basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

SENIOR CERTIFICATE/ NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P2

PREPARATORY EXAMINATION 2021

MARKS: 150

TIME: 3 hours

This question paper consists of 15 pages and 1 information sheet.

KWAZULU-NATAL

INSTRUCTIONS AND INFORMATION

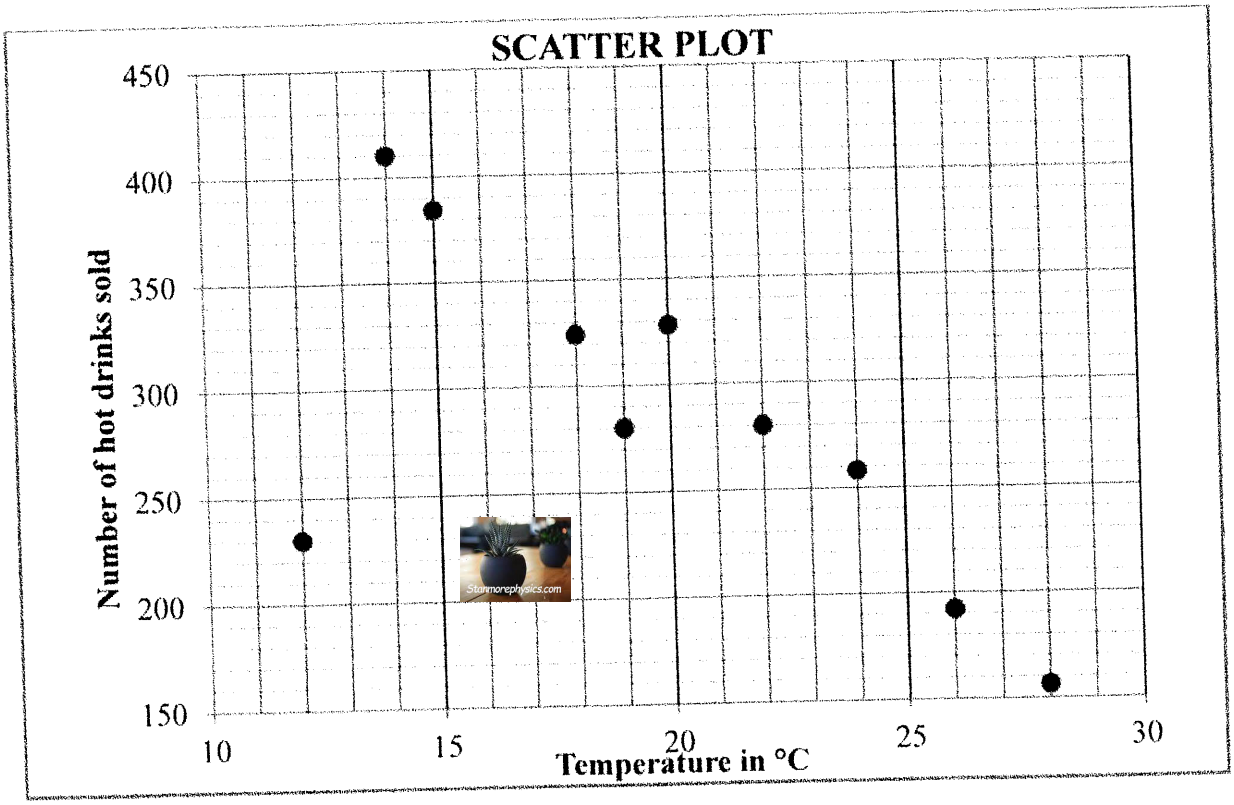
Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.
2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs, etc. that you have used in determining your answers.
4. Answers only will NOT necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round off answers correct to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. An information sheet with formulae is included at the end of the question paper.
9. Write neatly and legibly.

QUESTION 1

An annual sports festival is held over a period of 11 days. A tuckshop sells hot drinks at this festival. On each of the first 10 days, the owner of the tuckshop recorded the temperature at 13:00 and the number of cups of hot drinks sold. This information is presented in the table and scatter plot below.

Temperature (in °C)	14	24	26	18	20	28	22	15	12	19
Number of hot drinks sold	410	258	192	324	328	156	280	384	230	280



1.1 Describe the trend of the data. (1)

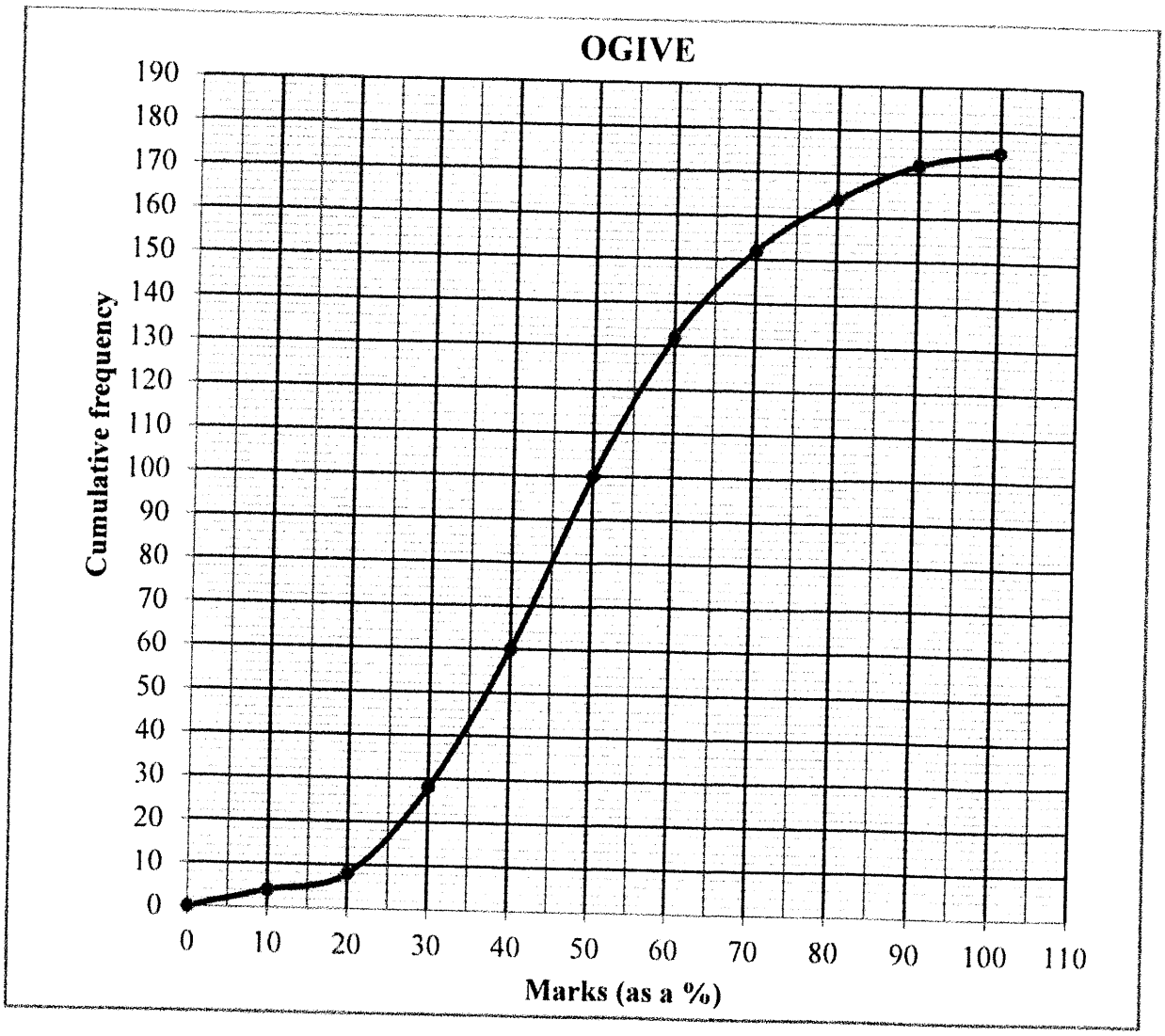
1.2 Determine the equation of the least squares regression line for the data. (3)

1.3 The owner observed that he had used one litre of milk for every 8 cups of hot drinks sold. If the temperature at 13:00 on the 11th day was expected to be 17 °C, predict the number of 1-litre boxes of milk the owner should buy for the 11th day. (3)

1.4 Identify an outlier in the data. (1)
[8]

QUESTION 2

2.1 Learners from various schools wrote an aptitude test in order to qualify for a bursary. Their marks (as a percentage) are represented in the ogive (cumulative frequency graph) below.



2.1.1 How many learners wrote the test? (1)

2.1.2 Write down the modal class of the data. (1)

2.1.3 The minimum mark to qualify for a bursary is 75%. How many learners qualified for a bursary? (2)

2.2 The table below shows the marks that 15 learners from one particular school obtained in the aptitude test.

Marks (as a %)	62	58	78	85	74	48	74	84	100	46	80	92	60	90	92

Calculate the:

2.2.1 Mean mark obtained by these learners (2)

2.2.2 Standard deviation of these learners' marks (1)

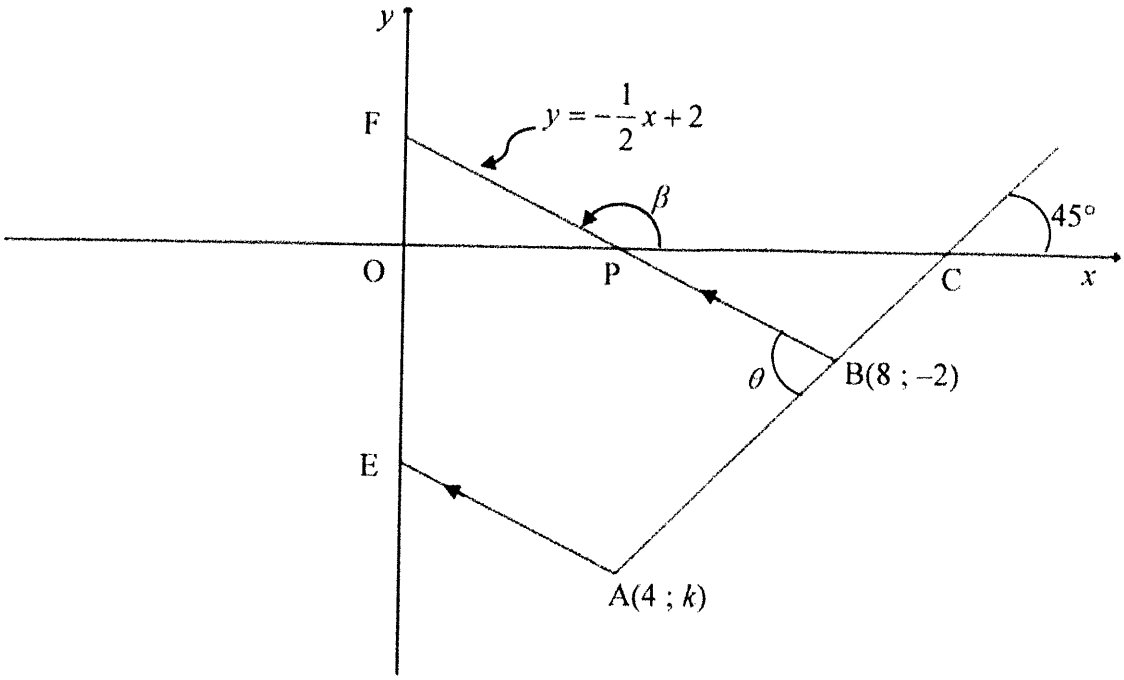
2.2.3 Number of these learners whose marks lie more than one standard deviation above the mean (2)

2.3 The final Grade 11 marks (as a percentage) obtained by a group of learners was analysed. The one standard deviation interval about the mean was calculated as (82,7 ; 94,1).

Calculate the standard deviation for the final Grade 11 marks. (3)
[12]

QUESTION 3

In the diagram below, the line BF is drawn from B(8 ; –2) to cut the x-axis at P and the y-axis at F. The inclination of BF is β and the equation of BF is $y = -\frac{1}{2}x + 2$. From A(4 ; k), another line is drawn parallel to BF and cuts the y-axis at E. The line passing through A and B has an inclination of 45° and cuts the x-axis at C. $A\hat{B}F = \theta$.



3.1 Calculate the gradient of AB. (1)

3.2 Show that the value of k is –6. (2)

3.3 Determine the equation of EA in the form $y = mx + c$. (3)

3.4 Calculate the:

3.4.1 Size of θ (3)

3.4.2 Length of BF (3)

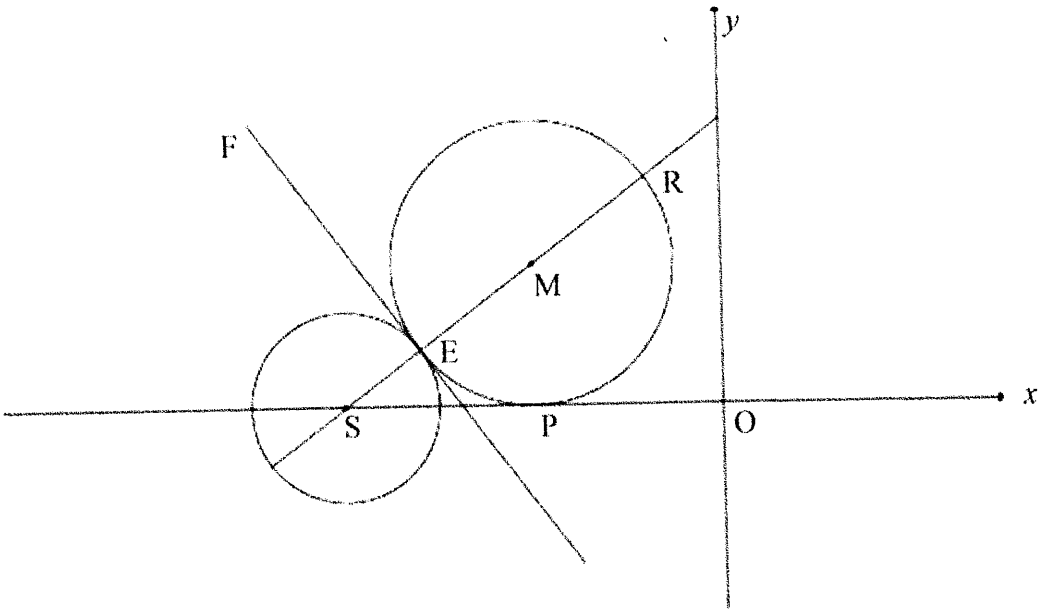
3.4.3 Area of ΔABF (4)

3.5 Let G be a point in the fourth quadrant such that APBG is a parallelogram. Calculate the size of $P\hat{A}G$. (4)

[20]

QUESTION 4

In the diagram below, S is a point on the x-axis. A circle centred at S and a circle centred at M are drawn. The two circles touch each other externally at E. FE is a common tangent to the circles at E. The circle centred at M, having ER as a diameter, touches the x-axis at P.



