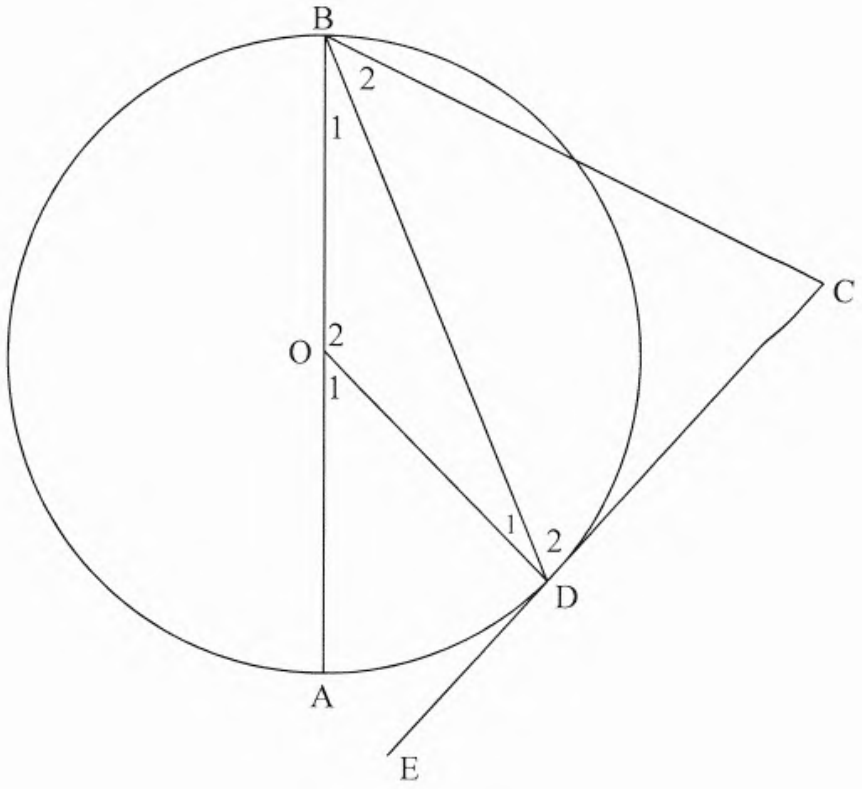
10.2 Calculate the size of $\hat{Q}_1$. (2)

10.3 Determine the size of $\hat{M}_2$. (3)

[9]

QUESTION 11

In the diagram below, O is the centre of the circle. CDE is a tangent to the circle at D. DB bisects $A\hat{B}C$. Let $\hat{B}_1 = x$



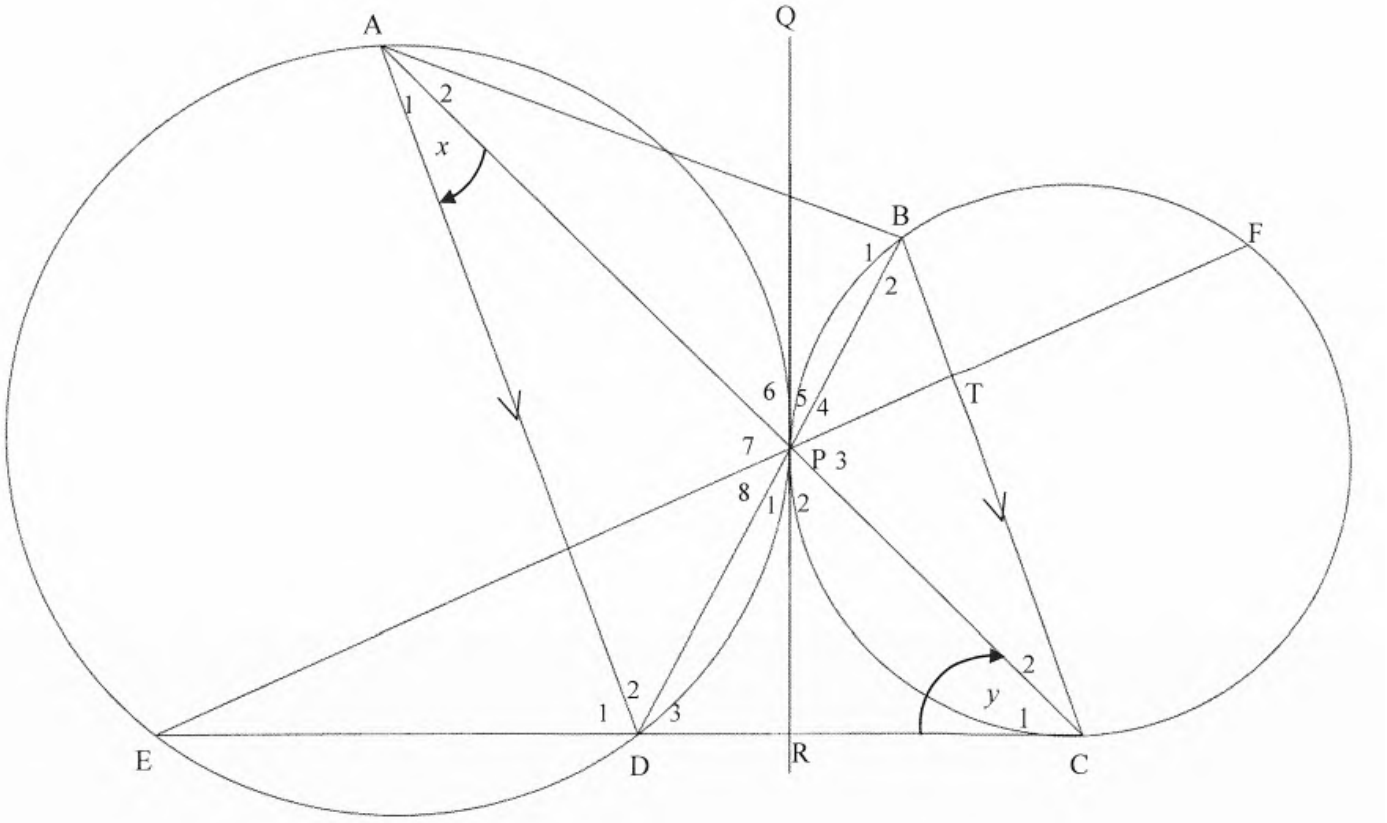
11.1 Prove that BC || OD (4)

11.2 Show that $\hat{C} = 90°$ (3)

[7]

QUESTION 12

In the diagram below, two circles touch each other externally at point P.
QPR is a common tangent to both circles at P. EDRC is a tangent to circle PBFC at C.
$R\hat{C}A = y$ and $D\hat{A}C = x$. AD || BC.



