LIMPOPO
PROVINCIAL GOVERNMENT
REPUBLIC OF SOUTH AFRICA

SEKHUKHUNE SOUTH DISTRICT

SENIOR CERTIFICATE

GRADE 12

MATHEMATICS
PAPER 2

PRE-TRIAL 2021

MARKS: 150

TIME: 3 HOURS

This question paper consists of 11 pages and 3-page diagram sheets.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.

2. Answer ALL the questions in the answer book.

3. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.

4. ANSWERS ONLY will not necessarily be awarded full marks.

5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

6. If necessary, round answers off to TWO decimal places, unless stated otherwise.

7. Diagrams are NOT necessarily drawn to scale.

8. A diagram sheets for questions 1.3, 1.5, 8.1 and 9 are included at the end of the question paper.

9. Number the answers correctly according to the numbering system used in this question paper.

10. Write legibly and present your work neatly.

QUESTION 1

Mathematical Literacy teachers usually complain about their learners' language and reading skills. The data below shows the percentages which 8 candidates obtained for English and Mathematical Literacy during the June Examination.

Mathematical Literacy	25	38	40	47	12	49	54	59
English	34	53	62	44	20	50	61	54

1.1 Calculate the:

1.1.1 mean percentage of Mathematical Literacy. (2)

1.1.2 standard deviation of Mathematical Literacy. (2)

1.2 Determine the number of learners whose percentages in Mathematical Literacy lie within ONE standard deviation of the mean. (3)

1.3 Use the grid provided to draw a scatter plot to represent the above data. (3)

1.4 Calculate an equation for the least squares regression line (line of best fit) for the data. (3)

1.5 Draw the regression line on the scatter plot. (2)

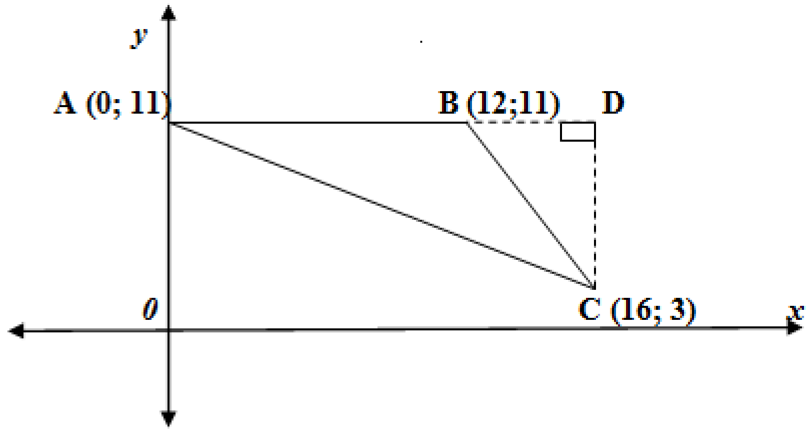
1.6 Describe the trend of the data by making use of the correlation coefficient. (3)

1.7 Estimate Mathematical Literacy mark a learner would get if his English mark is 58%. (2)

[20]

QUESTION 2

In the diagram below A(0;11), B(12;11) and C(16;3) are the vertices of Δ ABC, with height CD .



2.1 Write down the equation and the length of line AB. (3)

2.2 Write down the coordinates of point D. (2)

2.3 Determine the coordinates of M, the midpoint of AC. (2)

2.4 Determine the equation of the perpendicular bisector of AC. (4)

2.5 Does the line in 2.4 pass through B? Justify your answer with relevant calculations. (2)

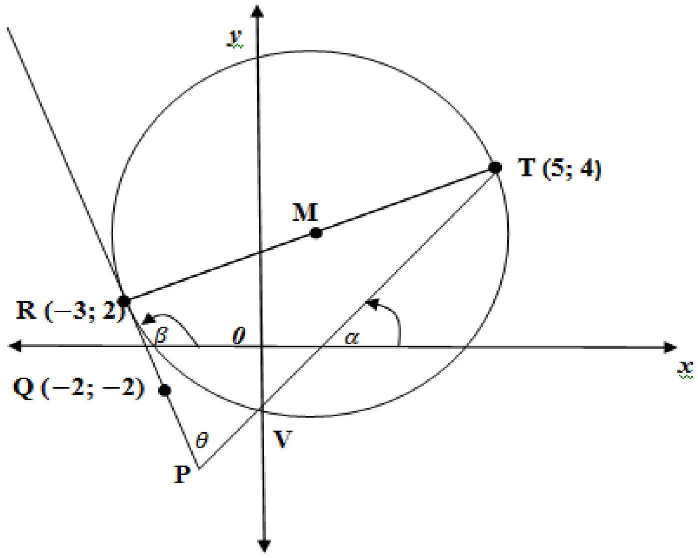
2.6 Determine the equation of the line parallel to AC, passing through D. (3)

2.7 Calculate the area of Δ ABC. (3)

[19]

QUESTION 3

In the diagram, the circle with centre M passes through points V, R(−3;2) and T(5;4).
Q is the point (−2;−2) and the lines through RQ and TV meet at P. The inclination angle of PT is α and the angle of inclination of PR is β.
V is the y-intercept of both the circle and line TP.



3.1	Determine the equation of the circle with centre M.	(5)
3.2	Show, using analytical methods, that PR is a tangent to the circle at R.	(3)
3.3	Determine the coordinates of V.	(4)
3.4	If $R\hat{P}T = \theta$, calculate θ to ONE decimal place.	(6)
		[18]

QUESTION 4

4.1.1 Simplify the following expression to a single trigonometric function:

$$\frac{2\sin(180°+x)\sin(90°+x)}{\cos^4 x - \sin^4 x}$$ (5)