4.1 The equation of the circle centred at S is $(x+8)^2 + y^2 = 4$.

4.1.1 Write down the coordinates of S. (2)

4.1.2 Show that the diameter of the circle centred at S is 4 units. (1)

4.2 If it is further given that SR = 8 units and $R\left(-\frac{8}{5}; \frac{24}{5}\right)$, calculate the:

4.2.1 Length of EM (2)

4.2.2 Gradient of the tangent FE (3)

4.2.3 Coordinates of M (4)

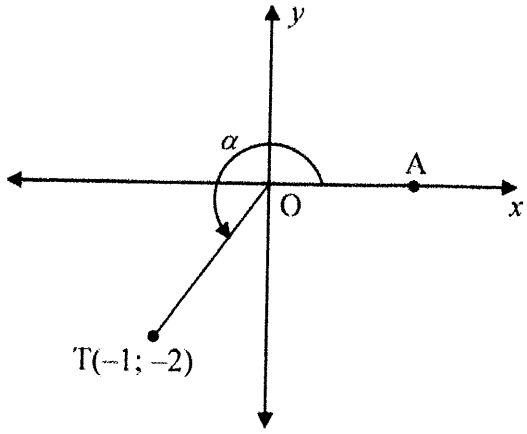
4.2.4 Coordinates of E (2)

4.3 The circle centred at M(–4 ; 3) is shifted 1 unit to the left and reflected in the x-axis to form a new circle centred at K. Determine whether the point (–8 ; 0) lies inside or outside the circle centred at K. Show ALL calculations. (5)

[19]

QUESTION 5

5.1 Point T(−1; −2) is given in the diagram below. A is a point on the x-axis such that reflex $A\hat{O}T = \alpha$.



Determine, **without using a calculator**, the value of each of the following:

5.1.1 $\tan\alpha$ (1)

5.1.2 $\cos\alpha$ (2)

5.1.3 $\cos(\alpha + 45°)$ in simplest form (4)

5.2 Determine, **without using a calculator**, the value of the following expression:

$2\sin(-20°).\sin 160° - \cos 40°$ (4)

5.3 Consider: $3\cos x.\sin x + \tan x.\cos^2(180° - x)$

5.3.1 Simplify the expression to a single trigonometric ratio. (4)

5.3.2 Hence, write down the range of:

$f(x) = 3\cos x.\sin x + \tan x.\cos^2(180° - x)$ (2)

5.4 Prove the identity: $\dfrac{\cos 3x}{\cos x} = 4\cos^2 x - 3$ (5)

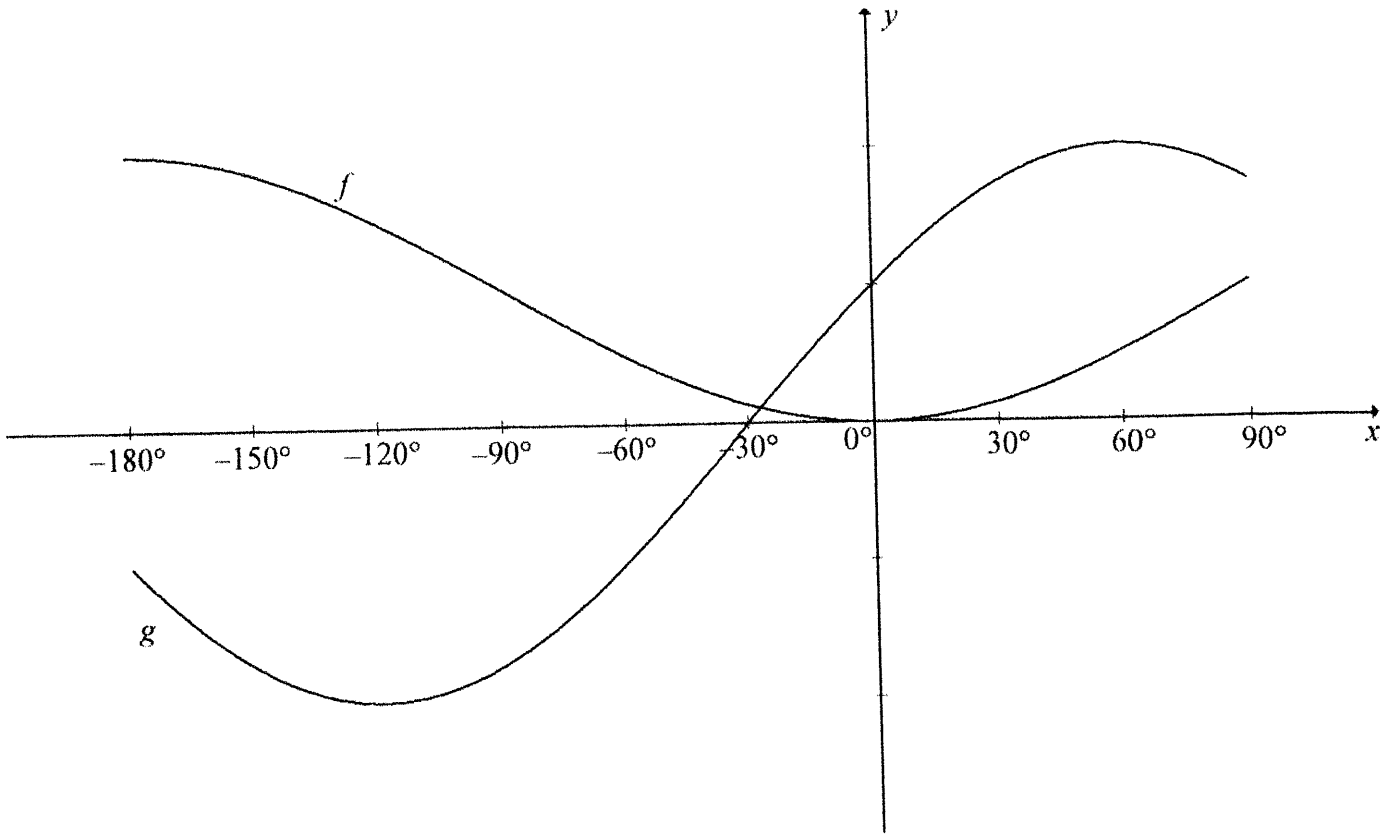
5.5 Determine the general solution of x in the following equation:

$3^{2\tan x} - 3^{\tan x + 1} = 54$ (5)

[27]

QUESTION 6

In the diagram, the graphs of $f(x) = -\cos x + 1$ and $g(x) = 2\sin(x + 30°)$ are drawn for the interval $x \in [-180°; 90°]$.



6.1 For which values of x, $x \in [-180° ; 90°]$, will:

6.1.1 $f(x).g(x) \geq 0$ (2)

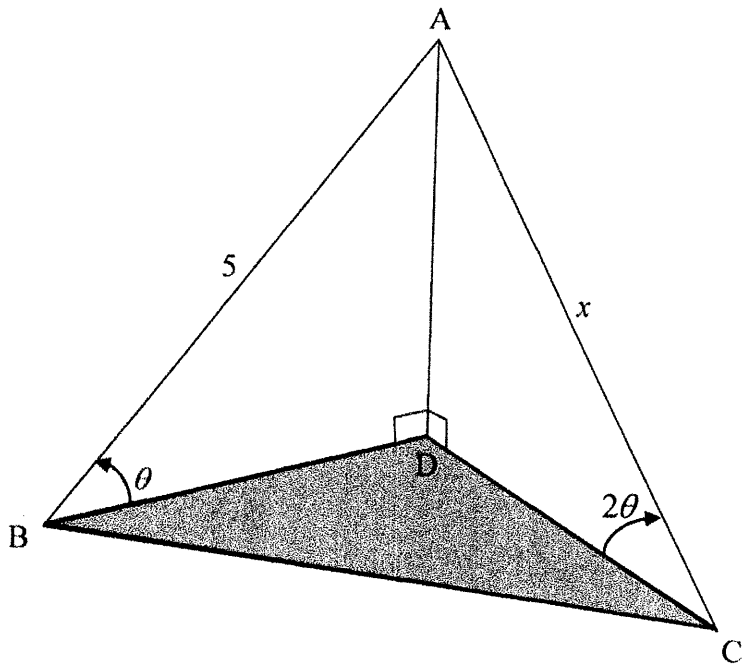
6.1.2 $g(x) = -1$ (2)

6.2 The y-axis is moved $90°$ to the right. Determine the new equation of the graph originally called f, in its simplest form. (2)

[6]

QUESTION 7

In the diagram, B, C and D are in the same horizontal plane. AD is a vertical pole anchored by two cables, AB and AC. The angles of elevation from B and C to A, the top of the pole, are θ and 2θ respectively. AB = 5 units and AC = x units.



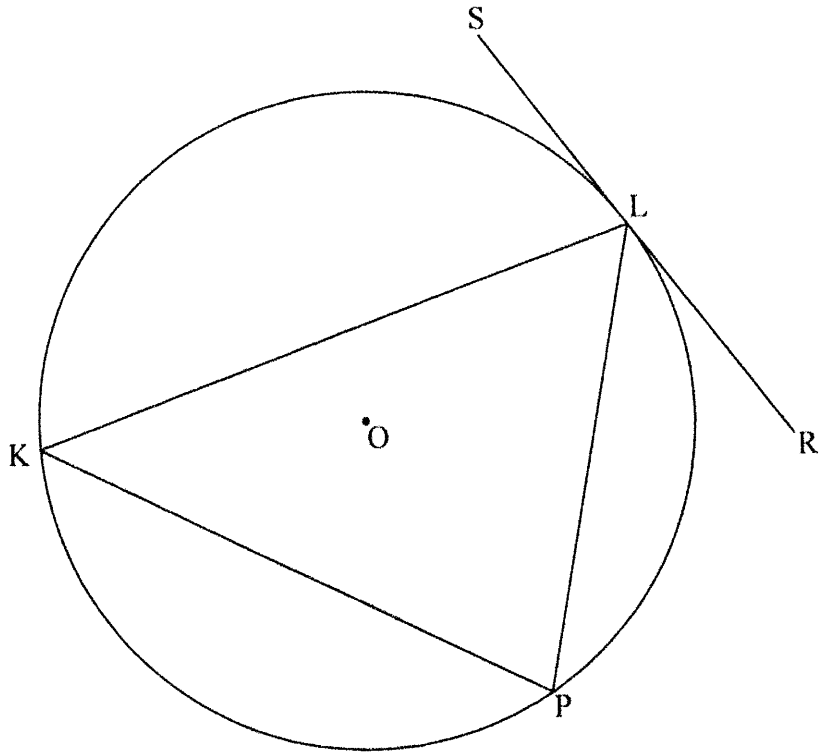
7.1 Show that $x = \dfrac{5}{2\cos\theta}$ (5)

7.2 Calculate the length of BC if it is given that $B\hat{A}C = 112°$ and $\theta = 30°$. (3)

[8]

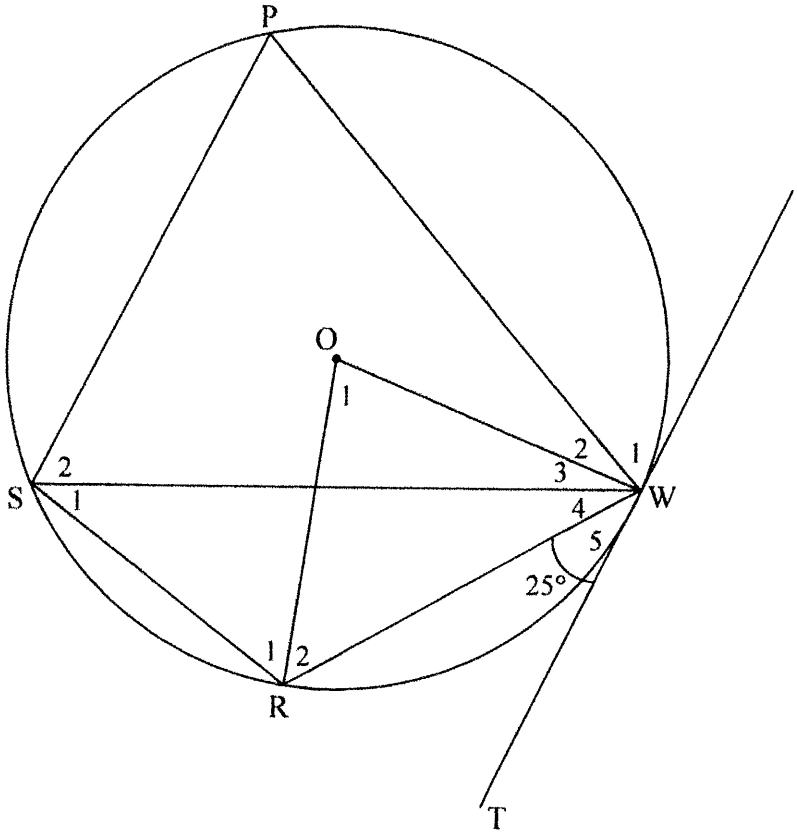
QUESTION 8

8.1 In the diagram, chords KL, LP and KP are drawn in a circle, centered at O. SLR is a tangent to the circle at L.



Prove the theorem which states that the angle between the tangent SLR and chord KL is equal to the angle in the alternate segment, that is prove that $S\hat{L}K = \hat{P}$. (6)

8.2 In the diagram below, PWRS is a cyclic quadrilateral in the circle, centered at O. ΔPSW is an equilateral triangle. TW is a tangent to the circle at W. Radii OR and OW are drawn. $\hat{W}_5 = 25°$.



8.2.1 Determine, giving reasons, the size of:

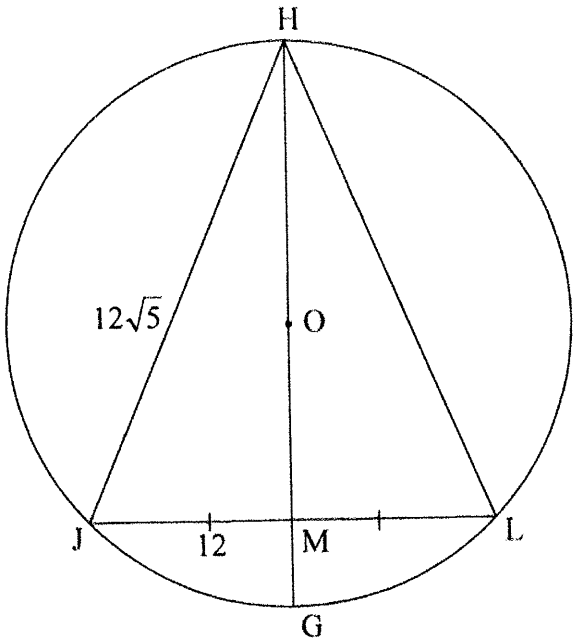
(a) $\hat{S}_1$ (2)

(b) $\hat{O}_1$ (2)

(c) $\hat{R}_1$ (5)

8.2.2 Prove that SP || TW. (3)

8.3 In the diagram below, a circle centered at O is drawn. H, J, G and L are points on the circle. ΔHJL is drawn. HOG bisects JL at M.
$HJ = 12\sqrt{5}$ units and $JM = 12$ units.

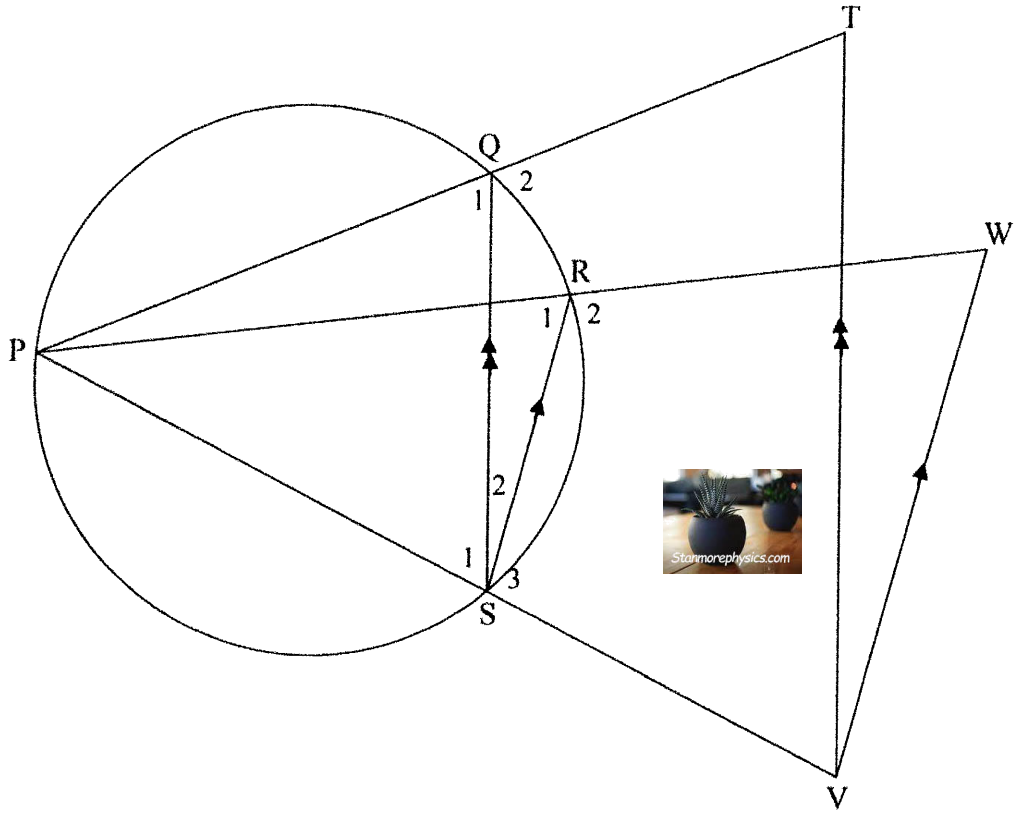


8.3.1 If $MG = 6$ units and $OM = x$, write HM in terms of x. (2)

8.3.2 Calculate, giving reasons, the length of the radius of the circle. (5)
[25]

QUESTION 9

In the diagram below, P, Q, R and S are points on a circle. PS, PQ and PR are produced to V, T and W respectively. VT || SQ and SR || VW.