12.1 Name, with reasons, FOUR other angles equal to x. (7)

12.2 Show that $E\hat{P}A = x + y$ (4)

12.3 Determine the numerical value of $x + y$, if it is given that DCTP is a cyclic quadrilateral. (4)

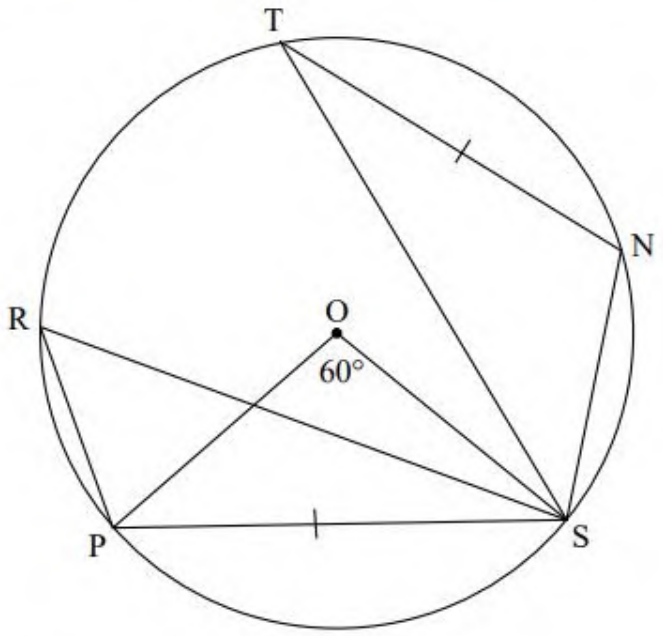
[15]

QUESTION 9

9.1 Complete the statement so that it is TRUE:

The angle subtended by an arc at the centre of a circle is ... (2)

9.2 O is the centre of circle TNSPR. $P\hat{O}S = 60°$ and PS = NT.



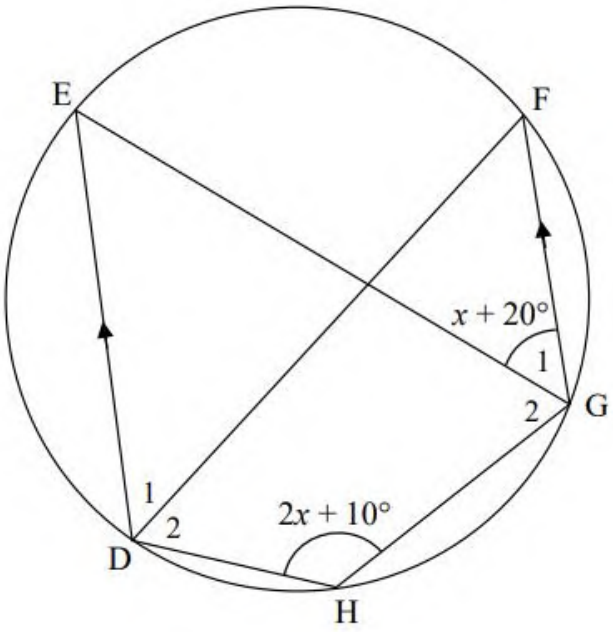
Calculate the size of:

9.2.1 $P\hat{R}S$ (2)

9.2.2 $N\hat{S}T$ (2)
[6]

QUESTION 10

D, E, F, G and H are points on the circumference of the circle.
$\hat{G}_1 = x + 20°$ and $\hat{H} = 2x + 10°$. DE || FG.

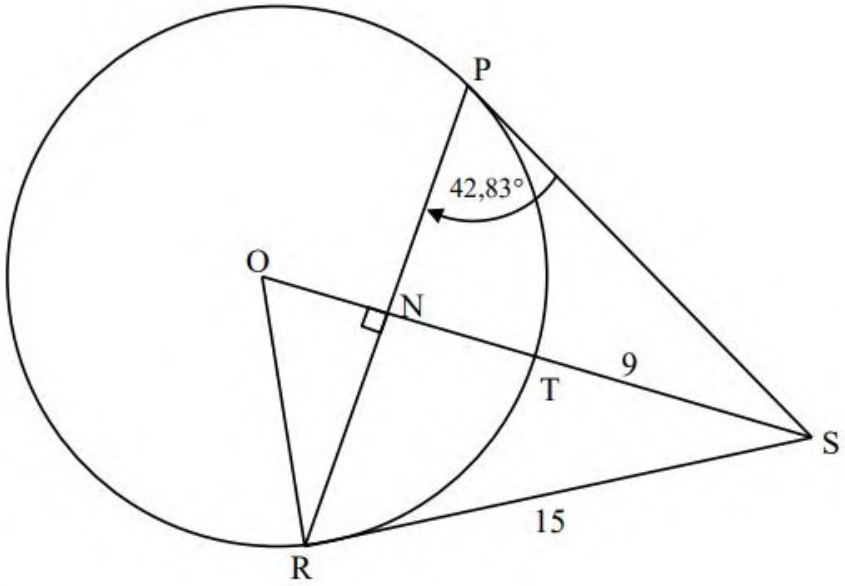


10.1 Determine the size of $D\hat{E}G$ in terms of x. (2)

10.2 Calculate the size of $D\hat{H}G$. (4)
[6]

QUESTION 11

O is the centre of the circle PTR. N is a point on chord RP such that ON ⊥ PR.
RS and PS are tangents to the circle at R and P respectively.
RS = 15 units; TS = 9 units; $R\hat{P}S = 42{,}83°$.

