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KWAZULU-NATAL PROVINCE

EDUCATION
REPUBLIC OF SOUTH AFRICA



**NATIONAL
SENIOR CERTIFICATE**

GRADE 12

**MATHEMATICS
JUNE COMMON TEST**

2021

MARKS: 75

TIME: 1½ hours

This question paper consists of 6 pages an information sheet and an answer sheet.

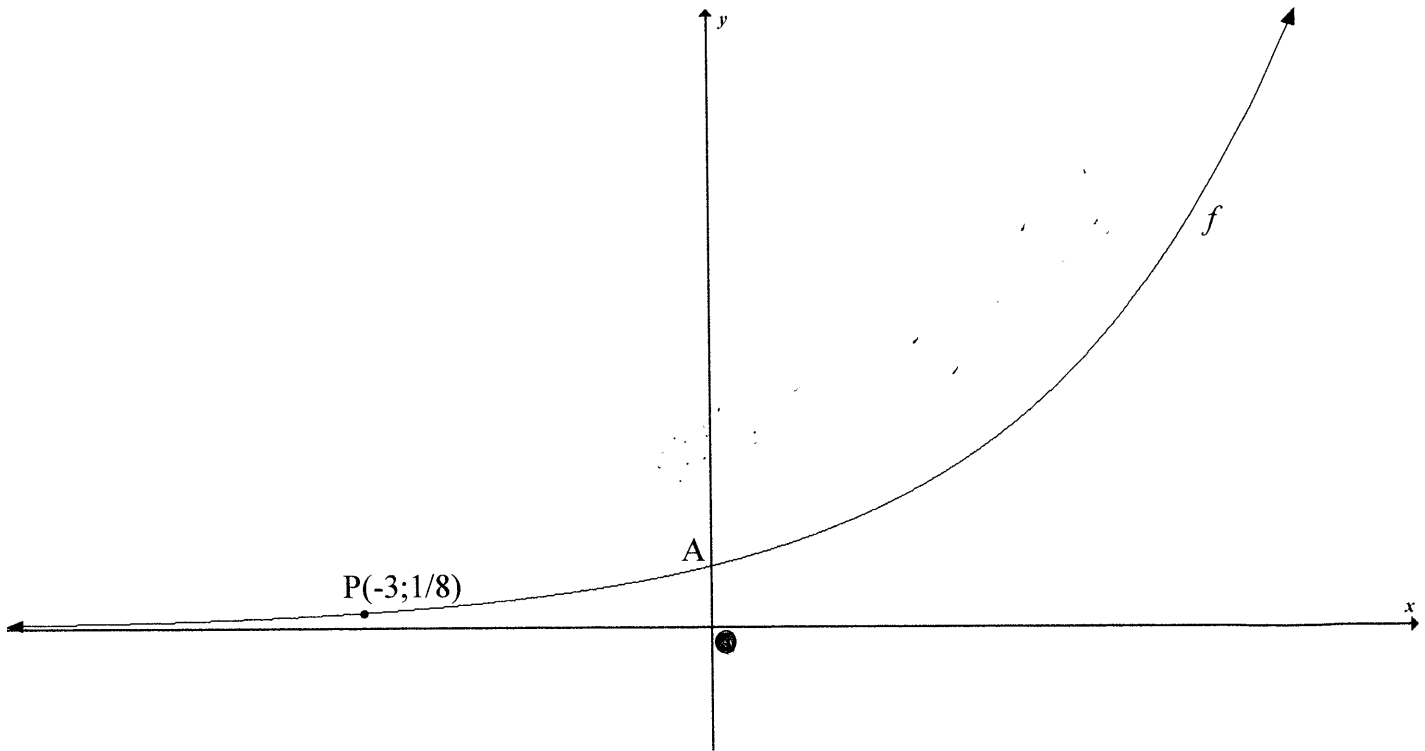
INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 5 questions.
2. Answer **ALL** questions.
3. Clearly show **ALL** calculations, diagrams, graphs, et cetera that you have used in determining your answers.
4. Answers only will not necessarily be awarded full marks.
5. An approved scientific calculator (non-programmable and non-graphical) may be used, unless stated otherwise.
6. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. Number the answers correctly according to the numbering system used in this question paper. Write neatly and legibly.