Prove, giving reasons, that:

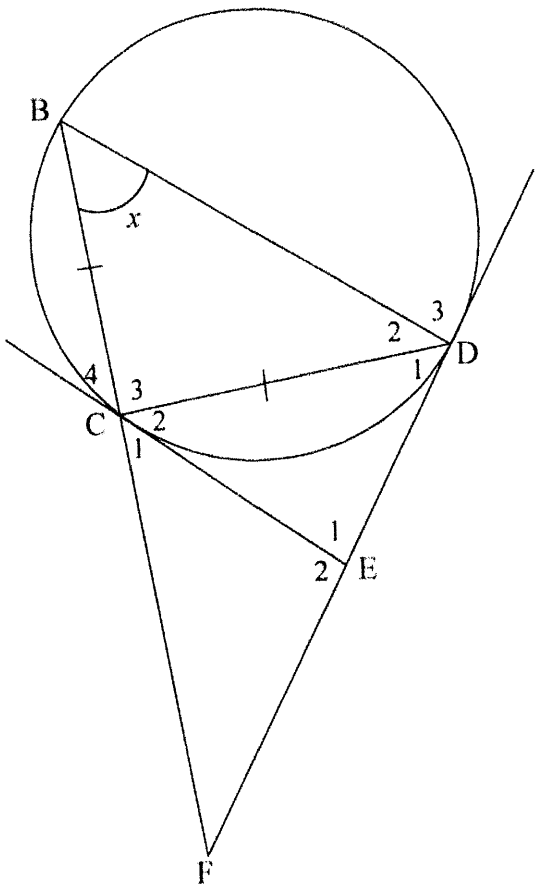
9.1 $\frac{TQ}{QP} = \frac{WR}{RP}$ (3)

9.2 TPVW is a cyclic quadrilateral (5)

[8]

QUESTION 10

In the diagram below, B, C and D are points on a circle such that BC = CD. EC and ED are tangents to the circle at C and D respectively. BC produced meets tangent DE produced at F. $\hat{B} = x$.



10.1 Prove, giving reasons, that:

10.1.1 $\hat{E}_1 = 180° - 2x$ (5)

10.1.2 $\Delta ECD \parallel\!| \Delta CBD$ (3)

10.2 Prove, giving reasons, that:

10.2.1 $CD^2 = CE.BD$ (3)

10.2.2 $\dfrac{CF^2}{EF^2} = \dfrac{BD}{DE}$ (6)

[17]

TOTAL: 150

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad A = P(1-ni)$ $\quad A = P(1-i)^n$ $\quad A = P(1+i)^n$

$T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}[2a + (n-1)d]$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a(r^n - 1)}{r-1}$; $r \neq 1$ $\quad S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $\quad P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad M\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$(x-a)^2 + (y-b)^2 = r^2$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc.\cos A$

area $\Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\quad \sin 2\alpha = 2\sin\alpha\cos\alpha$

$\bar{x} = \frac{\sum x}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $\quad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$

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Candidate Barcode label/Stafie

SC/NSC Answer Book
SS/NSS-antwoordeboek

Senior Certificate/National Senior Certificate/*Senior Sertifikaat/Nasionale Senior Sertifikaat (Grade 12/Graad 12)*

CENTRE NUMBER *SENTRUMNOMMER*	
EXAMINATION NUMBER *EKSAMENNOMMER*	
DATE *DATUM*	
BOOK NUMBER *BOEKNOMMER*	OF *VAN* BOOKS *BOEKE*
SUBJECT CODE *VAKKODE*	
PAPER NUMBER *VRAESTELNOMMER*	2
SUBJECT NAME *VAKNAAM*	**MATHEMATICS/*WISKUNDE***

MARKER/*NASIENER*			MODERATOR'S INITIALS IN RELEVANT BLOCK / *MODERATOR SE VOORLETTERS IN RELEVANTE BLOKKIE*									
Question *Vraag*	Marks *Punte*	Marker's Code & Initials *Nasiener se kode & Voorletters*	Marks *Punte*	SM	Marks *Punte*	DCM *AHN*	Marks *Punte*	CM *HN*	Marks *Punte*	IM	Marks *Punte*	EM
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CONTROLLED AND CERTIFIED CORRECT
(SURNAME AND INITIALS OF EA)
GEKONTROLEER EN AS KORREK GESERTIFISEER (VAN EN VOORLETTERS VAN EA)

READ INSTRUCTIONS ON THE NEXT PAGE
LEES INSTRUKSIES OP VOLGENDE BLADSY

This answer book consists of 23 pages./*Hierdie antwoordeboek bestaan uit 23 bladsye.*

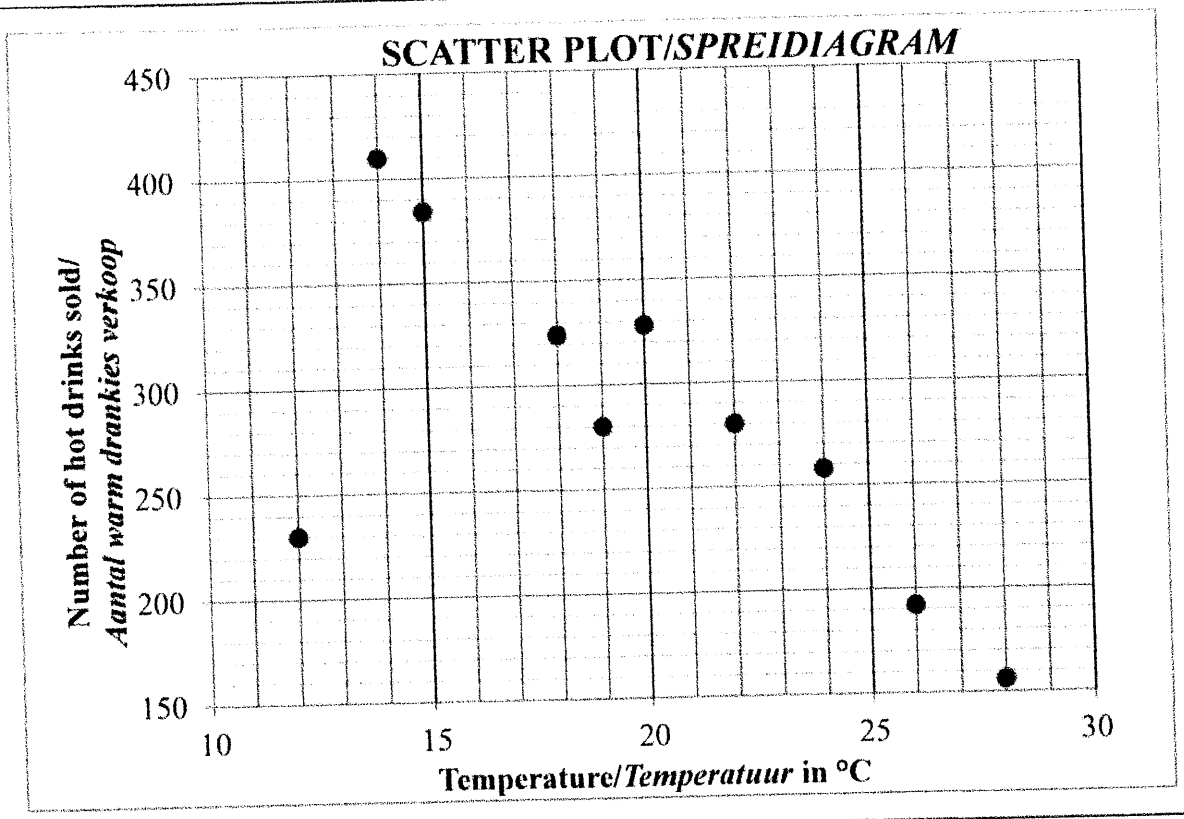
KWAZULU-NATAL

Please turn over/*Blaai om asseblief*

PLEASE FOLLOW THESE INSTRUCTIONS CAREFULLY	*FOLG ASSEBLIEF HIERDIE INSTRUKSIES NOUKEURIG*
1. Clearly write your examination number and centre number in the space provided and attach your barcode label in the space provided.	*1. Skryf jou eksamennommer en sentrumnommer duidelik in die ruimtes verskaf en plak jou stafieskodeplakker in die ruimte verskaf.*
2. Remember that your own name (or the name of your school) may NOT appear anywhere on or in this answer book.	*2. Onthou dat jou eie naam (of die naam van jou skool) NIE op of in hierdie antwoordeboek mag voorkom NIE.*
3. Answer ALL questions in the spaces provided.	*3. Beantwoord ALLE vrae in die ruimtes wat verskaf is.*
4. NO pages may be torn from this answer book.	*4. GEEN bladsye mag uit hierdie antwoordeboek geskeur word NIE.*
5. Read the instructions printed on your timetable carefully as well as any other instructions which may be given in each question paper.	*5. Lees die instruksies, wat op jou eksamenrooster gedruk is, sorgvuldig deur, asook enige ander instruksies wat op elke vraestel gegee word.*
6. Candidates may NOT retain an answer book or remove it from the examination room.	*6. GEEN antwoordeboek mag deur die kandidaat behou of uit die eksamenlokaal verwyder word NIE.*
7. Answers must be written in black/blue ink as distinctly as possible. Do NOT write in the margins.	*7. Skryf die antwoorde so duidelik moontlik met swart/blou ink. Laat die kantlyne oop.*
8. Write the numbers of the questions you have answered on the front cover of the answer book where marks are to be recorded.	*8. Skryf die nommers van die vrae wat jy beantwoord het op die voorblad van die antwoordeboek waar die punte aangebring word.*
9. If you require additional space for your answers: 9.1 Use the additional space provided at the end of the answer book. 9.2 When answering a question in the additional space, clearly indicate the question number in the column on the LHS. 9.3 Rule off after each answer.	*9. In geval jy bykomende ruimte benodig vir jou antwoorde:* *9.1 Gebruik die bykomende ruimte wat aan die einde van die antwoordeboek verskaf word.* *9.2 As 'n vraag in die bykomende ruimte beantwoord word, dui duidelik die vraagnommer in die kolom aan die LK aan.* *9.3 Trek 'n lyn na elke antwoord.*
10. Draw a neat line through any work/rough work that must not be marked.	*10. Trek 'n netjiese lyn deur enige werk/rofwerk wat nie nagesien moet word nie.*

QUESTION/*VRAAG* 1

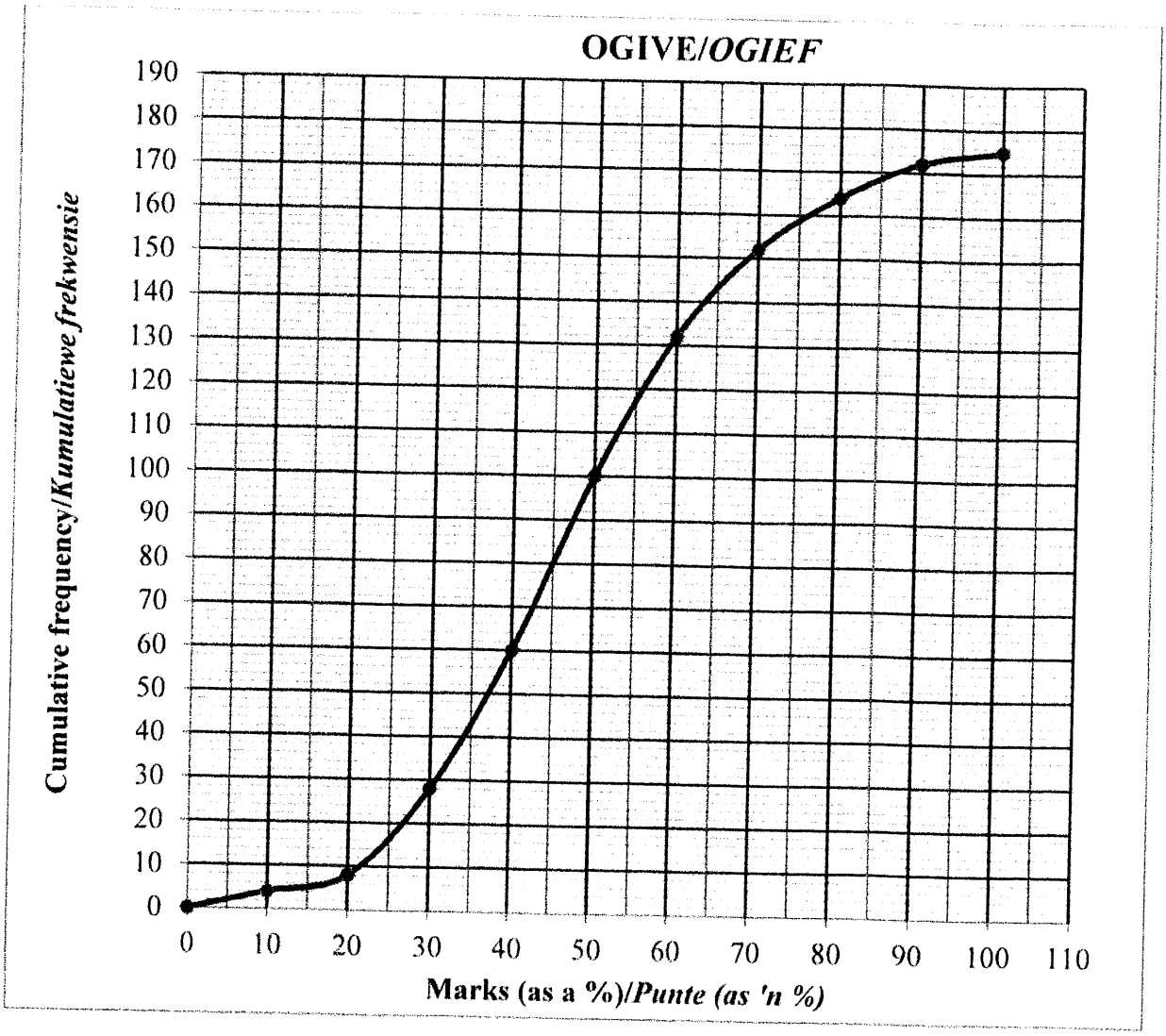
Temperature/*Temperatuur* (in °C)	14	24	26	18	20	28	22	15	12	19
Number of hot drinks sold ***Aantal warm drankies verkoop***	410	258	192	324	328	156	280	384	230	280



	Solution/*Oplossing*	**Marks** ***Punte***
1.1		(1)
1.2		(3)
1.3		(3)
1.4		(1)
		[8]

QUESTION/*VRAAG* 2

2.1



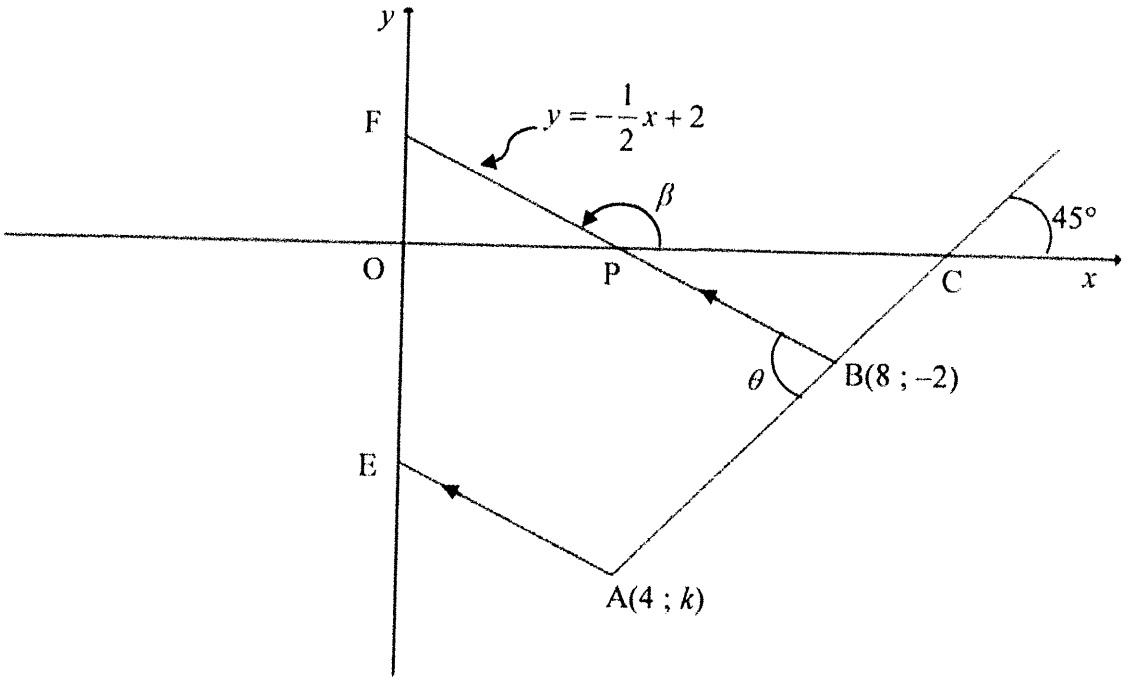
	Solution/*Oplossing*	Marks *Punte*
2.1.1		(1)
2.1.2		(1)
2.1.3		(2)