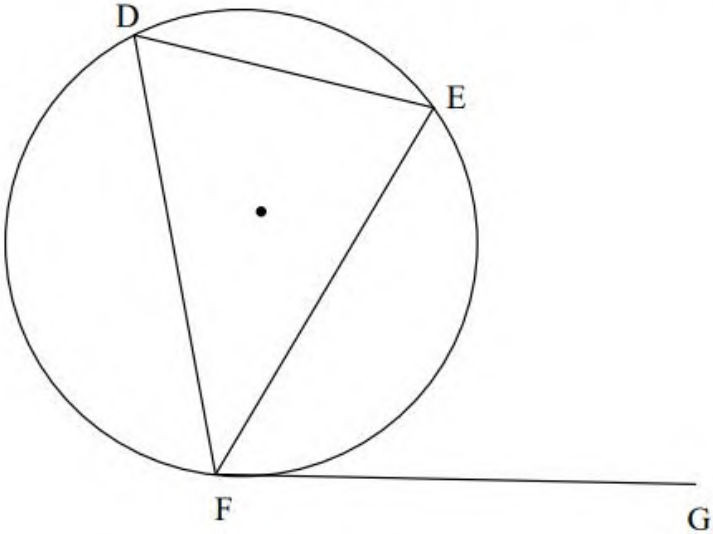


11.1 Calculate the size of $N\hat{O}R$. (5)

11.2 Calculate the length of the radius of the circle. (4)
[9]

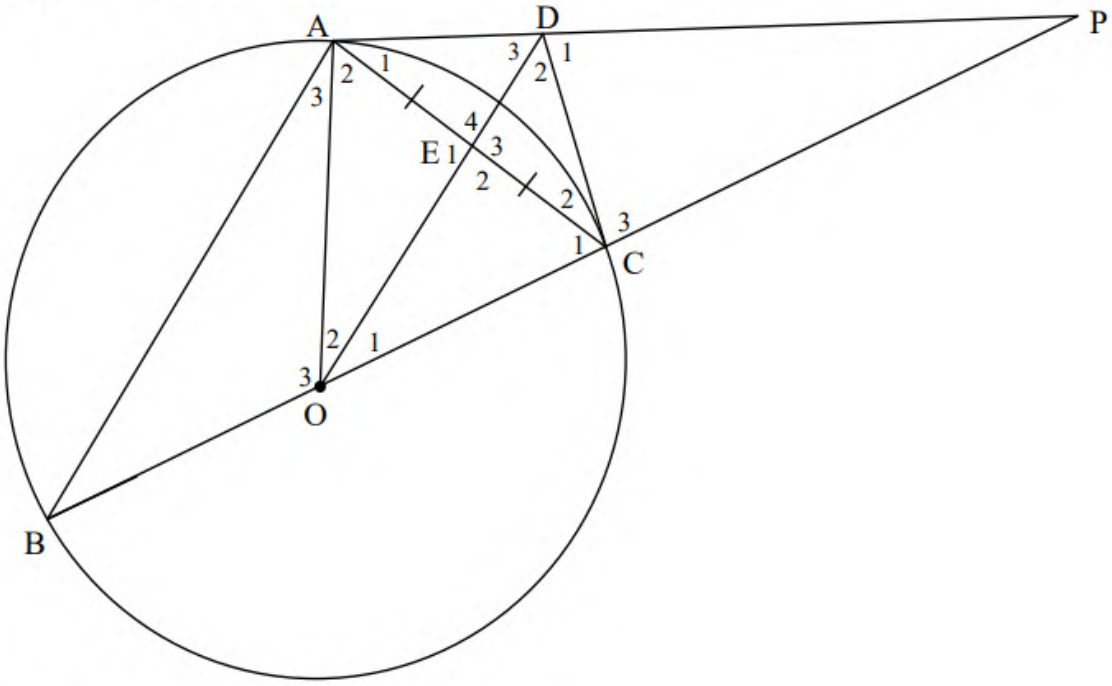
QUESTION 12

12.1 Use the diagram below to prove the theorem which states that $E\hat{F}G = E\hat{D}F$.



(5)

12.2 In the diagram below, BOC is a diameter of the circle. AP is a tangent to the circle at A and AE = EC.



Prove that:

12.2.1 BA || OD (4)

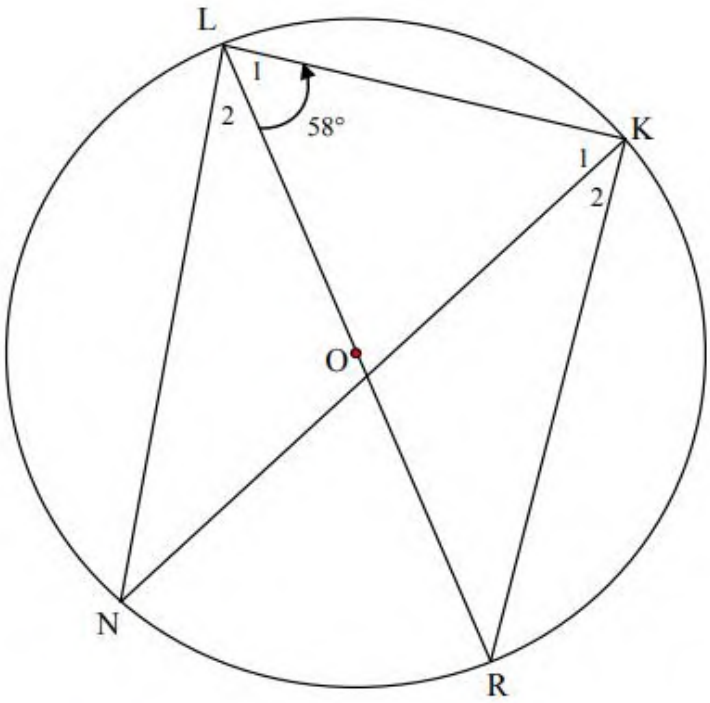
12.2.2 AOCD is a cyclic quadrilateral (5)

12.2.3 DC is a tangent to the circle at C (4)

[18]

QUESTION 9

In the diagram, O is the centre of the circle. Diameter LR subtends $L\hat{K}R$ at the circumference of the circle. N is another point on the circumference and chords LN and KN are drawn. $\hat{L}_1 = 58°$.