4.1.2 For which value(s) of x, $x \in [0°;360°]$ is the expression in 4.1 undefined? (3)

4.2 Evaluate, without using a calculator: $\dfrac{\cos 347°.\ \sin 193°}{\tan 315°.\ \cos 64°}$ (5)

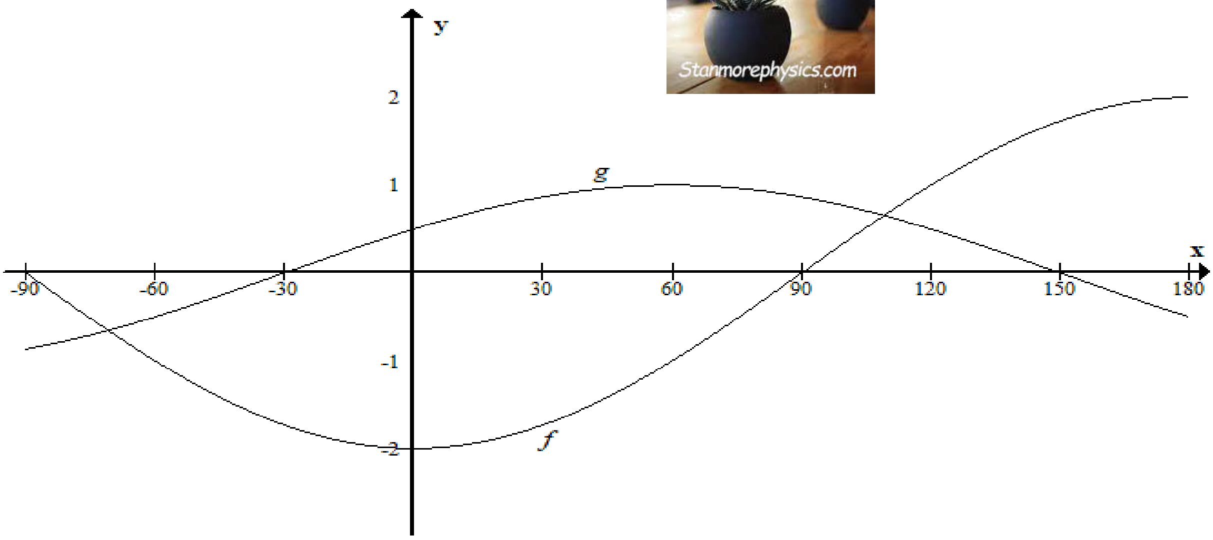
(5)

4.3 Prove the following identity:

$$\frac{cos3x}{cosx} = 2cos2x - 1$$

QUESTION 5

The graphs of $f(x) = -2\cos x$ and $g(x) = \sin(x + 30°)$ for $x \in [-90°; 180°]$ are drawn in the diagram below.



5.1 Determine the period of g. (1)

5.2 Calculate the x-coordinates of P and Q, the points where f and g intersect. (7)

5.3 Determine the x-values, $x \in [-90°; 180°]$, for which:

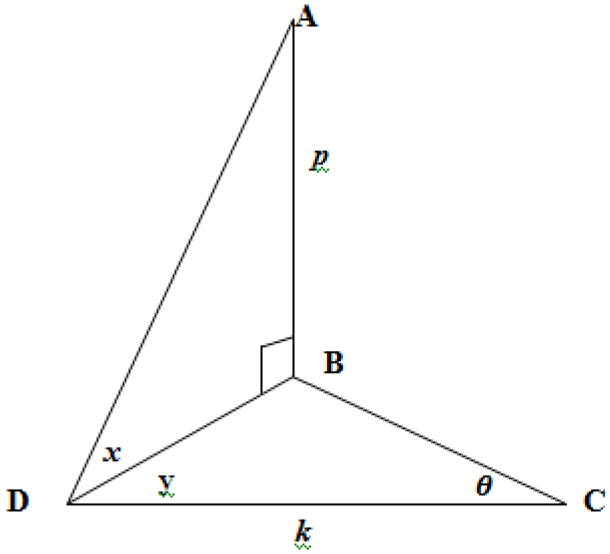
5.3.1 $g(x) \leq f(x)$ (3)

5.3.2 $f'(x).g(x) > 0$ (3)

[14]

QUESTION 6

AB is a vertical tower of p units high.
D and C are in the same horizontal plane as B, the foot of the tower.
The angle of elevation of A from D is x. B$\hat{D}$C = y and D$\hat{C}$B = θ.
The distance between D and C is k units.



6.1.1 Express p in terms of DB and x. (2)

6.1.2 Hence prove that: $p = \dfrac{k\sin\theta\tan x}{\sin y\cos\theta + \cos y\sin\theta}$ (5)

6.2 Find BC to the nearest meter if $x = 51,7°$, $y = 62,5°$, $p = 80\,m$ and $k = 95\,m$. (4)

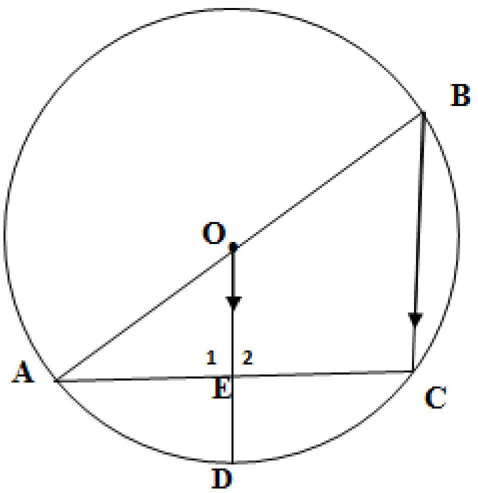
[11]

QUESTION 7

7.1 Complete the theorem that states: the line from the centre of the circle to the midpoint of the chord ... (1)

7.2 Write down the converse of the theorem in 7.1. (2)

7.3 AB is a diameter of circle O. OD is drawn parallel to chord BC and intersects AC at E.



The radius is 10 cm and AC = 16 cm.

7.3.1 Prove that AE = EC. (2)

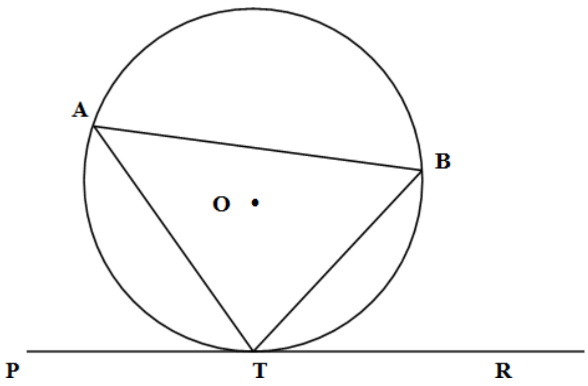
7.3.2 Prove that $\hat{E}_1 = 90°$. (2)

7.3.3 Hence calculate the length of ED. (3)

[10]