2.2

Marks (as a %) ***Punte (as 'n %)***	62	58	78	85	74	48	74	84	100	46	80	92	60	90	92

	Solution/*Oplossing*	**Marks** ***Punte***
2.2.1		(2)
2.2.2		(1)
2.2.3		(2)
2.3		(3)
		[12]

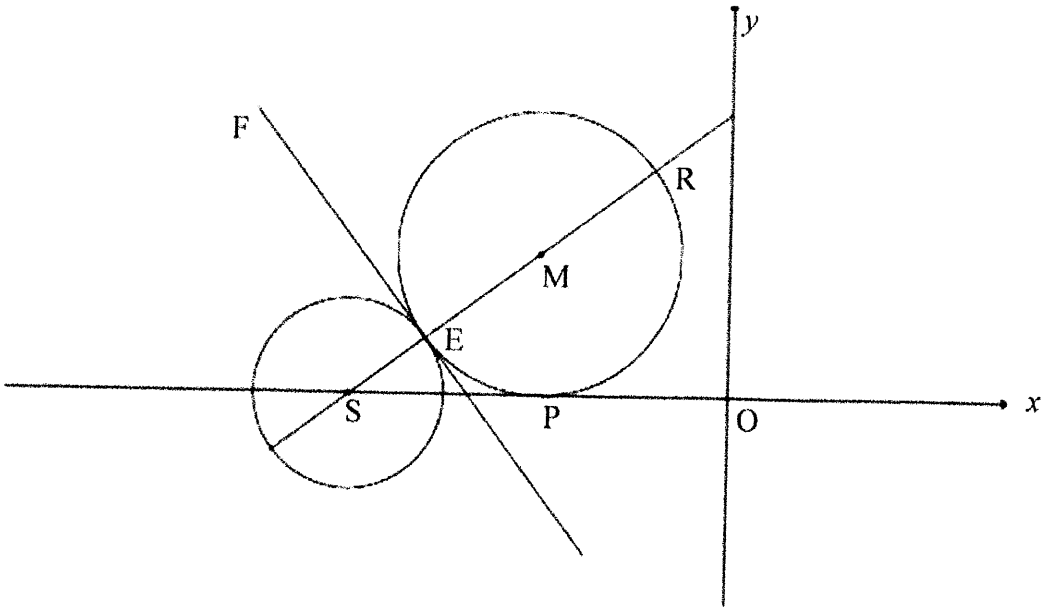
QUESTION/*VRAAG* 3



	Solution/*Oplossing*	Marks *Punte*
3.1		(1)
3.2		(2)
3.3		(3)

	Solution/*Oplossing*	Marks *Punte*
3.4.1		(3)
3.4.2		(3)
3.4.3		(4)
3.5		(4)
		[20]

QUESTION/*VRAAG* 4

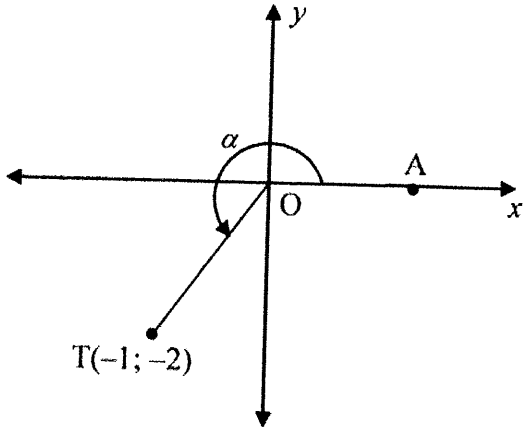


	Solution/*Oplossing*	**Marks** ***Punte***
4.1.1		(2)
4.1.2		(1)
4.2.1		(2)
4.2.2		

	Solution/*Oplossing*	Marks *Punte*
4.2.2 (cont./ *verv.*)		(3)
4.2.3		(4)
4.2.4		(2)
4.3		(5)
		[19]

QUESTION/*VRAAG* 5

5.1

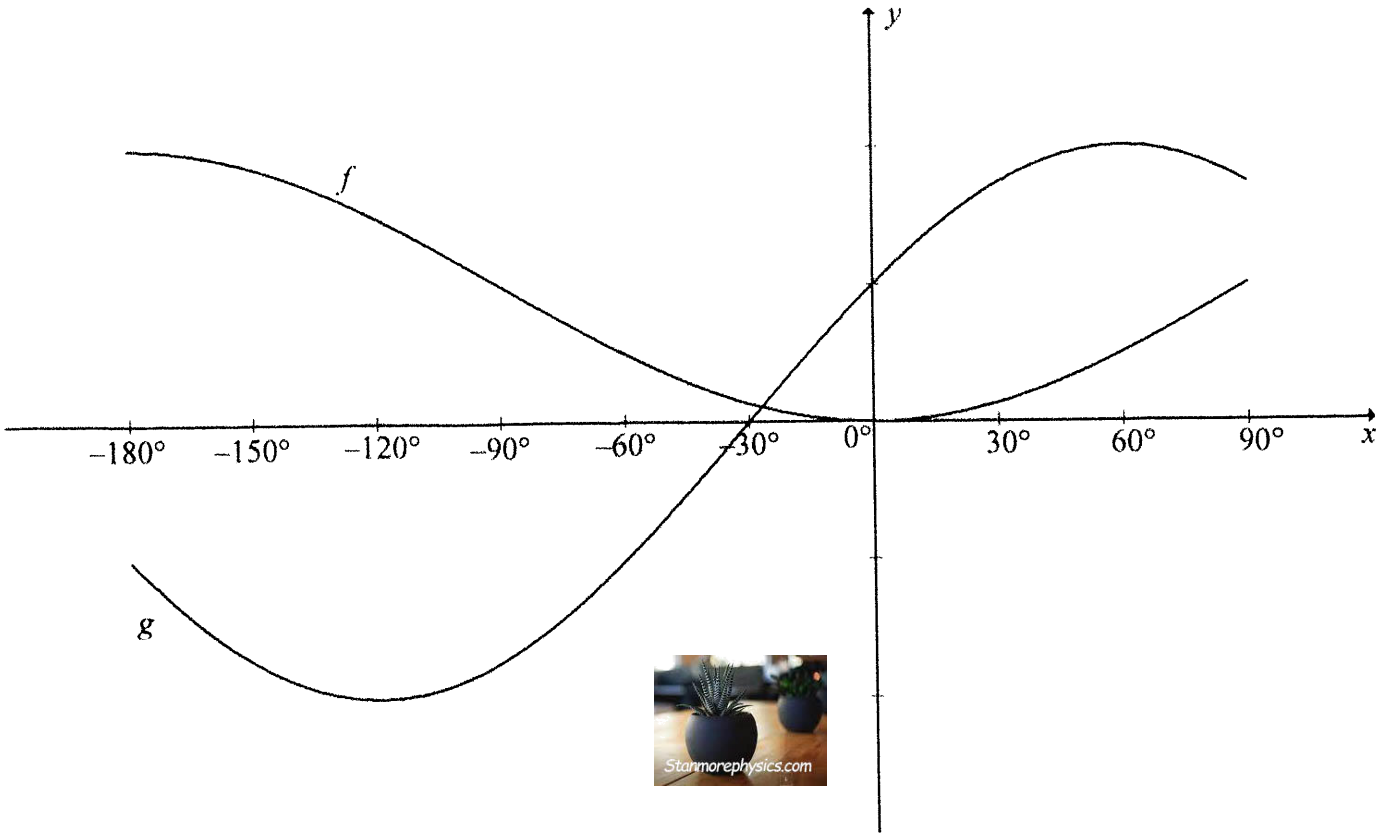


	Solution/*Oplossing*	**Marks** ***Punte***
5.1.1		(1)
5.1.2		(2)
5.1.3		(4)

	Solution/*Oplossing*	Marks *Punte*
5.2		(4)
5.3.1		(4)
5.3.2		(2)
5.4		

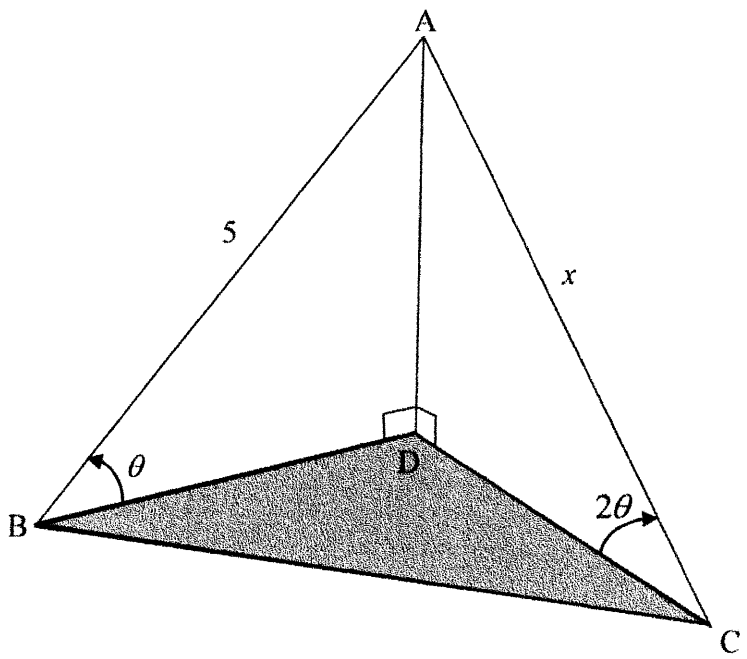
	Solution/*Oplossing*	**Marks** ***Punte***
5.4 (cont./ *verv.*)		
		(5)
5.5		
		(5)
		[27]

QUESTION/*VRAAG* 6



	Solution/*Oplossing*	**Marks** ***Punte***
6.1.1		(2)
6.1.2		(2)
6.2		(2)
		[6]

QUESTION/*VRAAG* 7

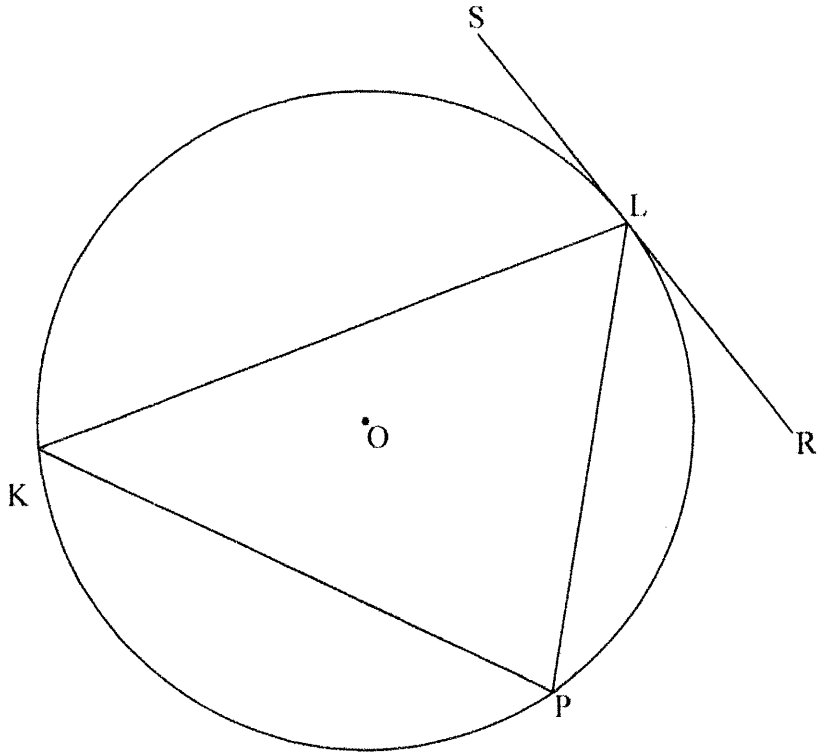


	Solution/*Oplossing*	**Marks** ***Punte***
7.1		(5)
7.2		(3)
		[8]

Give reasons for your statements in QUESTIONS 8, 9 and 10.
Verskaf redes vir jou bewerings in VRAAG 8, 9 en 10.

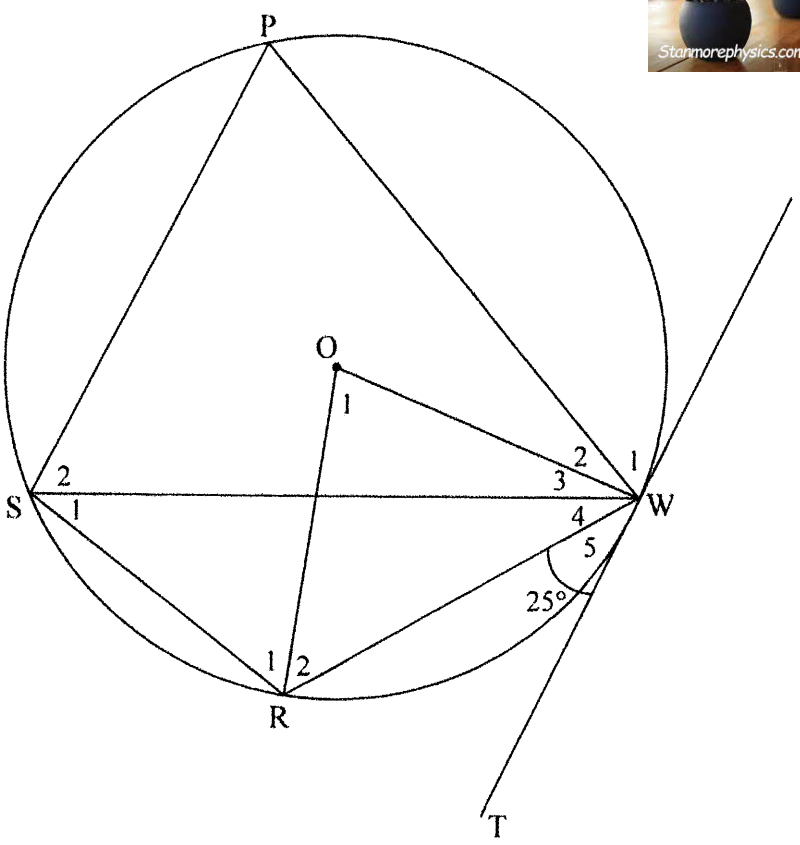
QUESTION/*VRAAG* 8

8.1



	Solution/*Oplossing*	**Marks** ***Punte***
		(6)

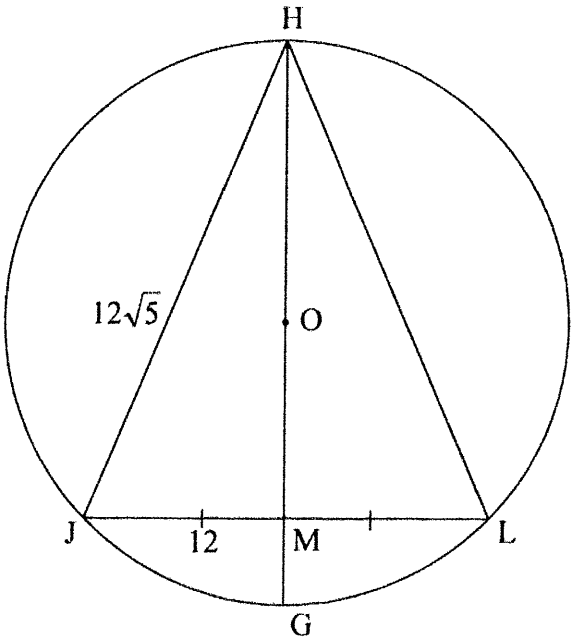
8.2



	Solution/*Oplossing*	Marks *Punte*
8.2.1(a)		(2)
8.2.1(b)		(2)