Calculate, giving reasons, the size of:

9.1 $L\hat{K}R$ (2)

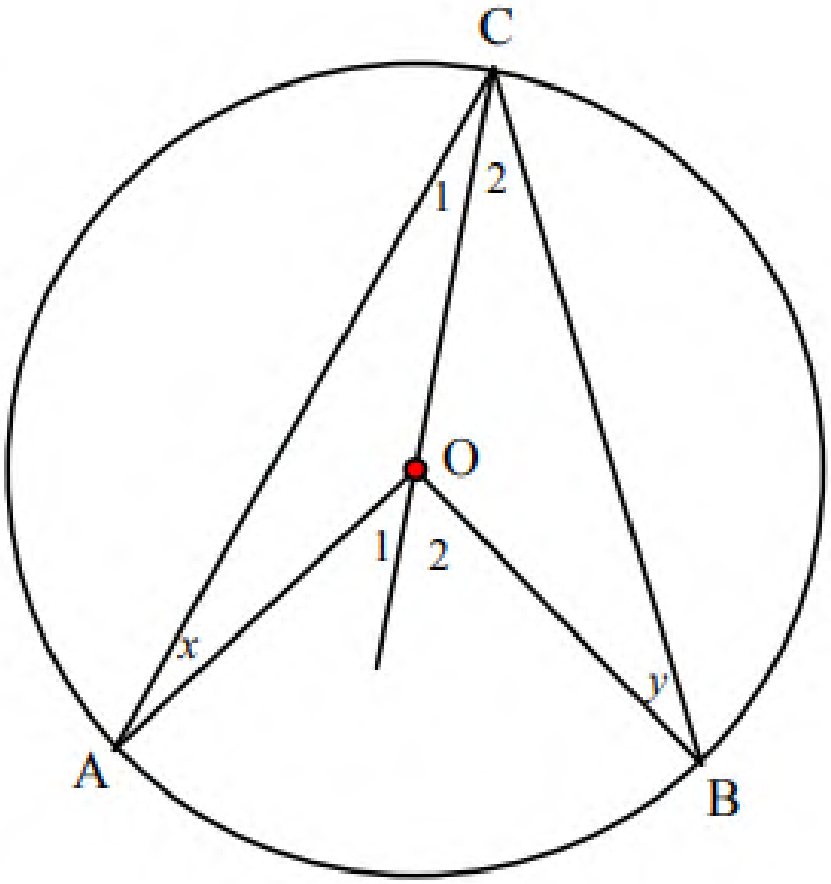
9.2 $\hat{R}$ (2)

9.3 $\hat{N}$ (2)

[6]

QUESTION 10

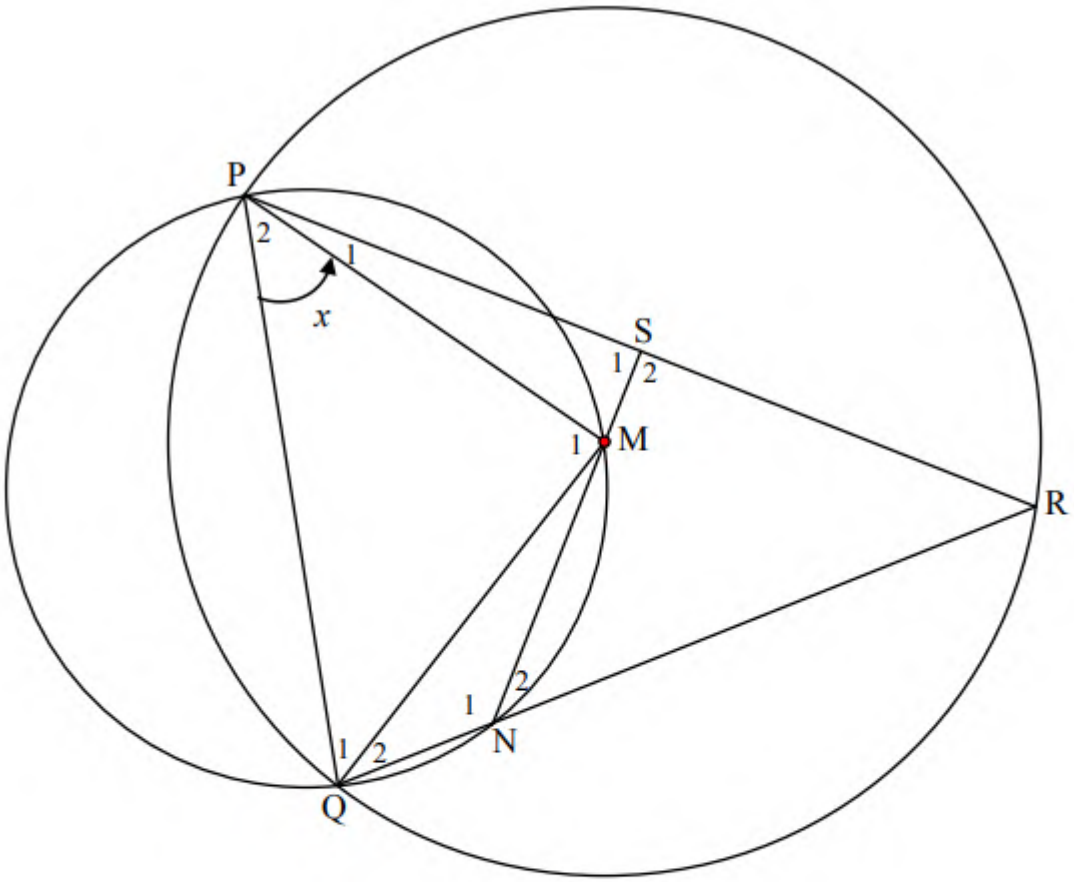
10.1 In the diagram, O is the centre of the circle. A, B and C are points on the circumference of the circle. Chords AC and BC and radii AO, BO and CO are drawn. $\hat{A} = x$ and $\hat{B} = y$.



10.1.1 Determine the size of $\hat{O}_1$ in terms of x. (3)

10.1.2 Hence, prove the theorem that states that the angle subtended by an arc at the centre is equal to twice the angle subtended by the same arc at the circumference, that is $A\hat{O}B = 2A\hat{C}B$. (3)

10.2 In the diagram, PQ is a common chord of the two circles. The centre, M, of the larger circle lies on the circumference of the smaller circle. PMNQ is a cyclic quadrilateral in the smaller circle. QN is produced to R, a point on the larger circle. NM produced meets the chord PR at S. $\hat{P}_2 = x$.