QUESTION 1

Given $f(x) = a^x$; $a \neq 1$; $a > 0$. $P\left(-3; \frac{1}{8}\right)$ is a point on the graph of f . The asymptote to the graph is the x -axis. The graph intersects the y -axis at A.



- 1.1 Write down the equation of the horizontal asymptote of f . (2)
- 1.2 Calculate the value of a . (3)
- 1.3 Write down the coordinates of A. (2)
- 1.4 Determine the equation of f^{-1} , the inverse of f , in the form $y = \dots$ (2)
- 1.5 If the point P is reflected about the line $y = x$. Write down the coordinates of the image point, P' . (2)
- 1.6 Use the system of axes provided, sketch the graph of f^{-1} , indicating the axis of symmetry to f and the coordinates of the point where the inverse graph intersects the x -axis. (3)
- 1.7 Use your graph to solve for x if $\frac{1}{3}f^{-1}(x) \leq 1$. (4)

**[18]**

QUESTION 2

- 2.1 Samuel invests R45 000 at an interest rate per annum compounded monthly which grew to R82 000 at the end of 7 years.
- 2.1.1 Determine the rate of interest offered by the institution. (Answer correct to two decimal places) (3)
- 2.1.2 If after 3 years, Samuel deposited an additional R20 000 for his studies. Determine the final amount he will now receive at the end of 7 years. (3)
- 2.2 Mrs Molly bought her car for R900 000. The value of the car depreciates at a rate of 9,2 % p.a. on a reducing balance method. Determine the value of the car after 9 years. (3)
- [9]**

QUESTION 3

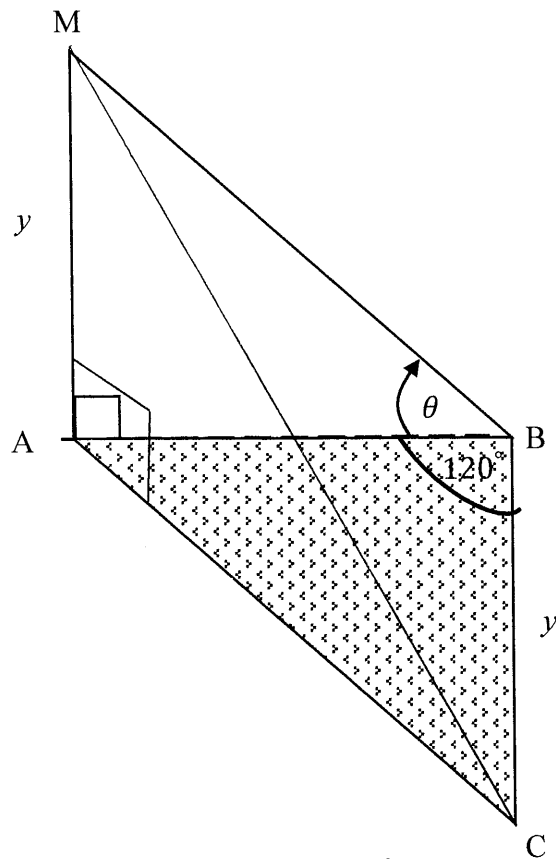
- 3.1 Determine the derivative of f , using first principle, if $f(x) = 2 - 5x^2$. (5)
- 3.2 Determine:
- 3.2.1 $g'(x)$ if $g(x) = (7x - 3)^2$ (3)
- 3.2.2 $D_x \left[\frac{x^3 + 4x^2 - 5}{\sqrt{x}} \right]$ (4)
- 3.2.3 $\frac{dy}{dx}$ if $y = \left[\frac{x^3 - 125}{5 - x} \right]$ (4)
- 3.3 Determine the derivative of f if: (4)

$$f(x) = \sum_{r=0}^3 r \cdot x^{3-r}$$

[20]

QUESTION 4

In the figure A, B and C are three points in the same horizontal plane. $\widehat{ABC} = 120^\circ$.
 M is a point directly above A such that $MA = BC = y$. $\widehat{MAB} = \widehat{MAC} = 90^\circ$.
 The angle of elevation of M from B is θ .