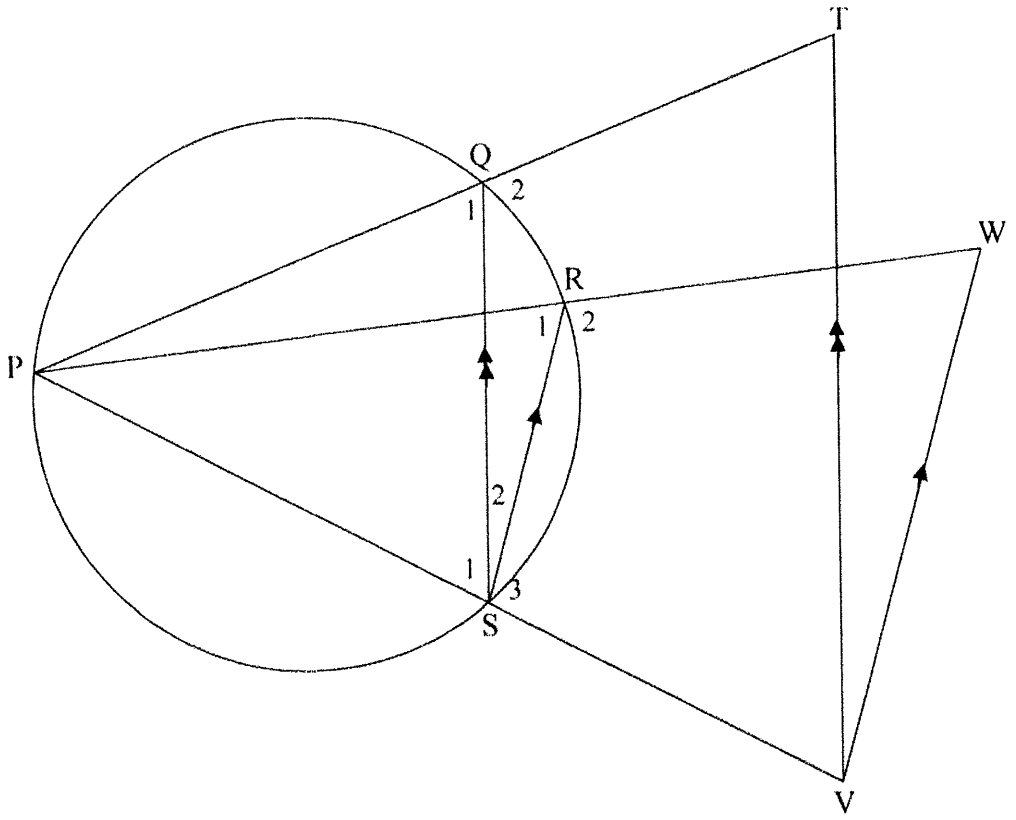
	Solution/*Oplossing*	Marks *Punte*
8.2.1(c)		(5)
8.2.2		(3)

8.3



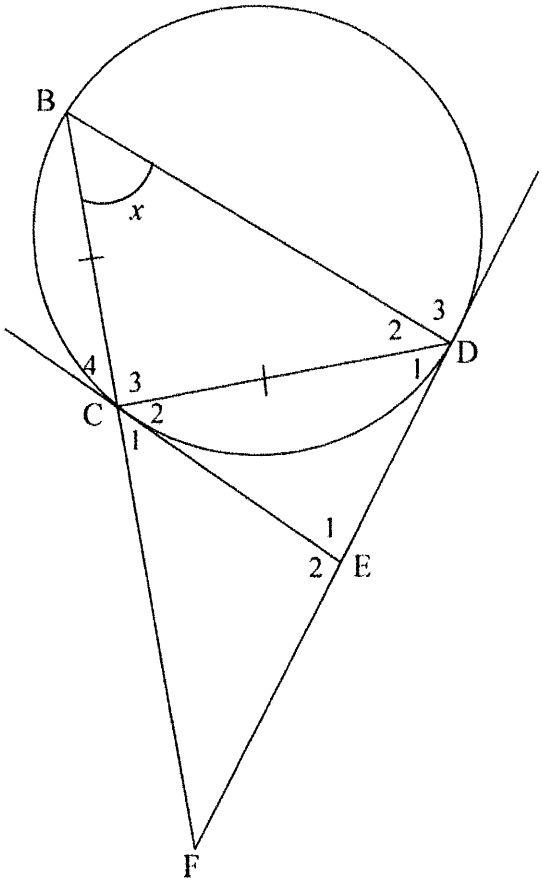
	Solution/*Oplossing*	Marks *Punte*
8.3.1		(2)
8.3.2		(5)
		[25]

QUESTION/*VRAAG* 9



	Solution/*Oplossing*	**Marks** ***Punte***
9.1		(3)
9.2		(5)
		[8]

QUESTION/*VRAAG* 10



	Solution/*Oplossing*	**Marks** ***Punte***
10.1.1		
		(5)
10.1.2		
		(3)

	Solution/*Oplossing*	Marks *Punte*
10.2.1		(3)
10.2.2		(6)
		[17]

	Additional space/*Bykomende ruimte*	**Marks** ***Punte***

	Additional space/*Bykomende ruimte*	Marks *Punte*

TOTAL/*TOTAAL*: 150

RE-MARK/RE-CHECK *HERMERK/HERSIEN*				
Question *Vraag*	Marks *Punte*			Initials *Voorletters*
TOTAL *TOTAAL*				
HASH TOTAL *KAF-TOTAAL*				

KWAZULU-NATAL

basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE/
NASIONALE SENIOR SERTIFIKAAT

GRADE/*GRAAD* 12

MATHEMATICS P2/*WISKUNDE V2*

SEPTEMBER 2021(2)

MARKING GUIDELINES/*NASIENRIGLYNE*

MARKS/*PUNTE*: 150

These marking guidelines consist of 24 pages.
Hierdie nasienriglyne bestaan uit 24 bladsye.

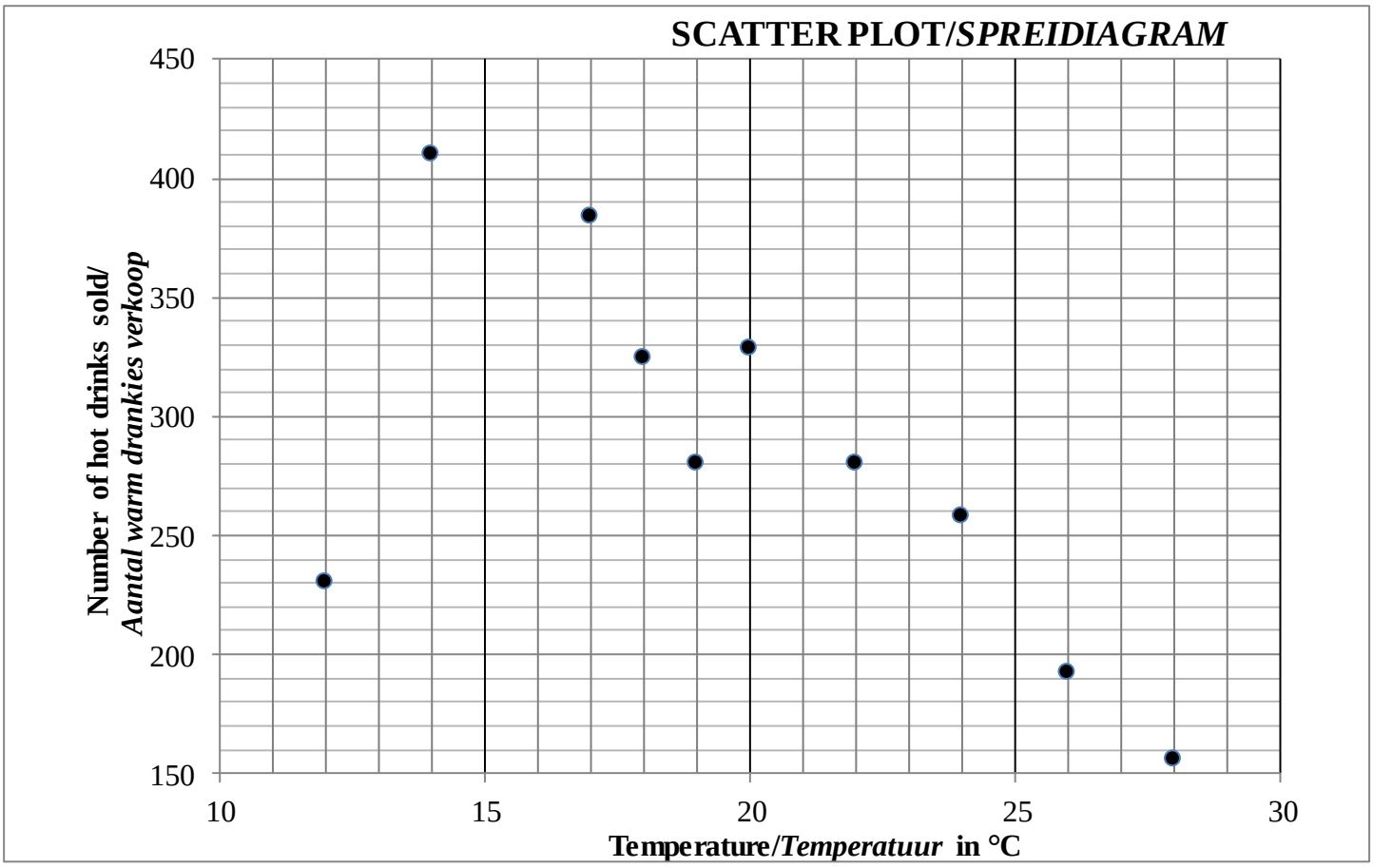
NOTE:
- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

NOTA:
- *As 'n kandidaat 'n vraag TWEE KEER beantwoord, merk slegs die EERSTE poging.*
- *As 'n kandidaat 'n antwoord van 'n vraag doodtrek en nie oordoen nie, sien die doodgetrekte poging na.*
- *Volgehoue akkuraatheid word in ALLE aspekte van die memorandum toegepas. Hou op nasien by die tweede berekeningsfout.*
- *Om antwoorde/waardes te aanvaar om 'n probleem op te los, word NIE toegelaat nie.*

GEOMETRY • *MEETKUNDE*	
S	**A mark for a correct statement** **(A statement mark is independent of a reason)**
	'n Punt vir 'n korrekte bewering ***('n Punt vir 'n bewering is onafhanklik van die rede)***
R	**A mark for the correct reason** **(A reason mark may only be awarded if the statement is correct)**
	'n Punt vir 'n korrekte rede ***('n Punt word slegs vir die rede toegeken as die bewering korrek is)***
S/R	**Award a mark if statement AND reason are both correct**
	Ken 'n punt toe as die bewering EN rede beide korrek is

QUESTION/*VRAAG 1*

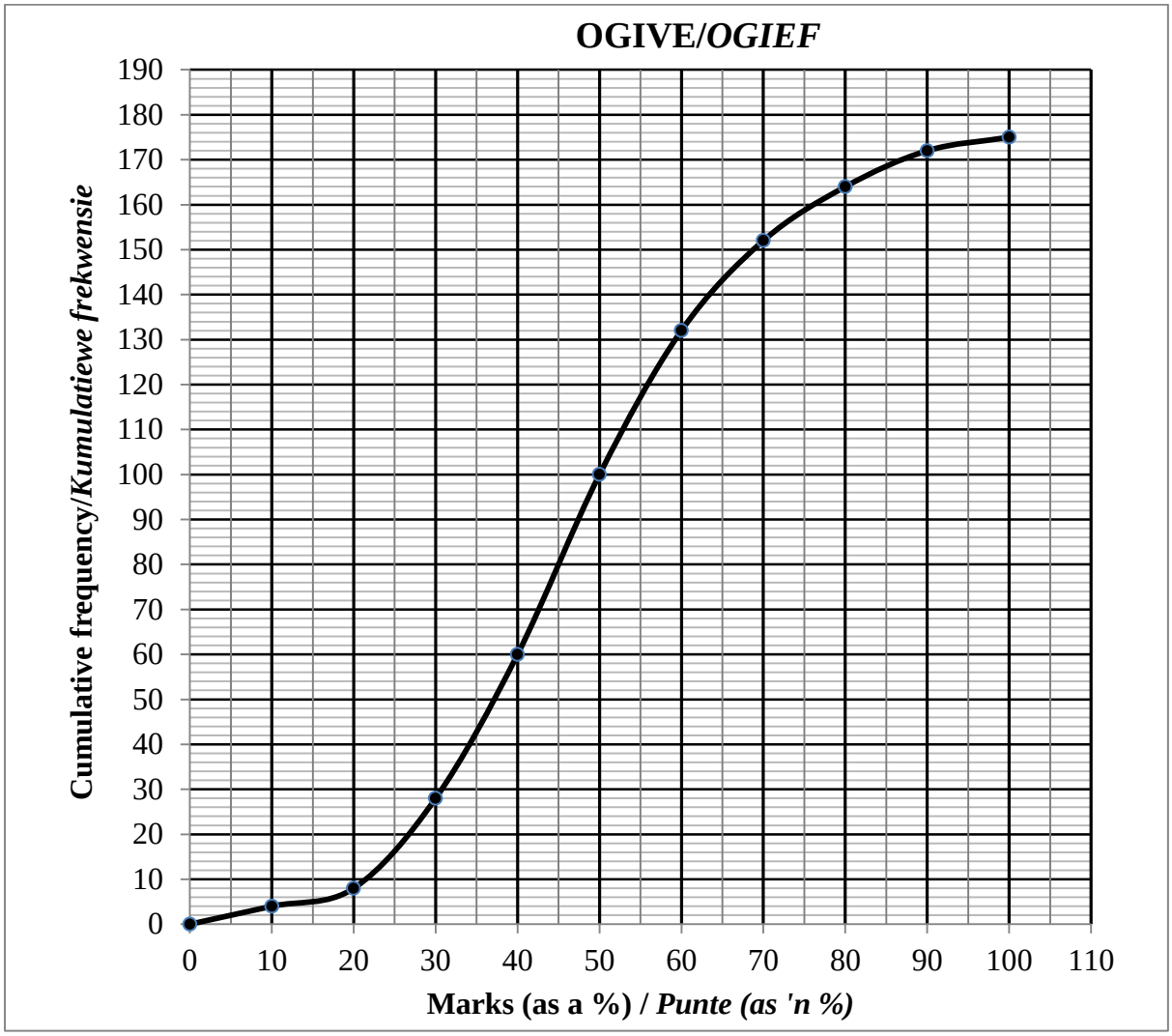
Temperature/ *Temperatuur* (in °C)	14	24	26	18	20	28	22	17	12	19
Number of hot drinks sold *Aantal warm drankies verkoop*	410	258	192	324	328	156	280	384	230	280



1.1	As the temperature increases the number of hot drinks sold decreases. / *Soos die temperatuur toeneem, neem die verkope van die warm drankies af.* **OR** As the temperature decreases the number of hot drinks sold increases. / *Soos die temperatuur afneem, neem die verkope van die warm drankies toe.*	✓ answer (1)
1.2	$a = 489{,}47$ $b = -10{,}37$ $\hat{y} = 489{,}47 - 10{,}37x$	✓ value of a ✓ value of b ✓ equation (3)

1.3	$\hat{y} = 489{,}47 - 10{,}37x$ $= 489{,}47 - 10{,}37(17)$ $= 313{,}18$ Number of hot drinks sold $= 314$ Number of litres of milk $= \frac{314}{8}$ $= 39{,}25$ $= 40$ boxes of 1ℓ	✓substitution ✓ 314 (accept 313) ✓ answer as N_0 (3)
1.4	The outlier is the point (12; 230).	✓(12; 230) (1)
		[8]