10.2.1 Give a reason why $\hat{N}_2 = x$. (1)

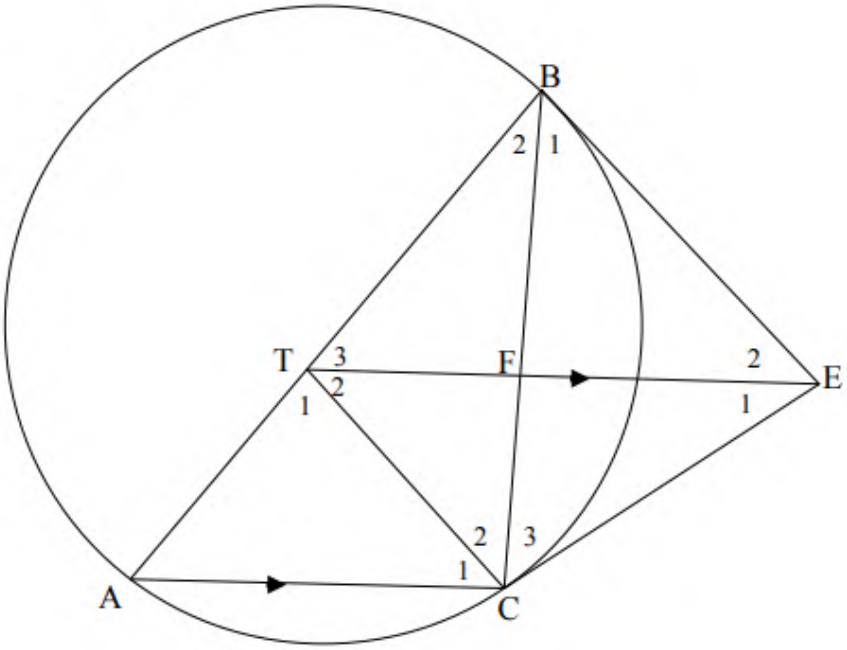
10.2.2 Write down another angle equal in size to x. Give a reason. (2)

10.2.3 Determine the size of $\hat{R}$ in terms of x. (3)

10.2.4 Prove that PS = SR. (3)
[15]

QUESTION 11

In the diagram, the vertices A, B and C of ΔABC are concyclic. EB and EC are tangents to the circle at B and C respectively. T is a point on AB such that TE | | AC. BC cuts TE in F.



11.1 Prove that $\hat{B}_1 = \hat{T}_3$. (4)

11.2 Prove that TBEC is a cyclic quadrilateral. (4)

11.3 Prove that ET bisects $B\hat{T}C$. (2)

11.4 If it is given that TB is a tangent to the circle through B, F and E, prove that TB = TC. (4)

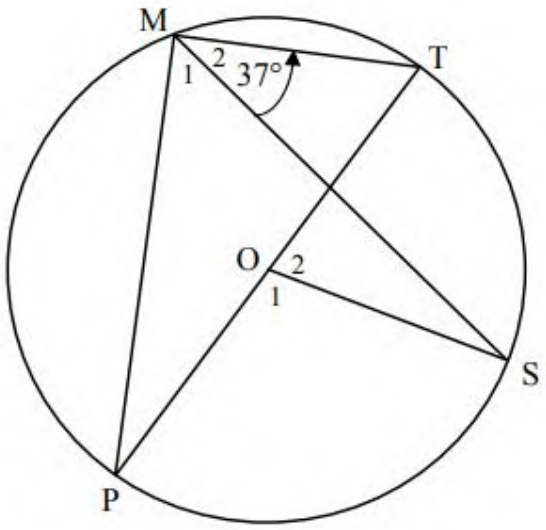
11.5 Hence, prove that T is the centre of the circle through A, B and C. (3)

[17]

QUESTION 8

8.1 In the diagram below, PT is a diameter of the circle with centre O. M and S are points on the circle on either side of PT.
MP, MT, MS and OS are drawn.

$\hat{M}_2 = 37°$

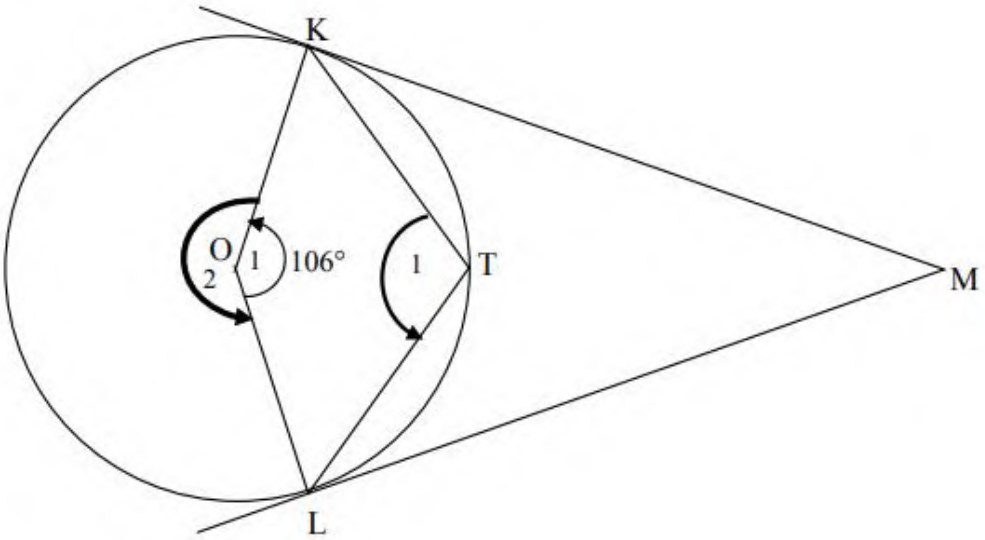


Calculate, with reasons, the size of:

8.1.1 $\hat{M}_1$ (2)

8.1.2 $\hat{O}_1$ (2)

8.2 In the diagram O is the centre of the circle. KM and LM are tangents to the circle at K and L respectively. T is a point on the circumference of the circle. KT and TL are joined. $\hat{O}_1 = 106°$.



8.2.1 Calculate, with reasons, the size of $\hat{T}_1$. (3)

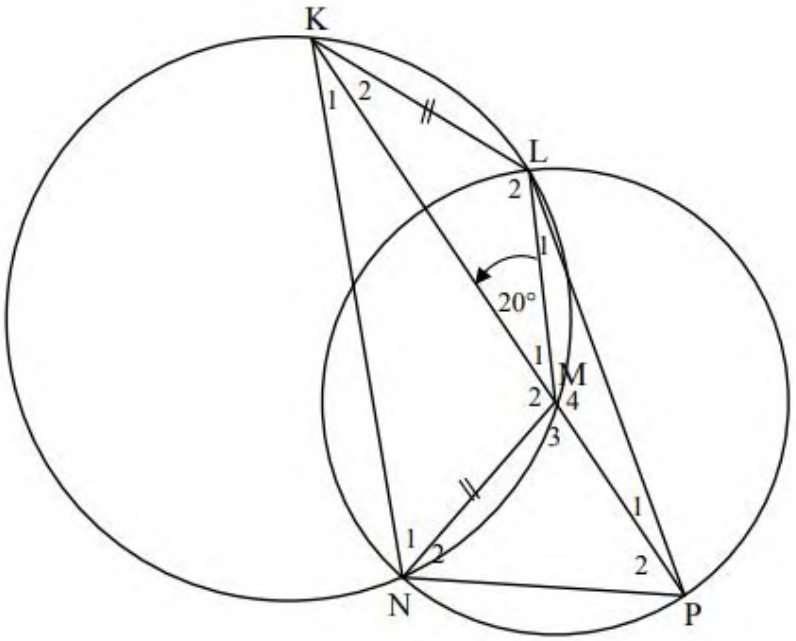
8.2.2 Prove that quadrilateral OKML is a kite. (3)