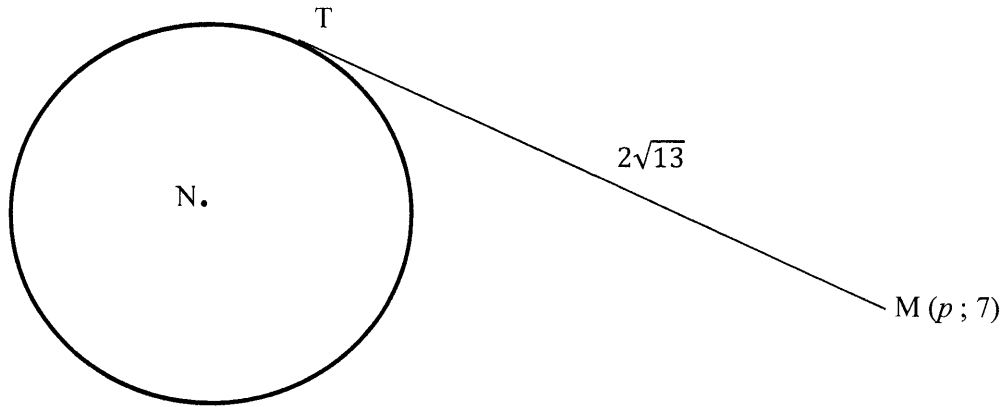


- 4.1 Show that $AB = \frac{y}{\tan \theta}$. (1)
 - 4.2 Determine AC^2 in terms of a trigonometric ratio of θ and y . (2)
 - 4.3 If it is given that $y = 15$ m and $\theta = 22^\circ$, then calculate the following:
 - 4.3.1 AC (2)
 - 4.3.2 \widehat{MCA} if MC is drawn. (2)
- [7]**

QUESTION 5

5.1

Given $x^2 + y^2 - 6x - 2y + 1 = 0$ is the equation of the circle, centre N. $M(p ; 7)$ is a point outside the circle and the length of the tangent to the point T on the circle is $2\sqrt{13}$ units.

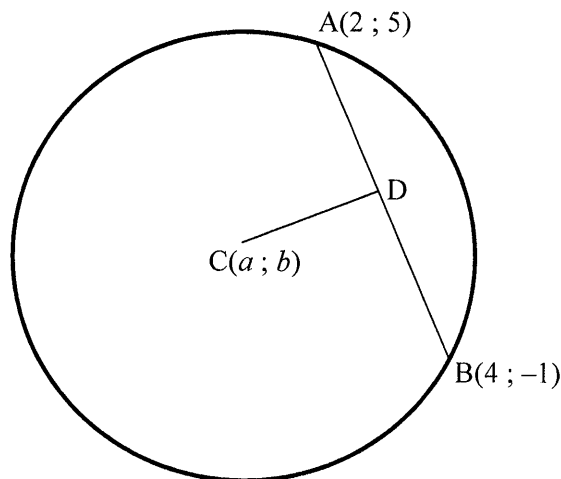


5.1.1 Determine the coordinates of the centre and the length of the radius, TN, of the circle. (4)

5.1.2 Calculate the value(s) of p . (6)

5.2

$A(2 ; 5)$ and $B(4 ; -1)$ are two points on a circle. $C(a ; b)$ is the centre of the circle. The centre of the circle lies on the line $2x - y + 1 = 0$. AB is a chord of the circle with $AD = \frac{1}{2} AB$.