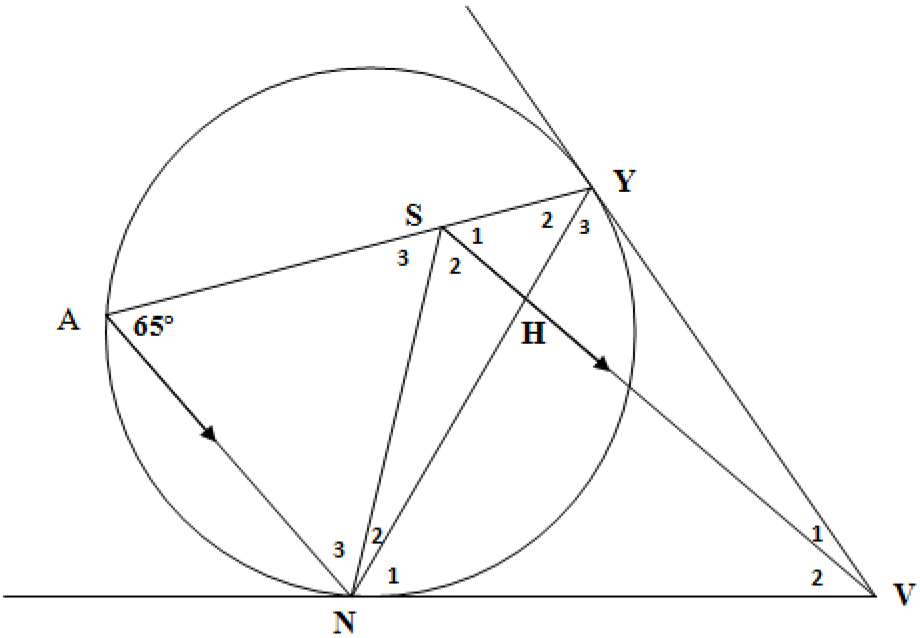
QUESTION 8

8.1 In the diagram, the circle with centre O passes through points A, B and T. PR is a tangent to the circle at T. AB, BT and AT are chords.



Prove that $B\hat{T}R = \hat{A}$. (6)

8.2 VN and VY are tangents to the circle at N and Y.
A is a point on the circle, and AN, AY and NY are chords so that $\hat{A} = 65°$.
S is a point on AY so that AN || SV. S and N are joined.



8.2.1 Write down, with reasons, THREE other angles each equal to 65°. (3)

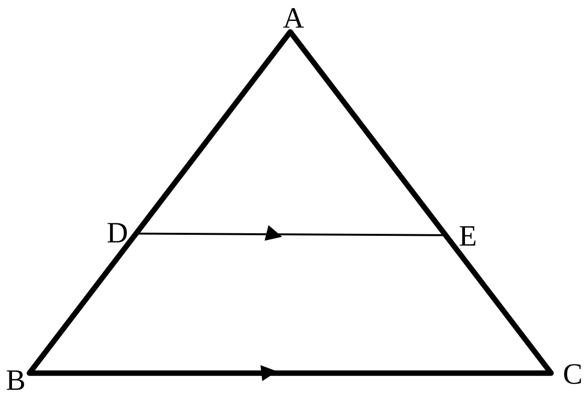
8.2.2 Prove that VYSN is a cyclic quadrilateral. (2)

8.2.3 Prove that ΔASN is isosceles. (5)

[16]

QUESTION 9

Use the diagram below to prove the theorem which states that if DE||BC then $\frac{BD}{AD} = \frac{EC}{AE}$.



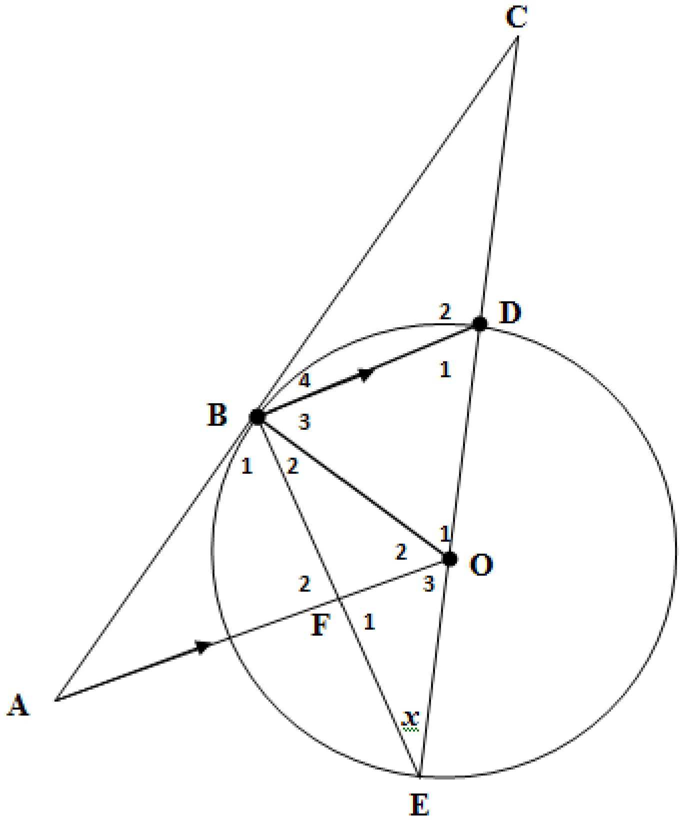
[6]

QUESTION 10

CE is a straight line passing through centre O of the circle.

CA is a tangent to the circle at B. AO intersects chord BE at F. BD || AO.

$\hat{E} = x$.



10.1 Give a reason why $\angle EBD = 90^\circ$ (1)

10.2 Give, with reasons, THREE other angles each equal to x. (3)

10.3 Give a reason why ABOE is a cyclic quadrilateral (1)

10.4 Express $C\hat{B}E$ in terms of x. (2)

10.5 Prove that:

10.5.1 Δ CBD ||| Δ CEB (2)

10.5.2 2EF. CB = CE.BD (5)

10.5.3 $\dfrac{2EF}{CE} = \dfrac{AO}{AC}$ (4)

[18]