8.2.3 Prove that quadrilateral OKML is a cyclic quadrilateral. (3)

8.2.4 Calculate, with reasons, the size of $\hat{M}$. (2)
[15]

QUESTION 9

In the diagram M is the centre of the circle passing through points L, N and P.
PM is produced to K. KLMN is a cyclic quadrilateral in the larger circle having KL = MN.
LP is joined. $K\hat{M}L = 20°$.



9.1 Write down, with a reason, the size of $N\hat{K}M$. (2)

9.2 Give a reason why KN | | LM. (1)

9.3 Prove that KL = LM. (2)

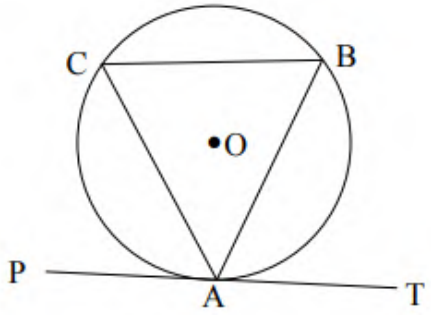
9.4 Calculate, with reasons, the size of:

9.4.1 $K\hat{N}M$ (4)

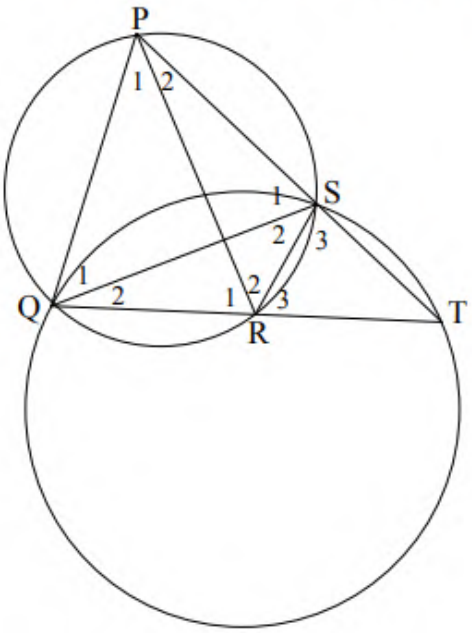
9.4.2 $L\hat{P}N$ (3)
[12]

QUESTION 10

10.1 Use the sketch on DIAGRAM SHEET 4 to prove the theorem which states that $B\hat{A}T = \hat{C}$. (6)



10.2 In the diagram PQ is a tangent to the circle QST at Q such that QT is a chord of the circle and TS produced meets the tangent at P. R is a point on QT such that PQRS is a cyclic quadrilateral in another circle. PR, QS and RS are joined.



10.2.1 Give a reason for each statement. Write down only the reason next to the question number.

Statement	Reason
$\hat{Q}_1 = \hat{T}$	10.2.1 (a)
$\hat{Q}_2 = \hat{P}_2$	10.2.1 (b)

(2)

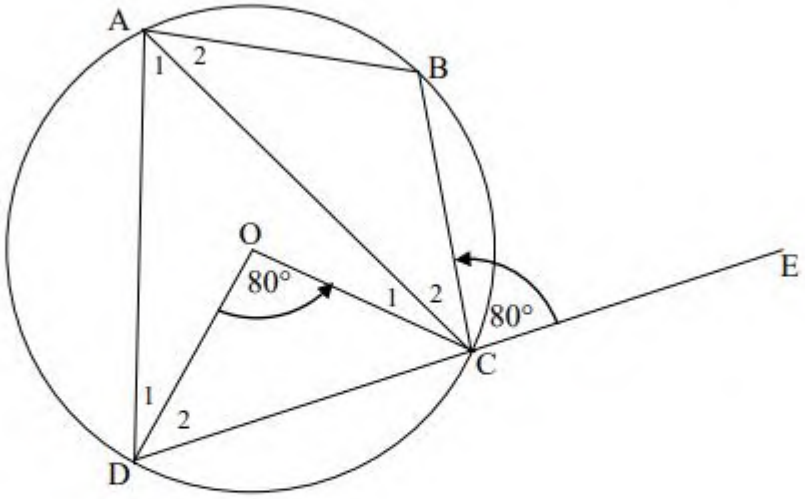
10.2.2 Prove that PQR is an isosceles triangle. (4)

10.2.3 Prove that PR is a tangent to the circle RST at point R. (3)

[15]

QUESTION 9

In the diagram, O is the centre of the circle. A, B, C and D are points on the circumference of the circle. Chord DC is produced to E. AC is drawn. $D\hat{O}C = 80°$ and $B\hat{C}E = 80°$.



9.1 Calculate the size of the following angles:

9.1.1 $D\hat{A}C$ (2)

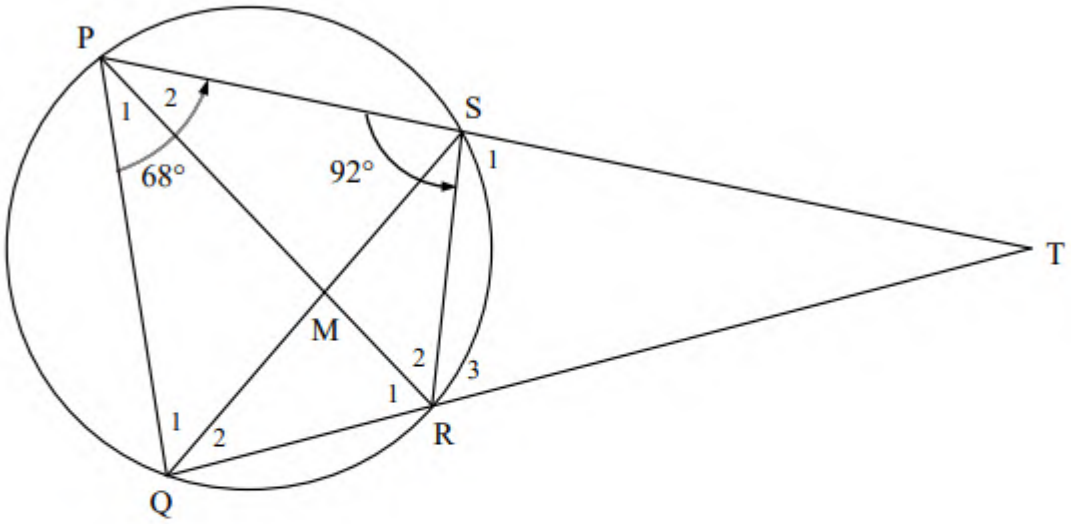
9.1.2 $D\hat{A}B$ (2)