5.2.1 Give a reason as to why $CD \perp AB$. (1)

5.2.2 Determine the equation of the circle. (5)

5.2.3 Determine the equation of the tangent to the circle at B. (5)

[21]

Total Marks : 75

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$A = P(1 - i)^n$$

$$A = P(1 + i)^n$$

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}; \quad r \neq 1$$

$$S_\infty = \frac{a}{1 - r}; \quad -1 < r < 1$$

$$F = \frac{x[(1+i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

$$\text{In } \triangle ABC: \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A \quad \text{area } \triangle ABC = \frac{1}{2} ab \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha$$

$$\bar{x} = \frac{\sum f \cdot x}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx$$

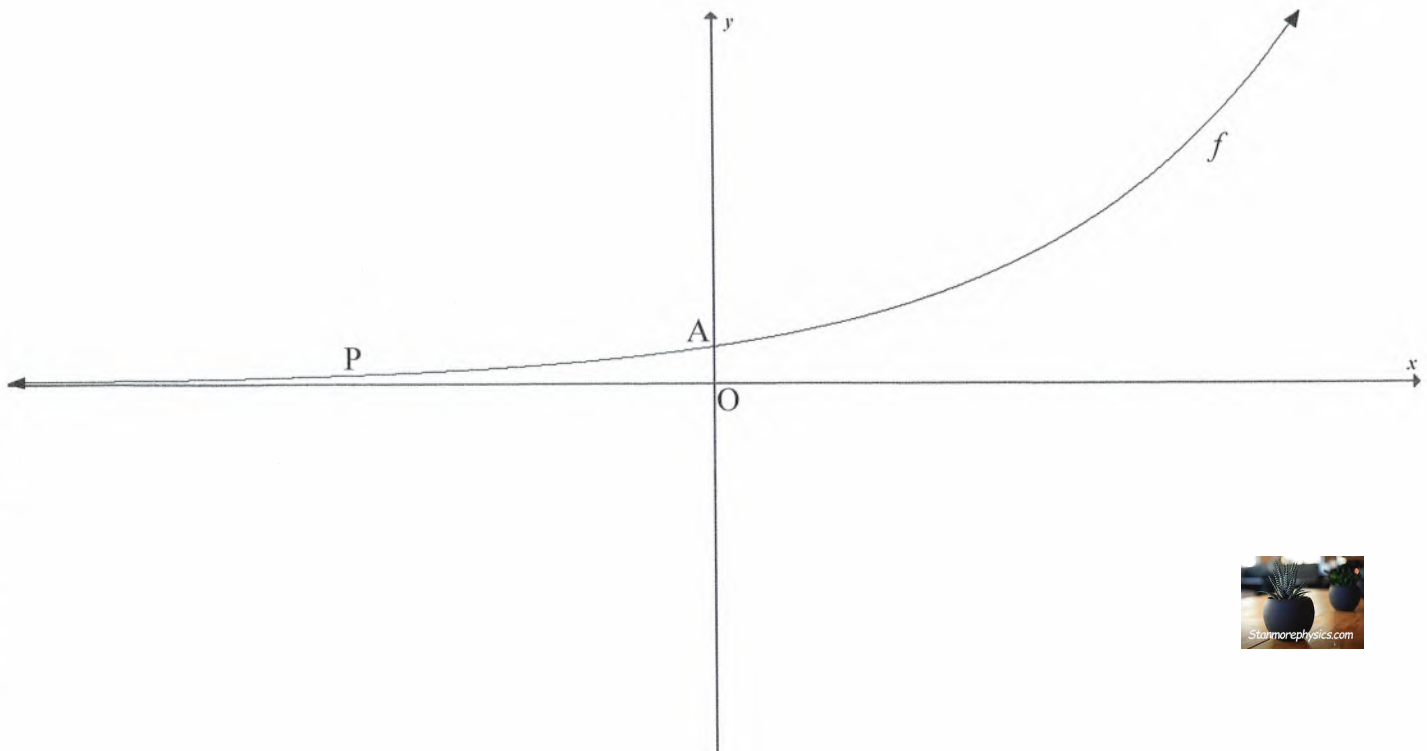
$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$