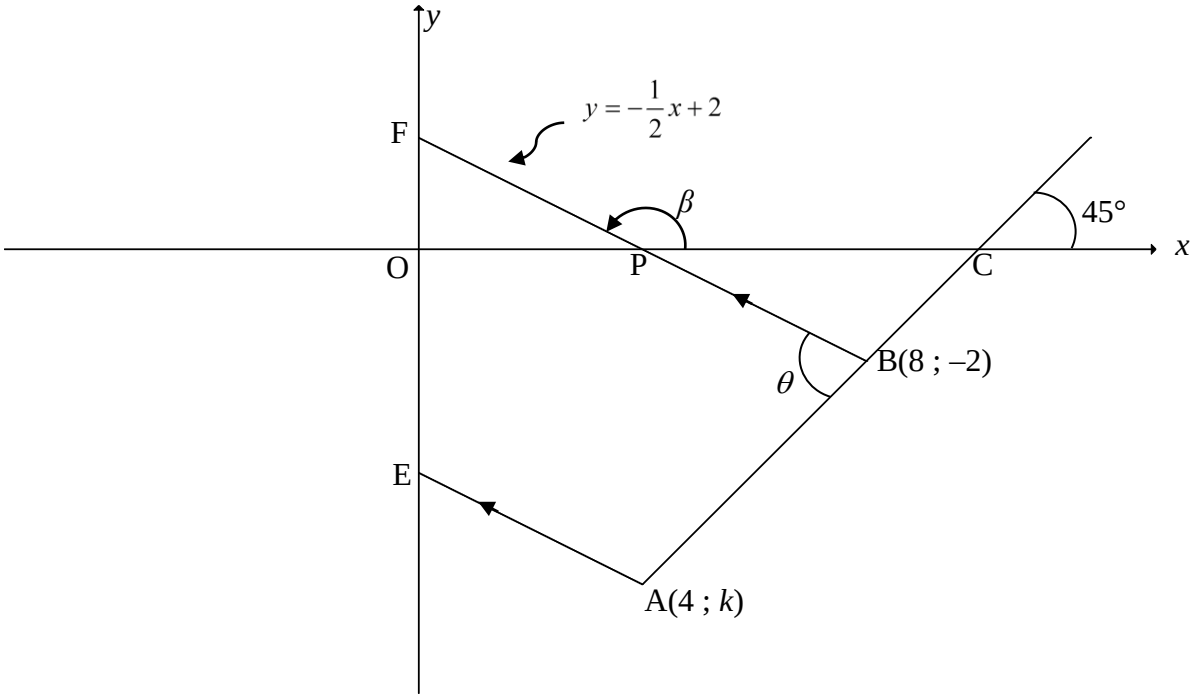
QUESTION/*VRAAG 2*



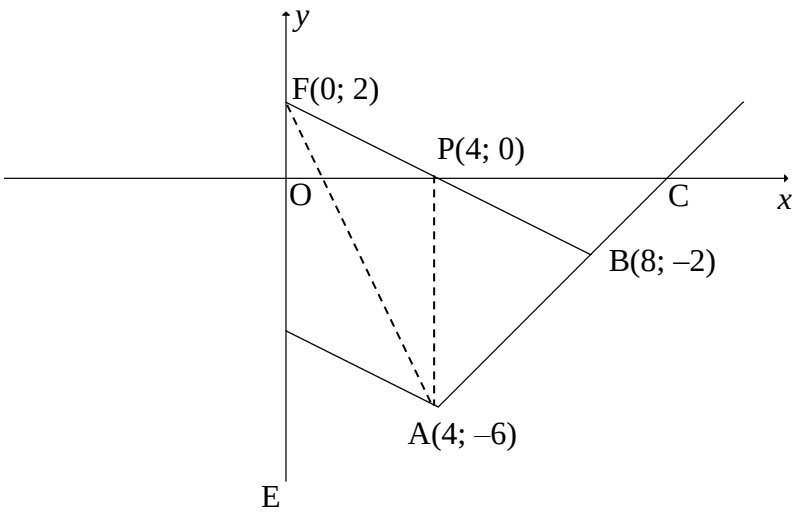
2.1.1	175	✓ answer (1)
2.1.2	$40 \le x < 50$ **OR** $40 < x \le 50$	✓ answer (1)
2.1.3	175 – 158 = 17	✓ 158 (accept 156 to 160) ✓ answer (accept 15 to 19) (2)
2.2.1	$\bar{x} = 74{,}87$	✓✓answer (2)
2.2.2	$\sigma = 16{,}12$	✓ answer (1)
2.2.3	$\bar{x} + \sigma = 74{,}87 + 16{,}12 = 90{,}99$ 3 learners	✓ 90,99 ✓ answer (2)

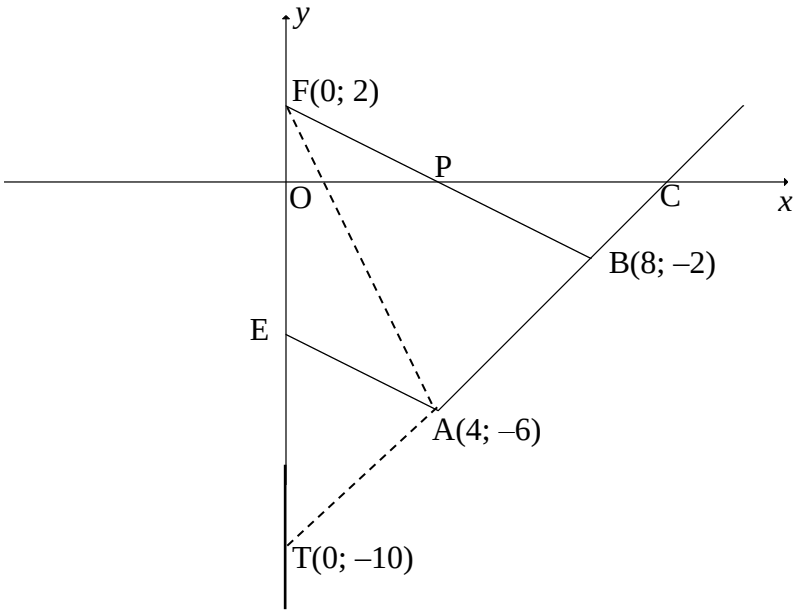
2.3	$\bar{x}-\sigma=82{,}7$ $\bar{x}+\sigma=94{,}1$ $2\bar{x}=176{,}8$ $\bar{x}=88{,}4$ $\sigma=88{,}4-82{,}7$ OR $\sigma=94{,}1-88{,}4$ $\sigma=5{,}7$ $\sigma=5{,}7$ **OR** $\bar{x}=\dfrac{82{,}7+94{,}1}{2}$ $\bar{x}=88{,}4$ $\sigma=88{,}4-82{,}7$ OR $\sigma=94{,}1-88{,}4$ $\sigma=5{,}7$ $\sigma=5{,}7$	✓✓ $\bar{x}=88{,}4$ ✓ answer (3) ✓✓ $\bar{x}=88{,}4$ ✓ answer (3)
		[12]

QUESTION/*VRAAG 3*



3.1	$m_{AB}=\tan 45°=1$	✓ $m_{AB}=\tan 45°=1$ (1)
3.2	$y=x+c$ $-2=8+c$ $c=-10$ $y=x-10$ $k=4-10$ $k=-6$ **OR** $\tan\theta=m_{AB}$ $1=\frac{k-(-2)}{4-8}$ $\frac{k+2}{-4}=1$ $k=-4-2$ $k=-6$	✓equation of AB ✓ substitute A in equation (2) ✓ substitute A & B into gradient formula ✓ equate to 1 (2)

3.3	$m_{FB} = m_{EA} = -\frac{1}{2}$ [FB ∥ EA] $y = -\frac{1}{2}x + c$ $y - y_1 = -\frac{1}{2}(x - x_1)$ $-6 = -\frac{1}{2}(4) + c$ **OR** $y - (-6) = -\frac{1}{2}(x - 4)$ $\therefore y = -\frac{1}{2}x - 4$	✓ $m_{EA} = -\frac{1}{2}$ ✓ substitution of (4; – 6) ✓equation (3)
3.4.1	$\tan\beta = -\frac{1}{2}$ $\beta = 153{,}43°$ $\theta = 26{,}565° + 45°$ [ext < of Δ] $= 71{,}57°$	✓ $\tan\beta = -\frac{1}{2}$ ✓ value of β ✓ value of θ (3)
3.4.2	F(0; 2) B(8; – 2) $BF = \sqrt{(8-0)^2 + (-2-2)^2}$ $BF = \sqrt{80} = 4\sqrt{5}$	✓ F(0; 2) ✓substitution ✓ answer (3)
3.4.3	 $0 = -\frac{1}{2}x + 2$ $x = 4$ ∴ P(4; 0) ∴ PA ∥ y-axis Area ΔABF = area ΔABP + area ΔAPF Area ΔABF $= \frac{1}{2}(6)(4) + \frac{1}{2}(6)(4)$ Area ΔABF = 24 units^2 **OR**	✓ P(4; 0) ✓ area of ΔABP ✓ area of ΔAPF ✓ answer (4)



$y = x + c$

$-2 = 8 + c$

$c = -10$

$\therefore$ T(0; –10) ✓ C(0; –10)

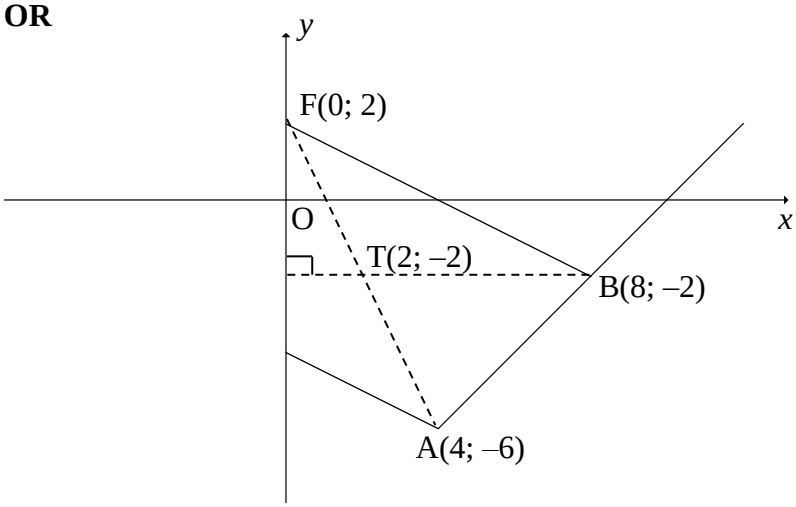
Area ΔABF = area ΔFBT – area ΔAFT

Area $\Delta ABF = \frac{1}{2}(8)(12) - \frac{1}{2}(12)(4)$ ✓ area of ΔABT ✓ area of ΔAFT

Area $\Delta ABF = 24$ units^2 ✓ answer

(4)

OR



$m_{AF} = \frac{-6-2}{4-0} = -2 \quad \therefore \; y = -2x + 2$

$-2 = -2x + 2$

$x = 2 \quad \therefore$ T(2; – 2) ✓ T(2; –2)

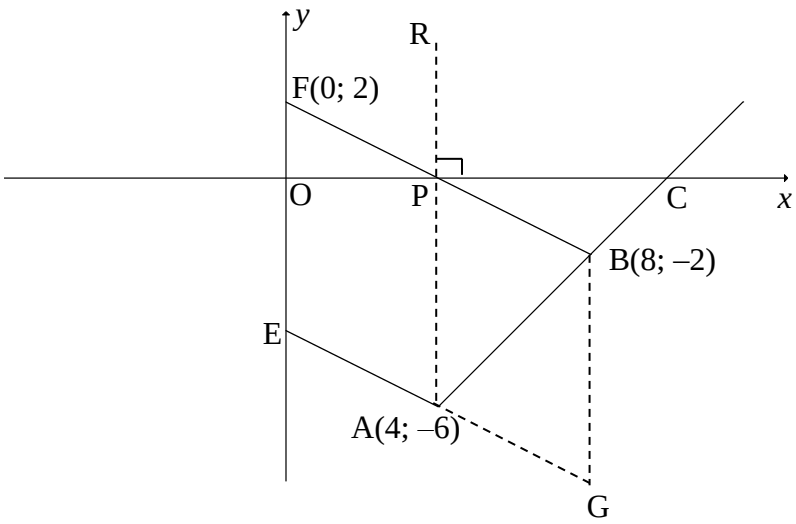
Area ΔABF = area ΔFTB + area ΔTBA

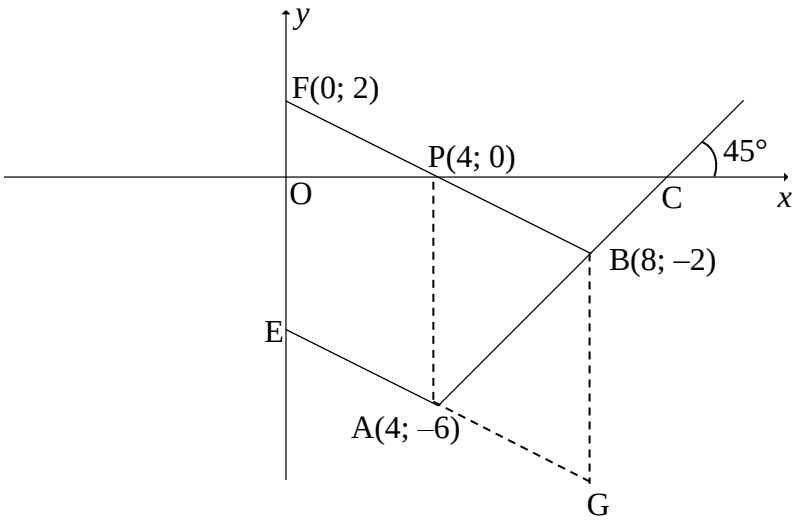
Area $\Delta ABF = \frac{1}{2}(6)(4) + \frac{1}{2}(6)(4)$ ✓ area of ΔFTB ✓ area of ΔTBA

Area $\Delta ABF = 24$ units^2 ✓ answer

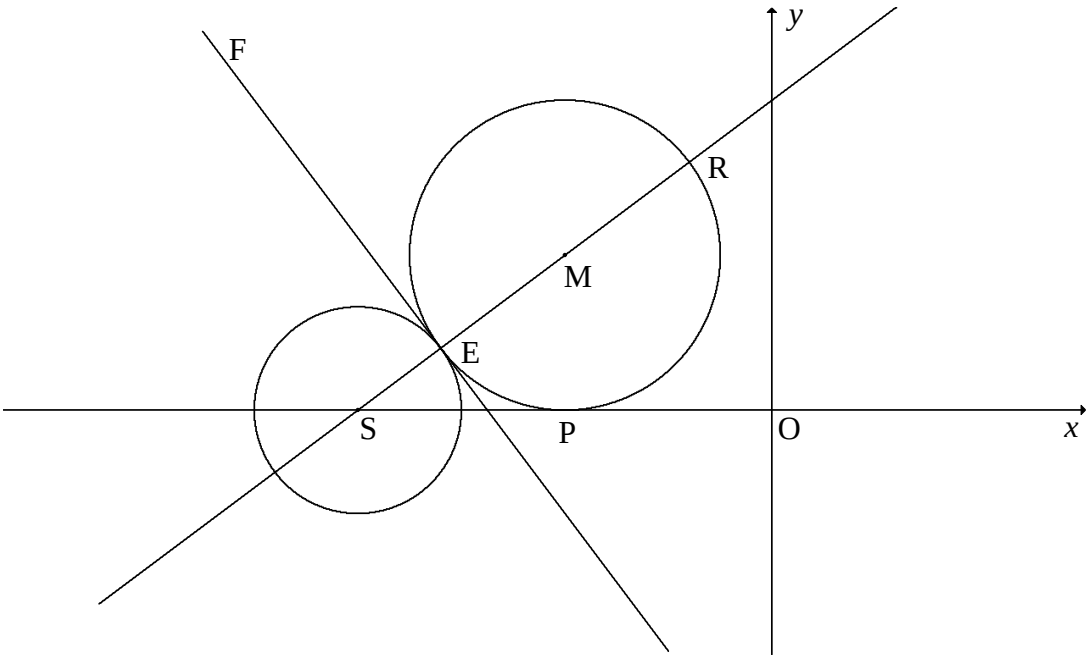
(4)