GRAND TOTAL: 150

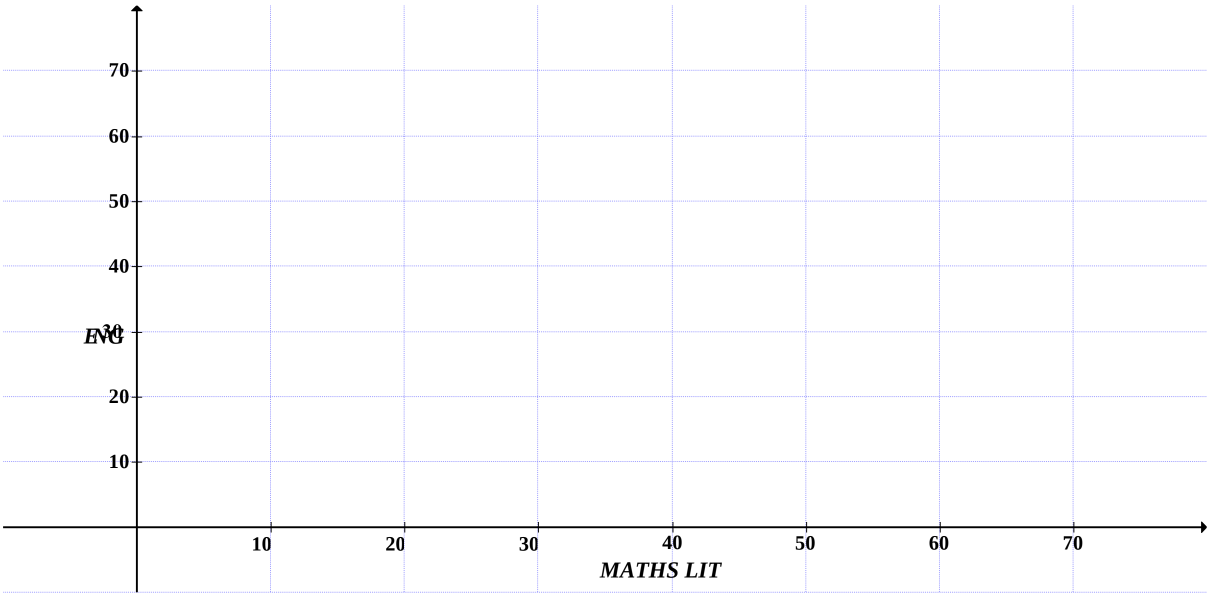
$——AO—$ (

DIAGRAMSHEET

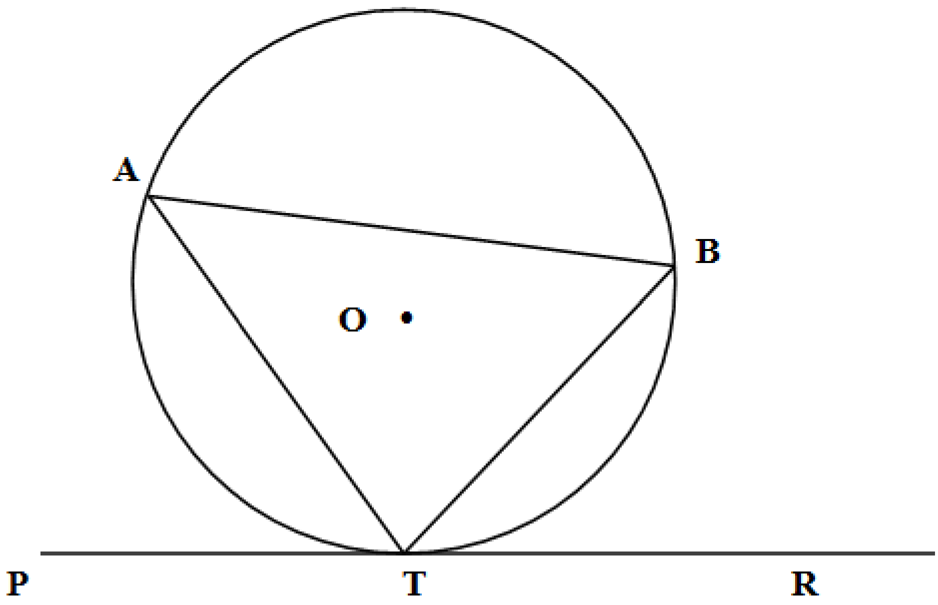
NAME:

QUESTION 1.3

MATHS LIT VS ENGLISH

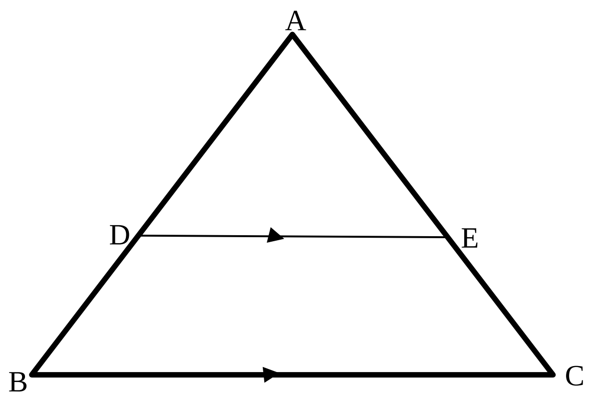


QUESTION 8.



QUESTION 9

Use the diagram below to prove the theorem which states that if DE||BC then $\frac{BD}{AD} = \frac{EC}{AE}$.



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GRADE 12

MATHEMATICS P2

MEMORANDUM

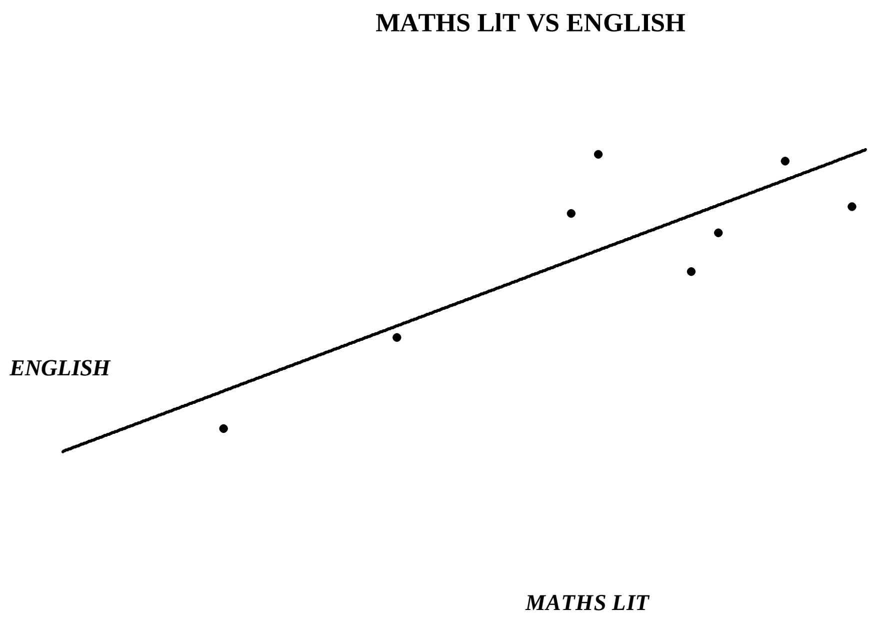
PRE-TRIAL 2021

MARKS: 150

This memorandum consists of 12 pages.