NAME: _____

GRADE: _____

ANSWER SHEET

Question 1.6



Detach and return







Education

KwaZulu-Natal Department of Education
REPUBLIC OF SOUTH AFRICA



MATHEMATICS

JUNE CONTROL TEST

MEMORANDUM

**NATIONAL
SENIOR CERTIFICATE**

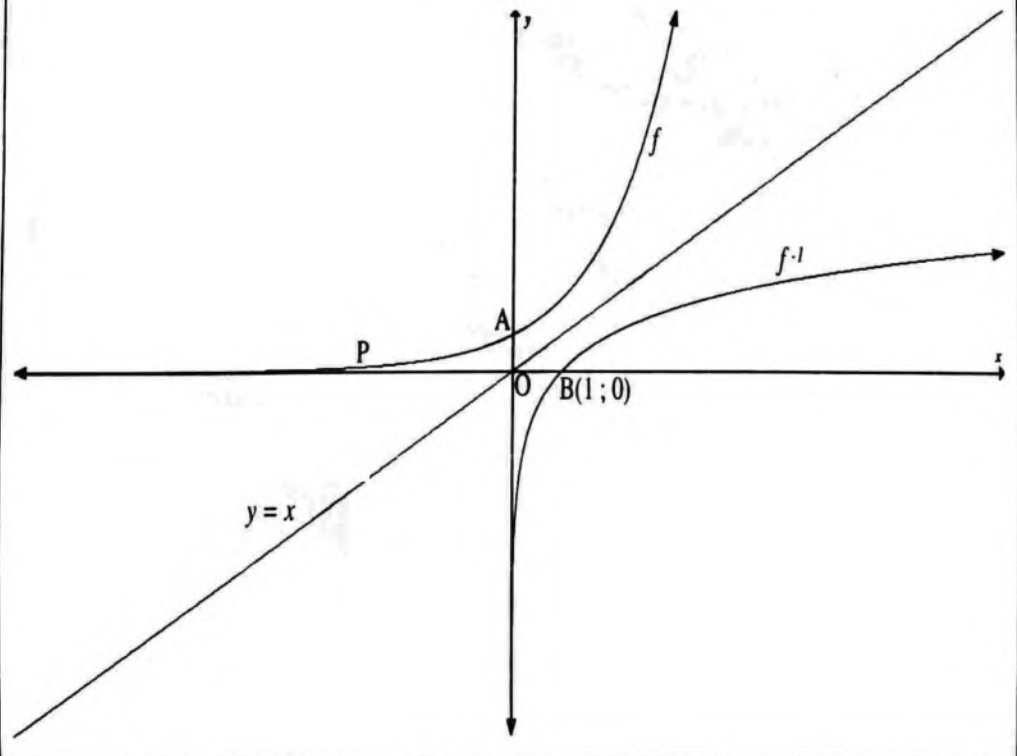
GRADE 12

MARKS: 75

TIME: 1.5 hours

This memorandum consists of 7 pages.

QUESTION 1

1.1	$y = 0$	AA✓✓ asymptote equation	(2)
1.2	$y = a^x$ $\frac{1}{8} = a^{-3}$ $2^{-3} = a^{-3}$ $a = 2$	A✓ substitution of $P(-3; \frac{1}{8})$ A✓ exponential form CA✓ a - value	(3)
1.3	$A(0; 1)$	AA✓✓ answer	(2)
1.4	$y = 2^x$ Interchanging x and y $x = 2^y$ $y = \log_2 x$	A✓ x form CA✓ answer	(2)
1.5	$P(\frac{1}{8}; -3)$	A✓ A✓ answer	(2)
1.6		A✓ shape A✓ coordinates of B A✓ $y = x$ line	(3)