OR

	$A(4;-6)$ $B(8;-2)$ $AB=\sqrt{(8-4)^2+(-2-(-6))^2}$ $AB=\sqrt{32}=4\sqrt{2}$ Area of $ABF=\frac{1}{2}(AB)(BF)\sin A\hat{B}F$ $=\frac{1}{2}(\sqrt{32})(\sqrt{80})\sin 71{,}57°$ $=24\text{units}^2$	✓ $AB=\sqrt{32}=4\sqrt{2}$ ✓ area formula ✓ substitution into area formula ✓ answer (4)
3.5	 RA ∥ *y*-axis $C\hat{P}B=26{,}57°$ $R\hat{P}B=90°+26{,}57°$ $R\hat{P}B=116{,}57°$ PB ∥ AG $\therefore P\hat{A}G=R\hat{P}B=116{,}57°$ [corresp ∠s; PB ∥ AG] **OR** $O\hat{F}P=153{,}43°-90°$ [ext ∠ of Δ] $O\hat{F}P=63{,}43°$ $F\hat{E}A=180°-63{,}43°$ [co-interior ∠ s; FB ∥ EA] $=116{,}57°$ $P\hat{A}G=116{,}57°$ [corresp ∠ s; FE ∥ PA]	✓ $C\hat{P}B=26{,}57°$ ✓ $R\hat{P}B=90°+C\hat{P}B$ ✓ $R\hat{P}B$ ✓ answer of $P\hat{A}G$ (4) ✓ $O\hat{F}P=63{,}43°$ ✓ $F\hat{E}A=180°-63{,}43°$ ✓ $=116{,}57°$ ✓ answer of $P\hat{A}G$ (4)

	 PA ∥ *y*-axis $P\hat{C}A = 45°$ [vert opp ∠s =] $P\hat{A}C = 45°$ [∠s of Δ] PA ∥ BG $B\hat{A}G = \theta = 71{,}57°$ [alt ∠s; PA ∥ BG] $P\hat{A}G = 45° + 71{,}57°$ $P\hat{A}G = 116{,}57°$	✓ $A\hat{P}C = 90°$ OR AP = PC ✓ $P\hat{A}C = 45°$ ✓ $B\hat{A}G = \theta = 71{,}57°$ ✓ answer of $P\hat{A}G$ (4)
		[20]

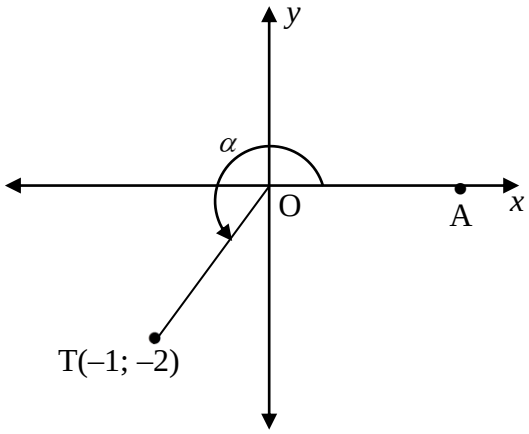
QUESTION/*VRAAG 4*



4.1.1	S(–8 ; 0)	✓ *x*-value ✓ *y*-value (2)
4.1.2	$r = 2$ $\therefore$ diameter = 4 units	✓ $r = 2$ (1)
4.2.1	ER = 6 units EM = 3 units	✓ length of ER ✓ answer (2)
4.2.2	$S(-8;0)\,;R\left(-\frac{8}{5}\,;\,\frac{24}{5}\right)$ $m_{SR} = \frac{0-(\frac{24}{5})}{-8-(-\frac{8}{5})}$ $= \frac{3}{4}$ $m_{FE} = \frac{-4}{3}$ [tan ⊥ rad]	✓ substitution ✓ m_{SM} ✓ answer (3)
4.2.3	EM = MP = 3 units [radii] SM = 5 units $SP^2 = 5^2 - 3^2$ [Pythagoras] SP = 4 units $\therefore$P(–4; 0) $\therefore$M(–4; 3)	✓ MP = 3 units ✓ length of SM ✓ length of SP ✓ coordinates of M (4)

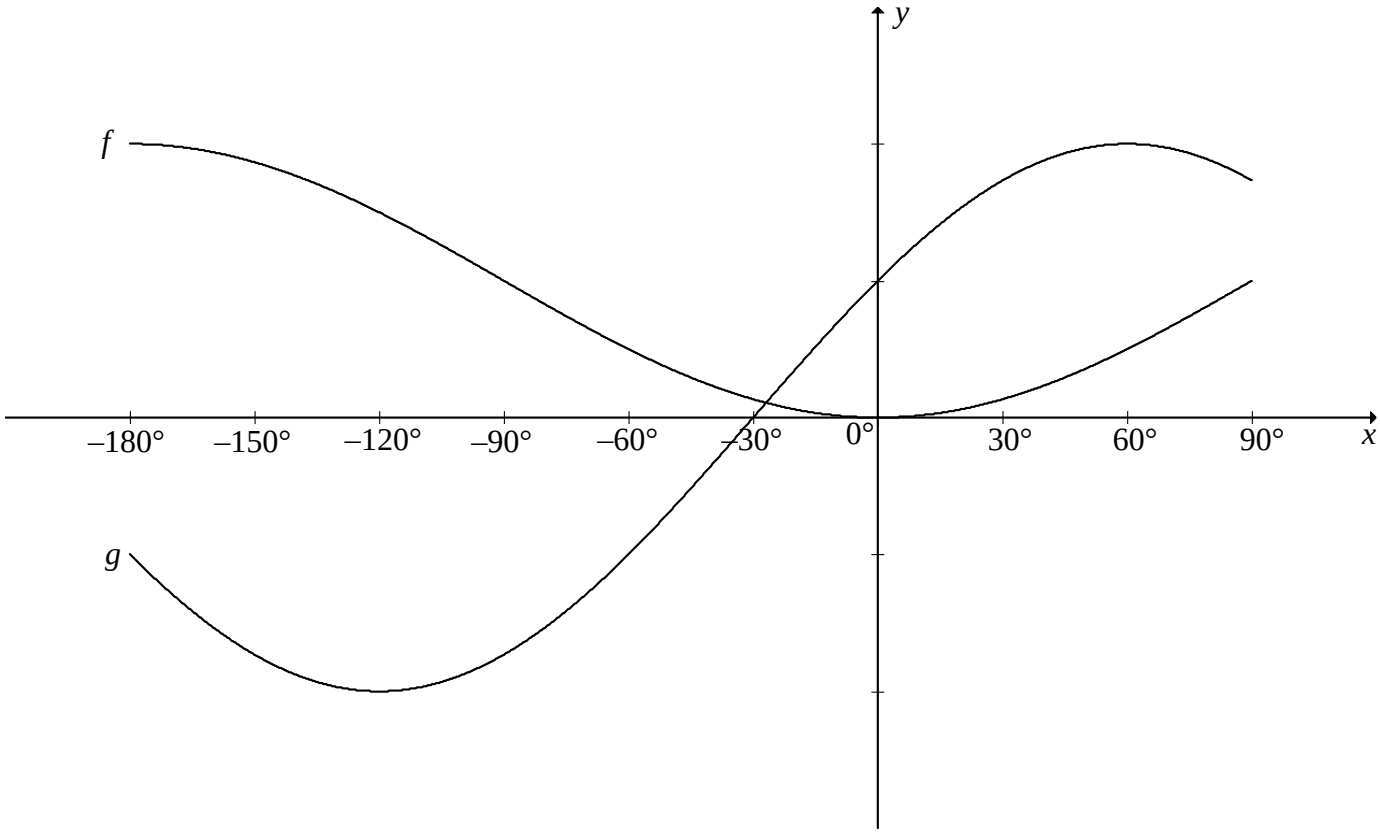
4.2.4	$\frac{x+\left(-\frac{8}{5}\right)}{2} = -4$ and $\frac{y+\frac{24}{5}}{2} = 3$ $x = \frac{-32}{5}$ $y = \frac{6}{5}$ $\therefore \text{E}\left(\frac{-32}{5};\frac{6}{5}\right)$ **OR** By translation: $\text{E}\left(\frac{-32}{5};\frac{6}{5}\right)$	✓ x_E ✓ y_E (2) ✓ x_E ✓ y_E (2)
4.3	$\text{K}(-5;-3)$ $\text{SK} = \sqrt{(-8-(-5))^2+(0-(-3))^2}$ $\text{SK} = \sqrt{18}$ $\text{SK} = 3\sqrt{2}$ SK > 3 (radius of circle) $\therefore$ S lies outside the circle	✓ *x*-value ✓*y*-value ✓ substitution ✓ length of SK ✓ conclusion (5)
		[19]

QUESTION/*VRAAG 5*

5.1.1	 $\tan\alpha = \frac{-2}{-1} = 2$	✓answer (1)
5.1.2	$OT = \sqrt{(-1)^2 + (-2)^2} = \sqrt{5}$ $\cos\alpha = \frac{-1}{\sqrt{5}}$	✓ $OT = \sqrt{5}$ ✓ answer (2)
5.1.3	$\cos(\alpha + 45°)$ $= \cos\alpha\cos 45° - \sin\alpha\sin 45°$ $= \left(\frac{-1}{\sqrt{5}}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{-2}{\sqrt{5}}\right)\left(\frac{\sqrt{2}}{2}\right)$ $= \frac{-\sqrt{2} + 2\sqrt{2}}{2\sqrt{5}}$ $= \frac{\sqrt{2}}{2\sqrt{5}}$  **OR** $\cos(\alpha + 45°)$ $= \cos\alpha\cos 45° - \sin\alpha\sin 45°$ $= \left(\frac{-1}{\sqrt{5}}\right)\left(\frac{1}{\sqrt{2}}\right) - \left(\frac{-2}{\sqrt{5}}\right)\left(\frac{1}{\sqrt{2}}\right)$ $= \frac{-1+2}{\sqrt{10}}$ $= \frac{1}{\sqrt{10}}$	✓expansion ✓ substitution of sin α ✓ special angle ratios ✓answer (4) ✓expansion ✓ substitution of sin α ✓ special angle ratios ✓answer (4)

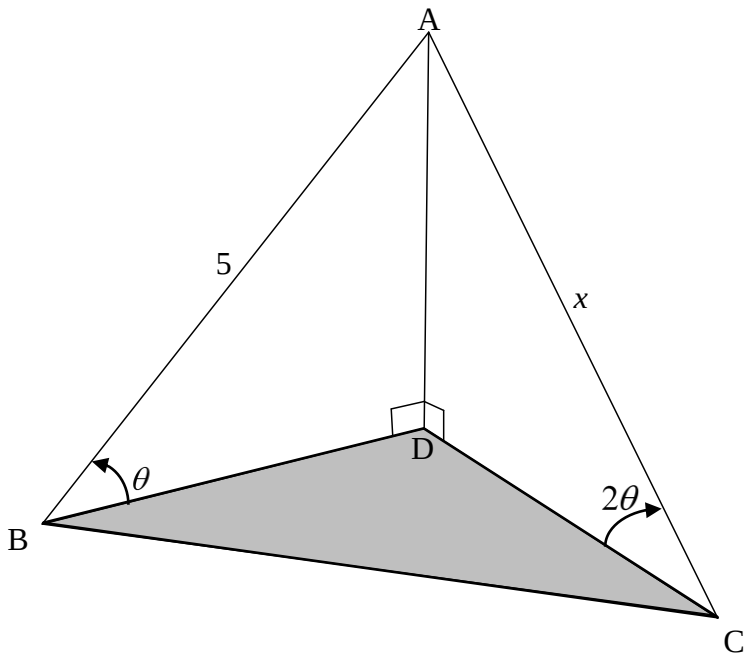
5.2	$2\sin(-20°).\sin 160° - \cos 40°$ $= 2(-\sin 20°).\sin 20° - \cos 40°$ $= -2\sin^2 20° - (1 - 2\sin^2 20°)$ $= -1$	✓ $-\sin 20°$ ✓ $\sin 20°$ ✓ $1 - 2\sin^2 20°$ ✓answer (4)
5.3.1	$3\cos x.\sin x + \tan x.\cos^2(180° - x)$ $= 3\cos x.\sin x + \tan x.(-\cos x)^2$ $= 3\cos x.\sin x + \frac{\sin x}{\cos x}.\cos^2 x$ $= 4\cos x.\sin x$ $= 2\sin 2x$	✓ reduction ✓ identity ✓ simplification ✓ single ratio (4)
5.3.2	$y \in [-2\ ;\ 2]$	✓critical values ✓ notation (2)
5.4	$\frac{\cos 3x}{\cos x} = 4\cos^2 x - 3$ $\text{LHS} = \frac{\cos 3x}{\cos x} = \frac{\cos(2x + x)}{\cos x}$ $= \frac{\cos 2x \cos x - \sin 2x \sin x}{\cos x}$ $= \frac{(2\cos^2 x - 1)\cos x}{\cos x} - \frac{2\sin x \cos x \sin x}{\cos x}$ $= 2\cos^2 x - 1 - 2\sin^2 x$ $= 2\cos^2 x - 1 - 2(1 - \cos^2 x)$ $= 2\cos^2 x - 1 - 2 + 2\cos^2 x$ $= 4\cos^2 x - 3$ $= \text{RHS}$	✓ compound identity ✓ $2\cos^2 x - 1$ ✓ $2\sin x \cos x$ ✓ $1 - \cos^2 x$ ✓expansion (5)
5.5	$3^{2\tan x} - 3^{\tan x + 1} = 54$ $3^{2\tan x} - 3.3^{\tan x} - 54 = 0$ $(3^{\tan x} - 9)(3^{\tan x} + 6) = 0$ $3^{\tan x} = 3^2$ or $3^{\tan x} = -6$ $\tan x = 2$ no solution $\therefore x = 63{,}43° + k.180°;\ k \in Z$ **OR** $\therefore x = 63{,}43° + k.360°;\ k \in Z$ or $x = 243{,}43° + k.360°;\ k \in Z$	✓ standard from ✓ factors ✓ both equations ✓ $\tan x = 2$ ✓ $x = 63{,}43° + k.180°;$ $k \in Z$ (5) **OR** ✓ $x = 63{,}43° + k.360°;$ $k \in Z$ & $243{,}43° + k.360°;$ $k \in Z$ (5)
		[27]

QUESTION/*VRAAG 6*



6.1.1	$x \in [-30° ; 90°]$	✓endpoints ✓notation (2)
6.1.2	$x = -180°$ or $-60°$	✓ $-180°$ ✓ $-60°$ (2)
6.2	$f(x) = -\cos(x+90°)+1$ $= \sin x + 1$	✓ $\cos(x+90°)$ ✓ answer (2)
		[6]

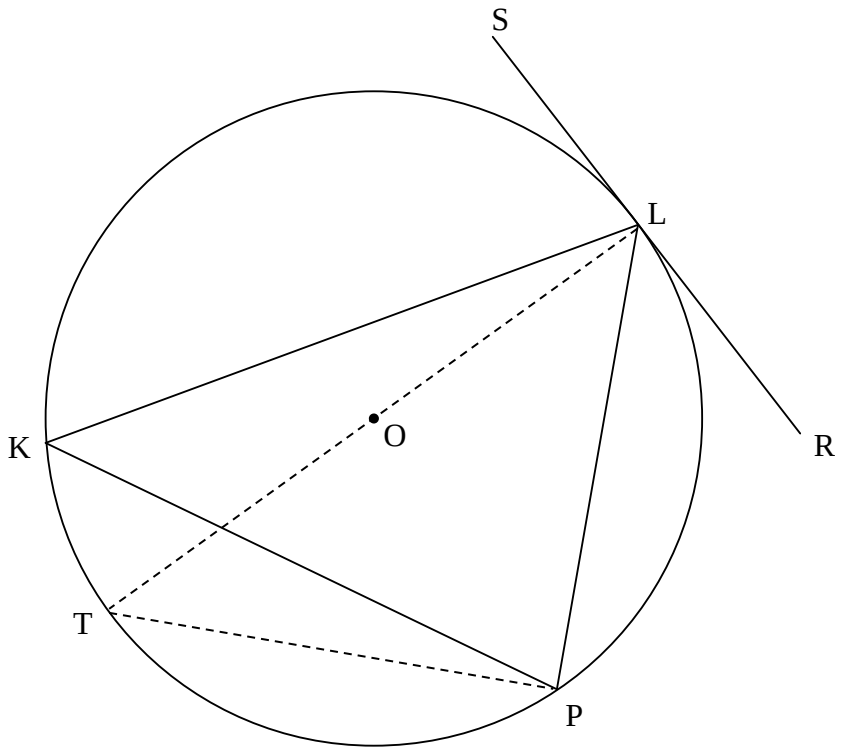
QUESTION/*VRAAG 7*



7.1	$\sin\theta = \frac{AD}{5}$ $AD = 5\sin\theta$ $\sin 2\theta = \frac{AD}{x}$ $AD = x\sin 2\theta$ $= x.2\sin\theta\cos\theta$ $x.2\sin\theta\cos\theta = 5\sin\theta$ $x = \frac{5\sin\theta}{2\sin\theta\cos\theta}$ $= \frac{5}{2\cos\theta}$	✓ trig ratio ✓ trig ratio ✓ $2\sin\theta\cos\theta$ ✓ equating AD ✓ x as subject (5)
7.2	$BC^2 = 5^2 + \left(\frac{5}{2\cos 30°}\right)^2 - 2(5)\left(\frac{5}{2\cos 30°}\right).\cos 112°$ $= 44{,}147$ $BC = 6{,}64$ units	✓ use area rule correctly ✓ substitution ✓ answer (3)
		[8]