9.1.3 $B\hat{A}C$ (1)

9.2 Hence, or otherwise, prove that DC = BC. (2)
[7]

QUESTION 10

In the diagram, PQRS is a cyclic quadrilateral. PS and QR are produced and meet at T. PR bisects $\hat{QPS}$. Also, $\hat{PSR} = 92°$ and $\hat{QPS} = 68°$.



Calculate the size of the following angles:

10.1 $R\hat{P}T$ (1)

10.2 $T\hat{Q}S$ (2)

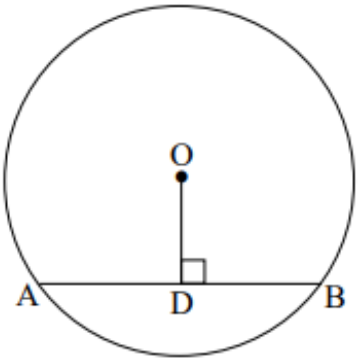
10.3 $P\hat{Q}S$ (3)

10.4 $\hat{T}$ (4)

[10]

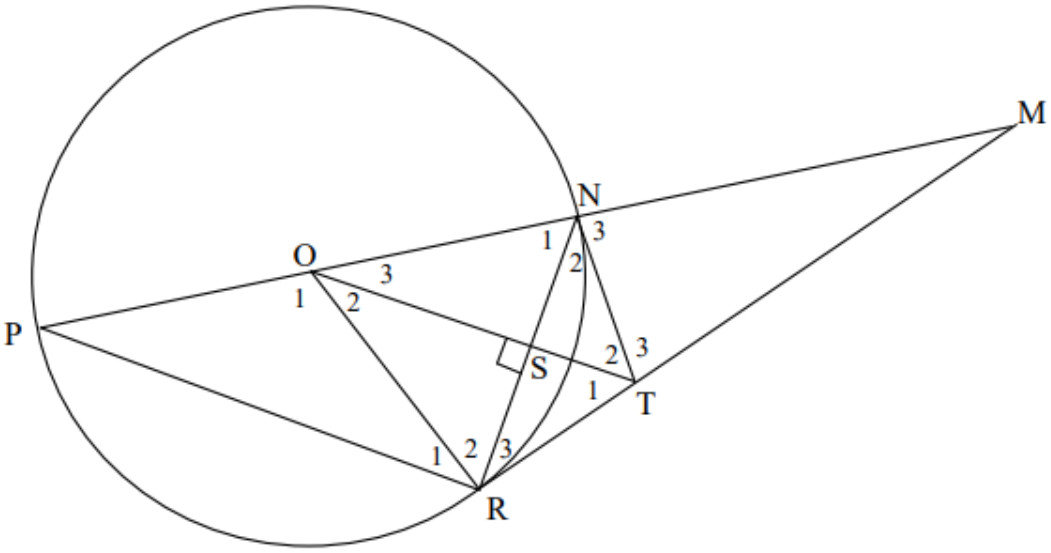
QUESTION 11

11.1 In the diagram, O is the centre of the circle and AB is a chord. D is a point on AB such that OD ⊥ AB. Use Euclidean geometry methods to prove the theorem which states that AD = DB.



(5)

11.2 In the diagram, PN is a diameter of the circle with centre O. RT is a tangent to the circle at R. RT produced and PN produced meet at M. OT is perpendicular to NR. NT and OR are drawn.



11.2.1 Prove that TO ∥ RP. (3)

11.2.2 It is further given that $T\hat{R}N = x$. Name TWO other angles each equal to x. (3)

11.2.3 Prove that NTRO is a cyclic quadrilateral. (2)

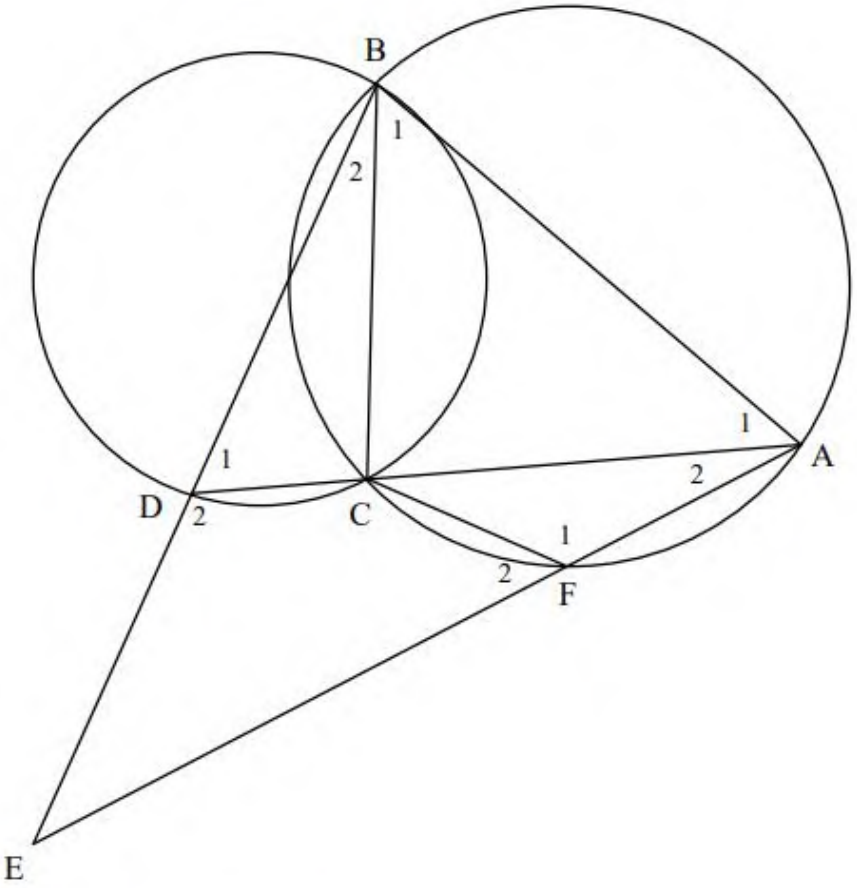
11.2.4 Calculate the size of $\hat{M}$ in terms of x. (3)

11.2.5 Show that NT is a tangent to the circle at N. (3)

[19]

QUESTION 12

In the diagram, ABCF is a cyclic quadrilateral. AB is a tangent to circle BCD at B.