1.7	$\frac{1}{3} \log_2 x \leq 1$ $\log_2 x = 3$ $x = 2^3$ $x = 8$ $0 < x \leq 8$	A✓ Equating CA✓ value 8 CA✓CA✓ answer (4)	
			[18]

QUESTION 2


2.1.1	$A = P(1 + i)^n$ $82\,000 = 45\,000 \left(1 + \frac{i}{12}\right)^{84}$ $\left(1 + \frac{i}{12}\right)^{84} = \frac{82\,000}{45\,000}$ $i = 12 \left(\sqrt[84]{\frac{82\,000}{45\,000}} - 1\right)$ $i = 0,0860 \dots$ 8.6 % p.a. compounded monthly.	A✓ substitution into formula CA✓ i - value CA✓ conclusion	(3)
2.1.2	$A = P(1 + i)^{n_1}(1 + i)^{n_2}$ $A = \left[45\,000 \left(1 + \frac{8.6\%}{12}\right)^{36} + 20\,000\right] \left(1 + \frac{8.6\%}{12}\right)^{48}$ $A = R\,110\,160,28$ OR $A = \left[45\,000 \left(1 + \frac{8.6\%}{12}\right)^{84}\right] + 20\,000 \left(1 + \frac{8.6\%}{12}\right)^{48}$ $A = R\,110\,160,28$	AA✓✓ substitution into formula CA✓ answer OR AA✓✓ substitution into formula CA✓ answer	(3) (3)
2.2	$A = P(1 - i)^n$ $A = 900\,000(1 - 9.2\%)^9$ $A = R\,377\,585,36$	AA✓✓ substitution into formula CA✓ answer	(3)
			[9]

**QUESTION 3** (penalize 1 mark once for incorrect notation in this question)

3.1	$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $f'(x) = \lim_{h \rightarrow 0} \frac{2 - 5(x+h)^2 - (2 - 5x^2)}{h}$ $f'(x) = \lim_{h \rightarrow 0} \frac{2 - 5x^2 - 10xh - 5h^2 - 2 + 5x^2}{h}$ $f'(x) = \lim_{h \rightarrow 0} \frac{-10xh - 5h^2}{h}$ $f'(x) = \lim_{h \rightarrow 0} \frac{h(-10x - 5h)}{h}$ $f'(x) = -10x$ <p>OR</p> $f(x+h) = 2 - 5(x+h)^2$ $= 2 - 5x^2 - 10xh - 5h^2$ $f(x+h) - f(x) = -10xh - 5h^2$ $\frac{f(x+h) - f(x)}{h} = \frac{h(-10x - 5h)}{h} = (-10x - 5h)$ $f'(x) = \lim_{h \rightarrow 0} (-10x - 5h)$ $f'(x) = -10x$	A✓ formula A✓ substitution CA✓ simplification of numerator CA✓ factorization CA✓ answer OR A✓ $f(x+h)$ value CA✓ $f(x+h) - f(x)$ value CA✓ $\frac{f(x+h)}{h}$ value A✓ formula CA✓ answer	(5)
3.2.1	$g(x) = (7x - 3)^2$ $g(x) = 49x^2 - 42x + 9$ $g'(x) = 98x - 42$	A✓ squaring CACA✓✓ derivatives	(3)