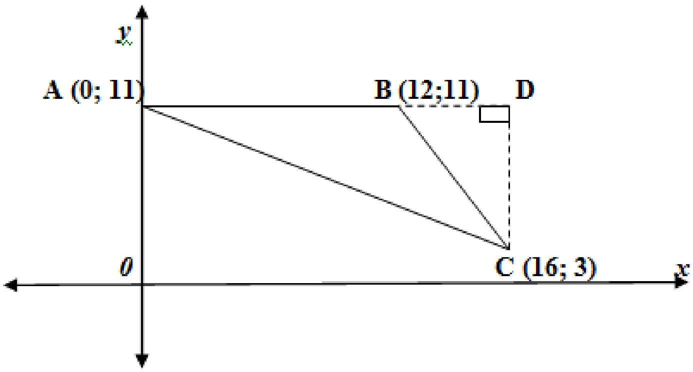
QUESTION 1

1.1



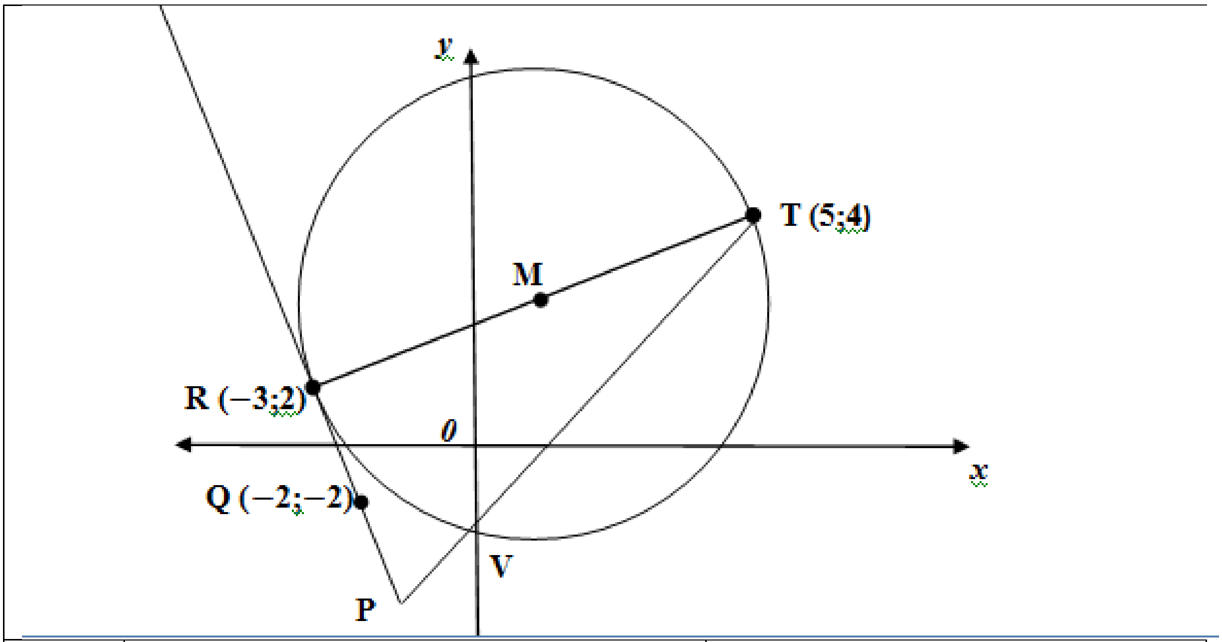
1.1.1	$\bar{x}=\frac{324}{8}$ $= 40,5$	✓$\frac{324}{8}$ ✓ 40,5 (2)
1.1.2	δ = 14,5688 = 14,57	✓ ✓ accuracy (2)
1.2	(40,5−14,57; 40,5 + 14,57) (25,93; 55,07) ∴ 5 learners.	✓method ✓(25,93; 55,07) ✓5 (3)
1.3	See scatter plot above	✓ 2-4 points ✓✓ 5-7pts correct ✓✓✓ all pts correct (3)
1.4	a =16,89 b = 0,75 y = 16,89 +0,75x	✓ a ✓ b ✓equation (3)
1.5	See above	✓ positive gradient ✓c-value betw 15 and 20 (2)
1.6	r = 0,82 It is a strong positive relationship	✓r = 0,82 ✓strong ✓positive (3)
1.7	54,81%	✓ ✓accuracy (2)
		[20]

QUESTION 2



2.1	$y = 11$ AB = 12	✓✓y = 11 ✓ AB = 12 (3)
2.2	D(16;11)	✓✓ (2)
2.3	M(8; 7)	✓✓ (2)
2.4	$m_{AC} = \frac{3-11}{16} = -\frac{8}{16} = -\frac{1}{2}$ $m_{line} = 2$ $y - 7 = 2(x - 8)$ $y = 2x - 9$	✓$-\frac{1}{2}$ ✓$m_{line} = 2$ ✓substitution ✓ equation (4)
2.5	$y = 2(12) - 9$ $= 15$ $\neq 11$ No, it does not pass through B	✓substitution ✓ $\neq 11$ No, it does not pass through B (2)
2.6	$\tan\theta = m_{BC} = \frac{11-3}{12-16}$ $\tan\theta = -2$ $\theta = 116{,}57°$	✓$\tan\theta$ ✓-2 ✓116,57° (3)
2.7	$m_{new\ line} = -\frac{1}{2}$ $y - 11 = -\frac{1}{2}(x - 16)$ $y = -\frac{1}{2}x + 19$	✓ $-\frac{8}{13}$ ✓substitution ✓equation (3)
2.8	Area $\Delta\, ABC = \frac{1}{2} base\ height$ $= \frac{1}{2} \times 12 \times 8$ = 48 sq units	✓h=8 ✓ substitution ✓answer (3)
		[22]

QUESTION 3



3.1	M(1; 3) $r^2 = (5-1)^2 + (4-3)^2$ $r^2 = 16 + 1 = 17$ $(x-1)^2 + (y-3)^2 = 17$	✓✓M ✓substitution ✓$r^2 = 17$ ✓$(x-1)^2 + (y-3)^2 = 17$ (5)
3.2	$m_{PR} = \frac{-2-2}{-2+3} = -4$ $m_{RT} = \frac{4-2}{5+3} = \frac{1}{4}$ $m_{PR} \times m_{RT} - 1$ PR is a tangent	✓m_{PR} ✓m_{RT} ✓product = -1 (3)
3.3	Y int: $(0-1)^2 + (y-3)^2 = 17$ $1 + y^2 - 6y + 9 = 17$ $y^2 - 6y - 7 = 0$ $(y-7)(y+1) = 0$ $y = -1$ or $y = 7$ V$(0; -1)$	✓let $x = 0$ ✓standard form ✓$y = -1$ or $y = 7$ ✓V$(0; -1)$ (4)
3.4	$m_{PT} = \frac{4+1}{5-0} = 1$ $\tan\alpha = 1$ $\alpha = 45°$ $\tan\beta = -4$ $\beta = 104°$ $\theta = 59°$	✓m_{PT} ✓$\tan\alpha = 1$ ✓$\alpha = 45°$ ✓$\tan\beta = -4$ ✓$\beta = 104°$✓ $\theta = 59°$ (6)
		[18]