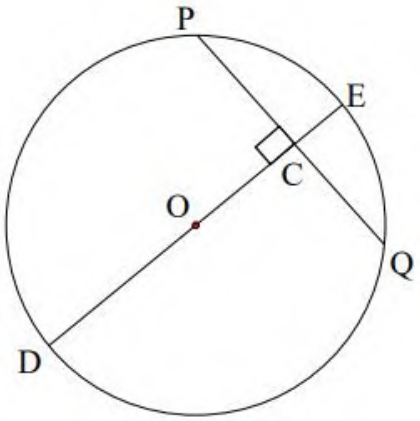
Prove that CDEF is a cyclic quadrilateral. **[5]**

QUESTION 9

9.1 Complete the statement so that it is valid:

The line drawn from the centre of the circle perpendicular to the chord … (1)

9.2 In the diagram, O is the centre of the circle. The diameter DE is perpendicular to the chord PQ at C. DE = 20 cm and CE = 2 cm.



Calculate the length of the following with reasons:

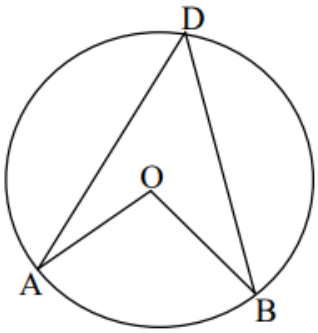
9.2.1 OC (2)

9.2.2 PQ (4)

[7]

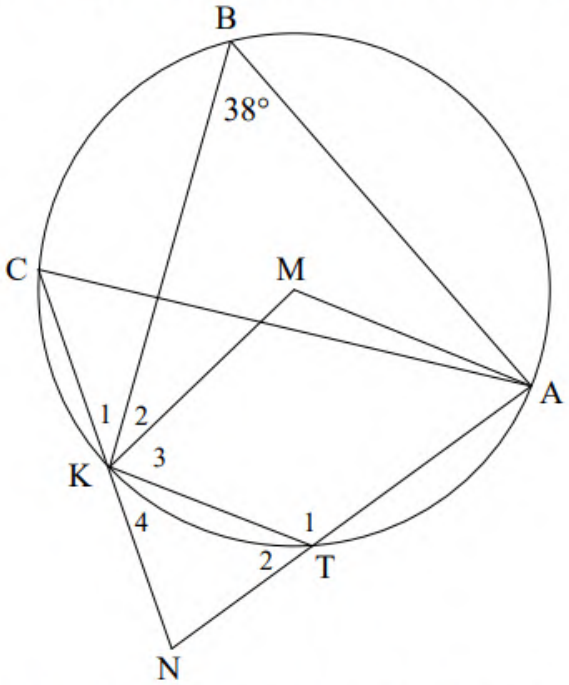
QUESTION 10

10.1 In the diagram, O is the centre of the circle and A, B and D are points on the circle. Use Euclidean geometry methods to prove the theorem which states that $A\hat{O}B = 2A\hat{D}B$.



(5)

10.2 In the diagram, M is the centre of the circle. A, B, C, K and T lie on the circle. AT produced and CK produced meet in N. Also NA = NC and $\hat{B} = 38°$.



10.2.1 Calculate, with reasons, the size of the following angles:

(a) $K\hat{M}A$ (2)

(b) $\hat{T}_2$ (2)

(c) $\hat{C}$ (2)

(d) $\hat{K}_4$ (2)

10.2.2 Show that NK = NT. (2)

10.2.3 Prove that AMKN is a cyclic quadrilateral. (3)

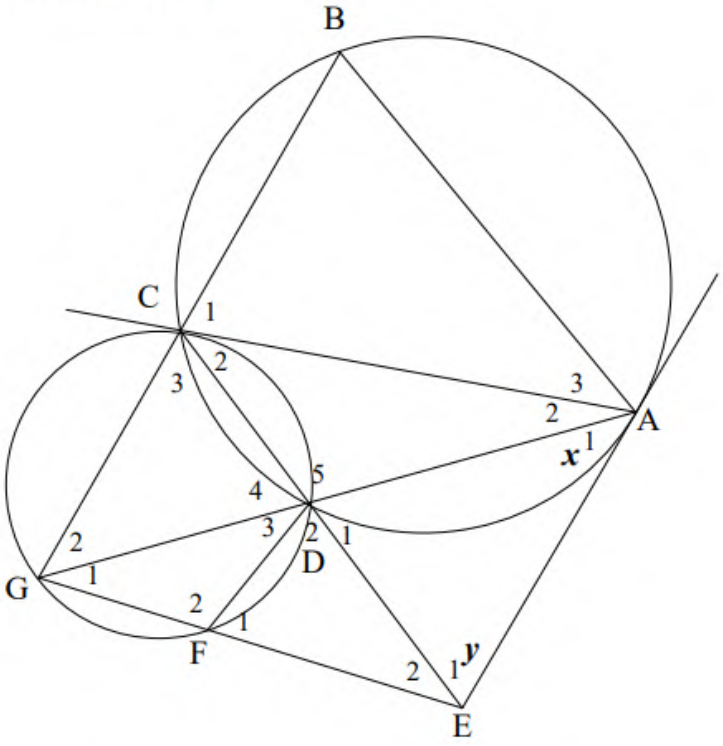
[18]

QUESTION 11

11.1 Complete the following statement so that it is valid:

The angle between a chord and a tangent at the point of contact is … (1)

11.2 In the diagram, EA is a tangent to circle ABCD at A.
AC is a tangent to circle CDFG at C.
CE and AG intersect in D.



If $\hat{A}_1 = x$ and $\hat{E}_1 = y$, prove the following with reasons:

11.2.1 BCG || AE (5)

11.2.2 AE is a tangent to circle FED (5)

11.2.3 AB = AC (4)
[15]