3.2.2	$D_x \left[\frac{x^3 + 4x^2 - 5}{\sqrt{x}} \right]$ $= D_x \left[x^{\frac{5}{2}} + 4x^{\frac{3}{2}} - 5x^{-\frac{1}{2}} \right]$ $= \frac{5}{2}x^{\frac{3}{2}} + 6x^{\frac{1}{2}} + \frac{5}{2}x^{-\frac{3}{2}}$	<p>A✓ writing in exponential form</p> <p>CACACA✓✓✓ answers</p>	(4)
3.2.3	$y = \left[\frac{x^3 - 125}{5 - x} \right]$ $y = \left[\frac{(x - 5)(x^2 + 5x + 25)}{-(x - 5)} \right]$ $= -x^2 - 5x - 25$ $\frac{dy}{dx} = -2x - 5$	<p>A✓ factorizing numerator</p> <p>CA✓ simplifying</p> <p>CACA✓✓ answers</p>	(4)
3.3	$\frac{d}{dx} [0 + x^2 + 2x^1 + 3]$ $= 2x + 2$	 <p>AA✓✓ generating terms (1 mark for 2 terms and 1 mark for all terms correct)</p> <p>CACA✓✓</p>	(4)
		[20]	

QUESTION 4

4.1	$\frac{y}{AB} = \tan \theta$ $AB = \frac{y}{\tan \theta}$ 	A✓trig. equation	(1)
4.2	$AC^2 = \frac{y^2}{\tan^2 \theta} + y^2 - 2\left(\frac{y}{\tan \theta}\right)(y) \cos 120^\circ$	AA✓✓Substitution into cosine formula	(2)
4.3.1	$AC^2 = \frac{(15)^2}{\tan^2 22^\circ} + (15)^2 - 2\left(\frac{15}{\tan 22^\circ}\right)(15) \cos 120^\circ$ $AC = 46,5 \text{ m}$	CA✓Substitution CA✓answer	(2)
4.3.2	$\frac{15}{46,5} = \tan \hat{MCA}$ $\hat{MCA} = 17,9^\circ$	CA✓trig. Equation CA✓answer	(2)
			[7]

QUESTION 5

5.1.1	$x^2 + y^2 - 6x - 2y + 1 = 0$ $x^2 - 6x + 9 + y^2 - 2y + 1 = -1 + 9 + 1 = 9$ $(x - 3)^2 + (y - 1)^2 = 9$ <p>Centre (3 ; 1) and Radius : 3 units</p>	A✓Completing the square CA✓Equation in centre – radius form CA✓Centre CA✓radius	(4)
5.1.2	<p>NT⊥TM ... (radius perp. tangent)</p> $MN^2 = 3^2 + (2\sqrt{13})^2 = 9 + 52 = 61$ $MN = \sqrt{(p - 3)^2 + (7 - 1)^2}$ $(p - 3)^2 + 36 = 61$ $(p - 3)^2 = 25$	A✓S/R CA✓ $MN^2 = 61$ CA✓length of MN in terms of p CA✓equating	

	$p - 3 = \pm 5$ $p = 8 \text{ or } -2$	CA✓CA✓answers	(6)
5.2.1	Line from centre to midpoint of chord	A✓R	(1)
5.2.2	The coordinates of C(a ; 2a + 1) CA = CB (Radii) $CA^2 = CB^2$ $(a - 2)^2 + (b - 5)^2 = (a - 4)^2 + (b + 1)^2$ $(a - 2)^2 + (2a + 1 - 5)^2 = (a - 4)^2 + (2a + 1 + 1)^2$ $(a - 2)^2 + (2a - 4)^2 = (a - 4)^2 + (2a + 2)^2$ $-4a + 4 - 16a + 16 = -8a + 16 + 8a + 4$ $-20a = 0$ $a = 0 \quad \therefore b = 1$ $r^2 = 4 + 16 = 20$ $x^2 + (y - 1)^2 = 20$	A✓Coordinates of centre CA✓Equating radii CA✓Centre CA✓radius value CA✓Equation	(5)
5.2.3	$m \text{ of radius} = \frac{-1-1}{4-0} = \frac{-2}{4} = -\frac{1}{2}$ $m \text{ of tangent} = 2$ Equation of tangent: $-1 = 2(4) + c$ $-9 = c$ $y = 2x - 9$	A✓gradient of radius CA✓gradient of tangent CA✓substitution of point B into equation of line CA✓c - value CA✓equation of line	(5)
			[21]