QUESTION 4

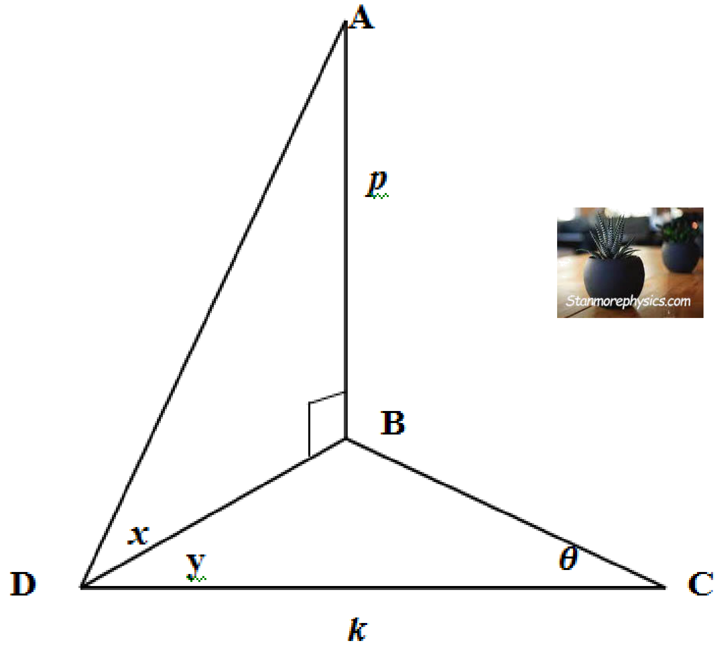
4.1.1	$\frac{2\sin(180°+x)\sin(90°+x)}{\cos^4x-\sin^4x}$ $=\frac{-2\sin x.\cos x}{(\cos^2x-\sin^2x)(\cos^2x+\sin^2x)}$ $=\frac{-\sin2x}{\cos2x.(1)}$ $=-\tan2x$	✓$-2\sin x$ ✓$\cos x$ ✓factorisation ✓$-\sin2x$ ✓$\cos2x$ (5)
4.1.2	At $\cos2x = 0$ $2x=90°$ or $2x=270°$ $x = 45°$ or $x = 135°$	✓$\cos2x = 0$ ✓$2x=90°$ or $2x=270°$ ✓$x = 45°$ $x = 135°$ (3)
4.2	$=\frac{(\cos13°)(-\sin13°)}{(-\tan45°).(\cos64°)}$ $=\frac{\cos13°.-\sin13°}{-1.\cos64°}$ $=\frac{2\times\sin13°\cos13°}{2\cos64°}$ $=\frac{\sin26°}{2\sin26°}$ $=\frac{1}{2}$	✓$\cos13°$ ✓$-\sin13°$ ✓$-\tan45°$ ✓multiply by 2 in numerator **and** denominator ✓$\frac{\sin26°}{2\sin26°}$ (5)
4.3	LHS: $\frac{\cos(2x+x)}{\cos x}$ $=\frac{\cos2x.\cos x-\sin2x.\sin x}{\cos x}$ $=\frac{\cos2x.\cos x-2\sin x\cos x.\sin x}{\cos x}$ $=\frac{\cos x(\cos2x-2\sin^2x)}{\cos x}$ $=\cos2x-1+1-2\sin^2x$ $=\cos2x-1+\cos2x$ $=2\cos2x-1$ **OR**	✓$\cos2x.\cos x-\sin2x.\sin x$ ✓replacing $\sin2x$ ✓factorise ✓ $+1-1$ ✓replacing $1-2\sin^2x$ (5)

	$\frac{\cos(2x+x)}{cosx}$ $= \frac{cos2x.cosx - sin2x.sinx}{cosx}$ $= \frac{cos2x.cosx - 2sinxcosx.sinx}{cosx}$ $= \frac{cosx(cos2x - 2sin^2x)}{cosx}$ $= cos2x - 2sin^2x$ $= 2cos^2x - 1 - 2sin^2x$ $= 2(cos^2x - sin^2x) - 1$ $= 2cos2x - 1$	✓$cos2x.cosx - sin2x.sinx$ ✓replacing sin2x ✓factorise ✓ replacing cos2x ✓replacing $cos^2x - sin^2x$ (5)
		[18]

QUESTION 5

5.1	360°	✓ (1)
5.2	$sin(x+30°) = -2cosx$ $sinxcos30° + cosxsin30° = -2cosx$ $\sin x\left(\frac{\sqrt{3}}{2}\right) + \cos x\left(\frac{1}{2}\right) = -2\cos x$ $\sqrt{3}$sin$x$+cos$x$=$-4cosx$ $\sqrt{3}$sin$x$ = $-5cosx$ tan$x = -\frac{5}{\sqrt{3}}$ x = 180° − 70,89°+k.180° x = 109.11° + k.180°, k∈ Z $x = -70,89°$ or x =109,11°  	✓equating f and g ✓expanding $sin(x+30°)$ ✓special angle values ✓tan$x = -\frac{5}{\sqrt{3}}$ ✓$x = -70,89°$ ✓x = 109.11° + k.180° ✓x =109,11° (7)
5.3.1	$x \in [-90°; -70,89°] \cup [109,11°; 180°]$	✓✓boundaries ✓correct notation (3)
5.3.2	$x \in (-90°; -30°) \cup (90°; 150°)$	✓$(-90°; -30°)$ ✓$(90°; 150°)$ ✓correct notation (3)
		[14]

QUESTION 6

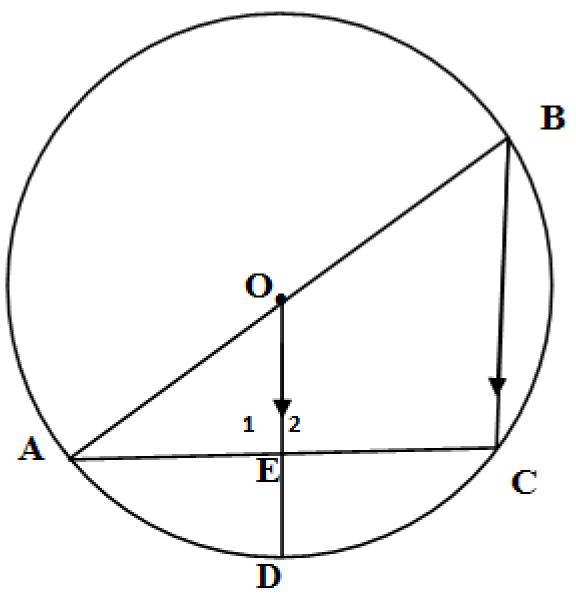


6.1.1	In $\Delta\, ABD$: $\tan x = \frac{p}{DB}$ $p = DB.\tan x$	✓$\tan x = \frac{p}{DB}$ ✓ $p = DB \tan x$ (2)
6.1.2	$\frac{DB}{sin\theta} = \frac{k}{\sin(180-(y+\theta)}$ $DB = \frac{k.sin\theta}{\sin(y+\theta)}$ $p = \frac{k.sin\theta}{\sin(y+\theta)} \times \tan x$ $= \frac{ksin\theta.tanx}{sinycos\theta+cosy.sin\theta}$	✓$B\hat{D}C = 180 - (y+\theta)$ ✓$\frac{DB}{sin\theta} = \frac{k}{\sin(180-(y+\theta)}$ ✓reduction formula ✓replacing DB ✓expanding $\sin(y+\theta)$ (5)
6.2	$tan51{,}7° = \frac{80}{DB}$ $DB = \frac{80}{tan51{,}7°} = 63{,}18\ m$ $BC^2 = (63{,}18)^2 + 95^2 - 2(63{,}18)(95)cos62{,}5°$ $= 7473{,}789697 \ldots$ $\therefore BC = 86{,}45 \approx 86\ m$	✓$tan51{,}7° = \frac{80}{DB}$ ✓$DB = 63{,}18\ m$ ✓application of cosine formula. ✓$86m$ (4)
		[11]

QUESTION 7

7.1	is perpendicular to the chord	✓ (1)
7.2	The line from the centre of the circle perpendicular to the chord, bisects the chord	✓The line from the centre of the circle perpendicular to the chord ✓ bisects the chord (2)

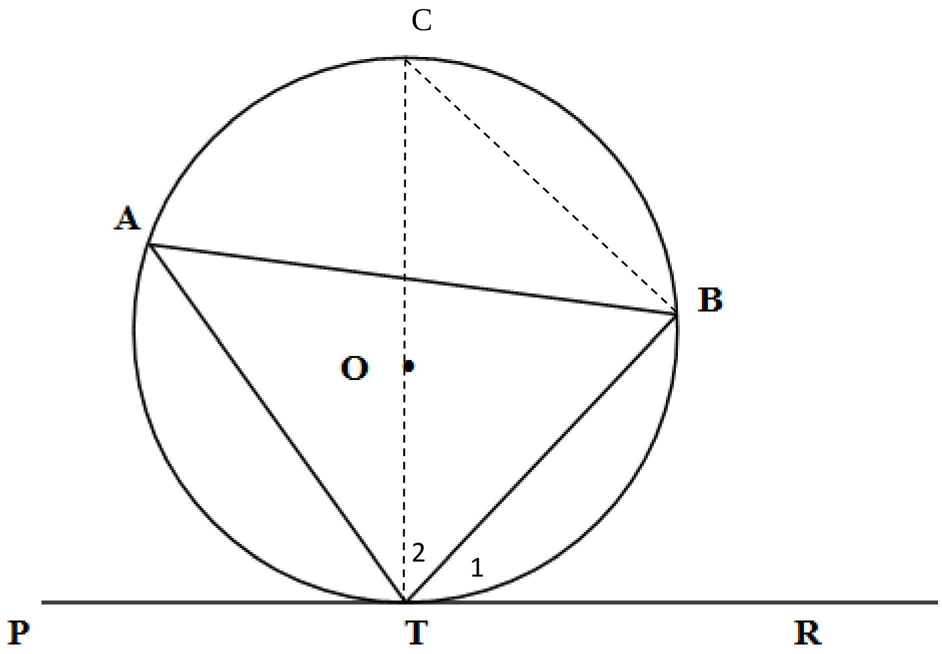
7.3



7.3.1	$\frac{AO}{OB}=\frac{AE}{EC}\cdots\cdots OE\,//\,BC$ AO=OB.....Radii $\Rightarrow$ AE = EC	✓S✓R (2)
7.3.2	$\hat{C}= 90°$ (angle in semi⊙) $\hat{E}_1 = 90°$ (corr. angles; OD || BC)	✓S/R ✓R (2)
7.3.3	$OE^2 = 10^2 - 8^2$ (theorem of Pyth) $OE^2 = 100 - 64 = 36$ OE = 6 cm ∴ED = 4 cm	✓S ✓OE = 6 cm ✓answer (3)
		[10]

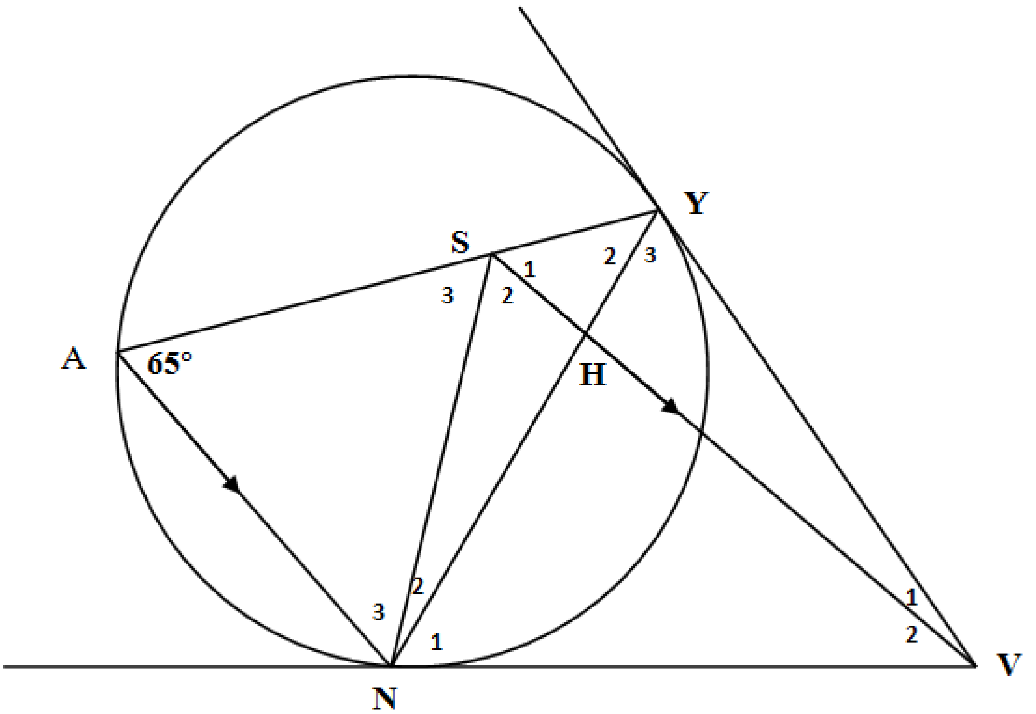
QUESTION 8

8.1



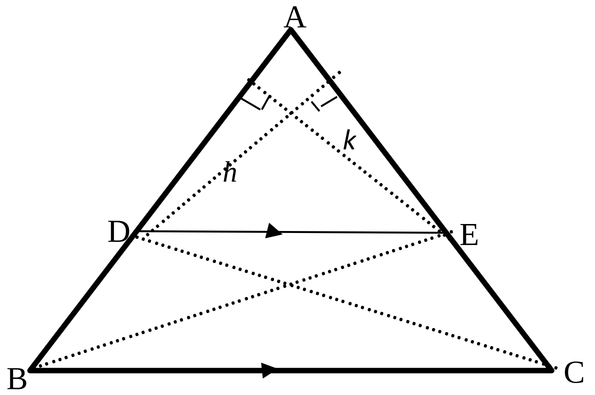
8.1	Construction: Draw diameter TC and join BC. $C\hat{B}T = 90°$ (∠ in semi ⊙) $\hat{C}+\hat{T}_2 = 90°$ (∠'s of Δ) $\hat{T}_1+\hat{T}_2 = 90°$ (tangent ⊥ r) $\therefore\ \hat{C} = \hat{T}_1$ But $\hat{C} = \hat{A}$ (∠'s in same segment) $\therefore \hat{T}_1 = \hat{A}$	✓construction ✓S / R ✓S ✓S/ R ✓S/ R ✓conclusion (6)

8.2

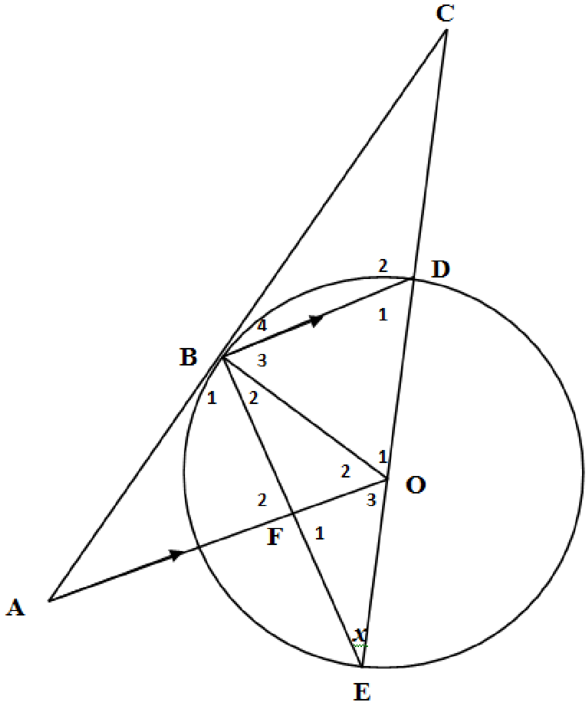


8.2.1	$\hat{S}_1 = 65°$ (corr ∠'s; AN || SV) $\hat{Y}_3 = 65°$ (tan-chord th) $\hat{N}_1 = 65°$ (tan-chord th)	✓S R ✓S R ✓S R (3)
8.2.2	$\hat{S}_1 = \hat{N}_1$ VYSN is a cyclic quad (YV subtends equal angles)	✓$\hat{S}_1 = \hat{N}_1$ ✓YV subtends equal angles (2)
8.2.3	$\hat{S}_2 = 65°$ (∠'s in same segment) $\hat{N}_3 = 65°$ (alt. ∠'s; AN || SV) $\therefore \hat{A} = \hat{N}_3$ AS = SN (sides opp equal angles)	✓S✓R ✓S✓R ✓R (5)
		[16]

QUESTION 9

<table>
<tr><td></td><td>Use the diagram below to prove the theorem which states that if
DE‖BC then $\frac{BD}{AD} = \frac{EC}{AE}$.</td><td></td></tr>
<tr><td></td><td>
</td><td>✓Construction</td></tr>
<tr><td></td><td>Construction: In ΔADE draw $altitudes\ h\ and\ k$
$\frac{area\ \Delta BDE}{area\ \Delta ADE} = \frac{\frac{1}{2}BD \times k}{\frac{1}{2}AD \times k}$
$= \frac{BD}{AD}$
$\frac{area\ \Delta CED}{area\ \Delta ADE} = \frac{\frac{1}{2}EC \times h}{\frac{1}{2}AE \times h}$
$= \frac{EC}{AE}$
But $area\ \Delta BDE = area\ \Delta CED$ $Same\ base, same\ height$
$\therefore \frac{area\ \Delta BDE}{area\ \Delta ADE} = \frac{area\ \Delta CED}{area\ \Delta ADE}$
$\therefore \frac{BD}{AD} = \frac{EC}{AE}$</td><td>✓ S
✓ S
✓ S
✓S & R
✓ S
[6]</td></tr>
</table>

QUESTION 10



10.1	Subtended by a diameter / Angle in a semi-circle	✓ Answer (1)
10.2	$\hat{B}_2 = x$ (radii =) $\hat{B}_4 = x$ (tan-chord th $\hat{A} = x$ (corr ∠'s; BD ǁ AO)	✓ S ✓SR ✓S (3)
10.3	$\hat{A} = \hat{E} = x$ Converse $\angle's$ subtended by the same cord	✓ Answer (1)
10.4	$\hat{B}_2 + \hat{B}_3 = 90°$ (∠ in semi ⊙) $C\hat{B}E = 90° + x$	✓ R ✓$90° + x$ (2)
10.5.1	In Δ CBD and Δ CEB: $\hat{C} = \hat{C}$ $\hat{B}_4 = \hat{E} = x$ $\hat{D}_2 = C\hat{B}E$ ∴ Δ CBD ǀǀǀ Δ CEB (∠∠∠)	✓S ✓S (2)
10.5.2	$\frac{CB}{CE} = \frac{BD}{EB}$ (ǀǀǀ triangles) EB.CB = CE. BD $\hat{F}_1 = 90°$ (corr ∠'s; BD ǁ AO) BF = FE (line from centre to mdpt of chord) ∴ BE = 2EF ∴ 2EF.CB = CE.BD	✓S✓R ✓SR ✓SR ✓replacing BE (5)
10.5.3	$\frac{2EF}{CE} = \frac{BD}{BC}$ out of 10.4 But Δ BCD ǀǀǀ Δ ACO (∠∠∠) $\therefore \frac{BD}{AO} = \frac{BC}{AC}$ $\frac{BD}{BC} = \frac{AO}{AC}$ $\frac{2EF}{CE} = \frac{AO}{AC}$	✓S ✓SR ✓S ✓S (4)