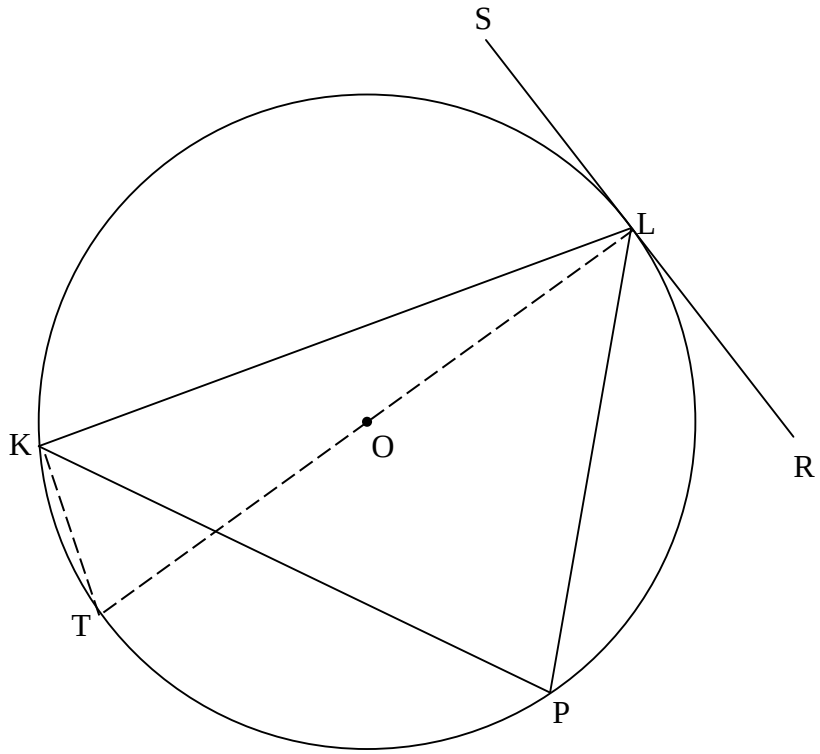
QUESTION/*VRAAG 8*

8.1



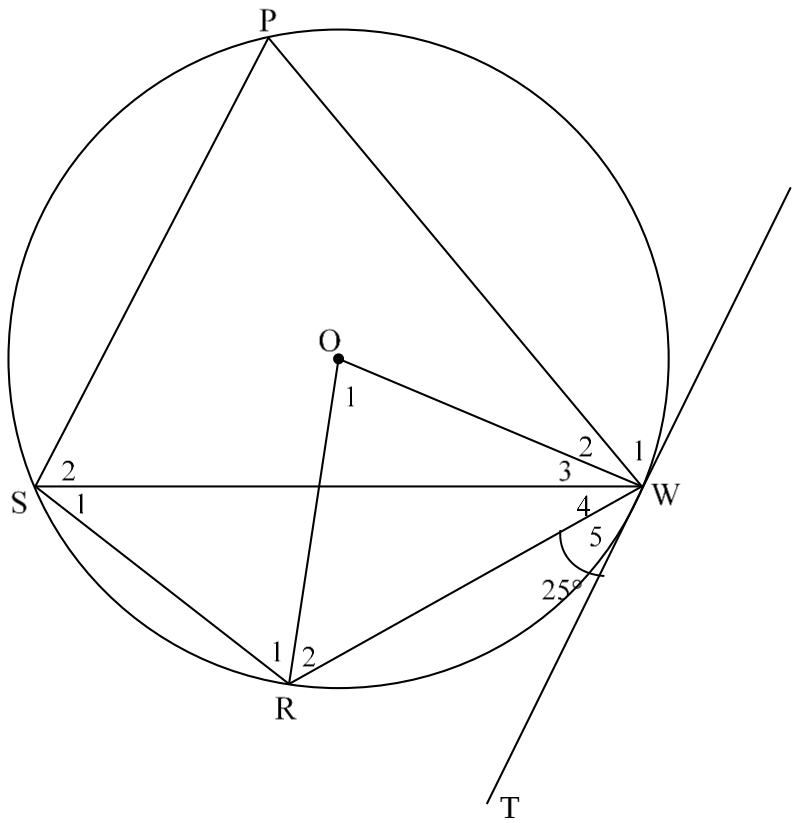
8.1	Construction: Draw diameter LT and draw TP *Konstruksie: Trek middellyn* LT *en verbind* TP $S\hat{L}K = 90° - T\hat{L}K$ [radius ⊥ tangent/*raaklyn*] $T\hat{P}L = 90°$ [∠ in semi-circle/*semi-sirkel*] $\therefore K\hat{P}L = \hat{P} = 90° - T\hat{P}K$ $= 90° - T\hat{L}K$ [∠s same segment/∠*e dieselfde segment*] $\therefore S\hat{L}K = \hat{P}$	✓Constr ✓S ✓R ✓S /R ✓S ✓R (6)

OR



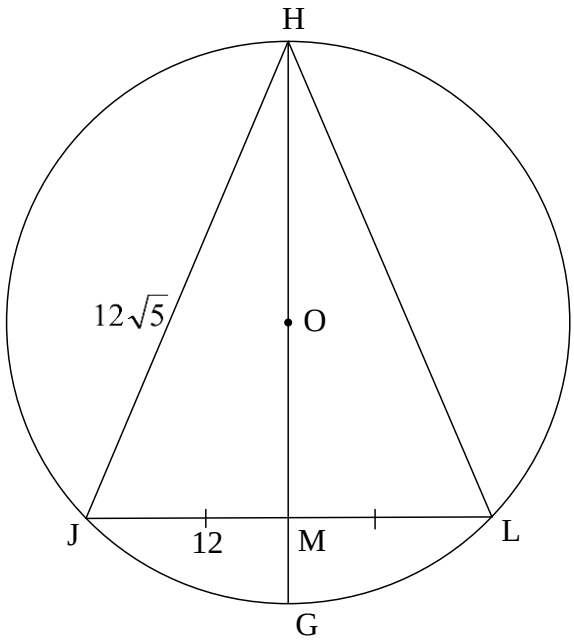
8.1	Construction: Draw diameter LT and draw KT *Konstruksie: Trek middellyn LT en verbind KT* $S\hat{L}K = 90° - T\hat{L}K$ [radius ⊥ tangent/*raaklyn*] $L\hat{K}T = 90°$ [∠ in half circle/*semi-sirkel*] $\therefore \hat{P} = K\hat{T}L$ [∠s same segment/∠*e dieselfde segment*] $= 90° - T\hat{L}K$ $\therefore S\hat{L}K = \hat{P}$	✓ construction ✓ S /R ✓ S✓ R ✓ S ✓ S / R (6)

8.2



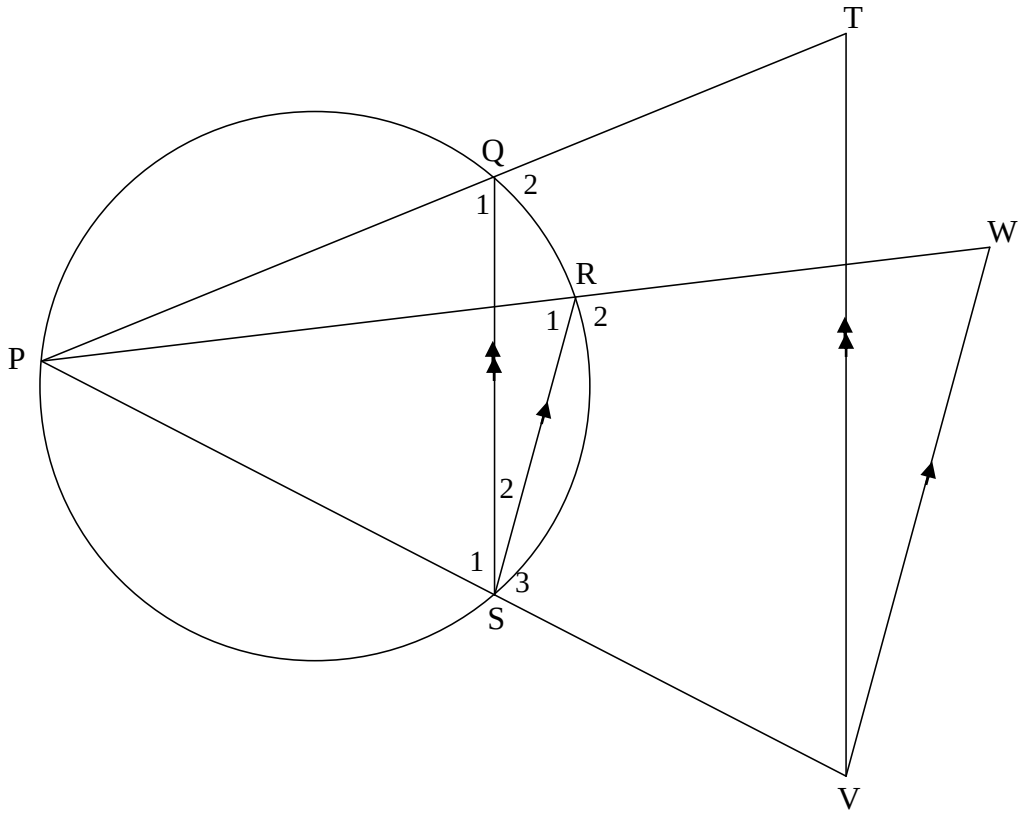
8.2.1(a)	$\hat{S}_1 = 25°$ [tan chord theorem/ ∠ *tussen raaklyn en koord*]	✓S ✓R (2)
8.2.1(b)	$\hat{O}_1 = 50°$ [∠ at centre = 2×∠ at circumference / *midpts.* ∠ = *2 × omtreks* ∠]	✓S ✓R (2)
8.2.1(c)	$\hat{R}_2 = \hat{W}_3 + \hat{W}_4 = 65°$ [∠s opp = radii / ∠*e teenoor = radiusse*] $\hat{P} = 60°$ [∠s of equilateral Δ / ∠*e van gelyksydige* Δ] $\hat{R}_1 = 55°$ [opp ∠ of cyclic quad / teenoorst. ∠*e van kvh*]	✓ S ✓ R ✓ S / R ✓ S ✓ R (5)
8.2.2	$\hat{W}_1 = \hat{S}_2 = 60°$ [tan chord theorem / ∠ *tussen en koord*] $\hat{P} = 60°$ [∠s of equilateral Δ /∠*e van gelyksydige* Δ] $\therefore \hat{W}_1 = \hat{P} = 60°$ SP ‖ TW [alt ∠s = / *verwisselende ∠e gelyk*]	✓ S / R ✓ S ✓ R (3)

8.3



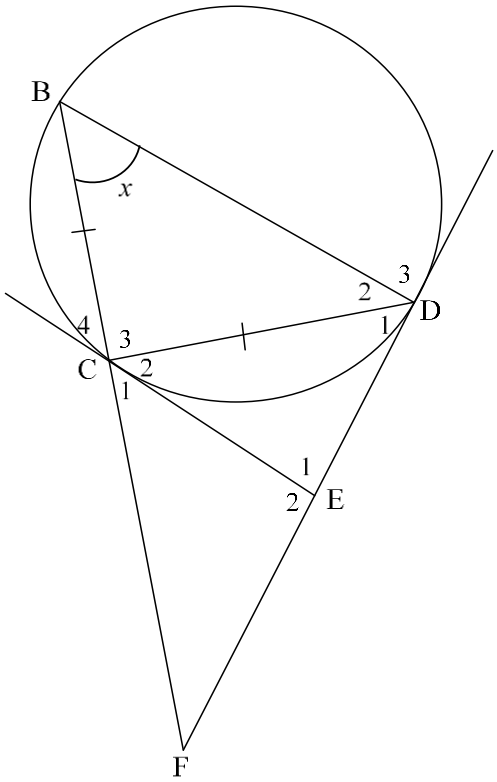
8.3.1	OG = x + 6 ∴ HM = 2x + 6	✓ S ✓ S (2)
8.3.2	OM ⊥ JL [line from centre to midp of chord/*lyn van midpt halv kd*] $OJ^2 = JM^2 + OM^2$ [Pythagoras] $(x+6)^2 = 12^2 + x^2$ $x^2 + 12x + 36 = 144 + x^2$ $x = 9$ $r = 15$ units **OR** OM ⊥ JL [line from centre to midp of chord/*lyn van midpt halv kd*] $HJ^2 = HM^2 + JM^2$ [Pythagoras] $(12\sqrt{5})^2 = (2x+6)^2 + 12^2$ $720 = 4x^2 + 24x + 36 + 144$ $0 = 4x^2 + 24x - 540$ $0 = x^2 + 6x - 135$ $0 = (x-9)(x+15)$ $x = 9$ $r = 15$ units	✓ S ✓R ✓ subst into Pyth ✓ value of x ✓ length of radius (5) ✓ S ✓R ✓ subst into Pyth ✓ value of x ✓ radius (5)
		[25]

QUESTION/*VRAAG 9*



9.1	$\frac{TQ}{QP}=\frac{VS}{SP}$ [Prop Th , TV ∥ QS / *Lyn* ∥ *een sy van* Δ] $\frac{VS}{SP}=\frac{WR}{RP}$ [Prop Th , RS ∥ VW / *Lyn* ∥ *een sy van* Δ] $\therefore \frac{TQ}{QP}=\frac{WR}{RP}$	✓S ✓ R ✓ S/R (3)
9.2	$\hat{Q}_1=\hat{R}_1$ [∠s in the same segment / *∠e in dieselfde sirkel segment*] $\hat{R}_1=\hat{W}$ [corres ∠s, RS ∥ VW / *ooreenkomstige ∠e,* RS ∥ VW] $\therefore \hat{Q}_1=\hat{W}$ $\hat{Q}_1=\hat{T}$ [corres ∠s ,TV ∥ QS / *ooreenkomstige ∠e,* TV ∥ QS] $\therefore \hat{T}=\hat{W}$ ∴ TPVW is a cyclic quad [converse ∠s in the same segment / *lyn onderspan gelyke hoeke*]	✓S ✓R ✓S/R ✓S ✓R (5)
		[8]

QUESTION/*VRAAG 10*



10.1.1	$\hat{D}_1 = x$ [tan chord theorem / ∠ *tussen en raaklyn koord*] $\hat{C}_2 = \hat{D}_1 = x$ [Tans from common pt / *Rklyne vanuit dies punt*] $\hat{E}_1 = 180° - 2x$ [sum of int ∠s Δ; ∠*e* Δ] **OR** $\hat{D}_1 = x$ [tan chord theorem / *raaklyn koordst.*] $\hat{C}_2 = x$ [tan chord theorem / *raaklyn koordst.*] $\hat{E}_1 = 180° - 2x$ [sum of int ∠s Δ; ∠*e* Δ]	✓ S ✓ R ✓ S✓ R ✓ R (5) ✓ S ✓ R ✓ S✓ R ✓ R (5)
10.1.2	In ΔECD and ΔCBD $\hat{C}_2 = \hat{B} = x$ [tan chord theorem / *raaklyn koordst.*] $\hat{D}_2 = \hat{B} = x$ [∠s opp equal sides / ∠ *teenoor gelyke sye*] $\therefore \hat{D}_1 = \hat{D}_2 = x$ ∴ ΔECD ||| ΔCBD [∠, ∠, ∠] **OR** In ΔECD and ΔCBD $\hat{C}_2 = \hat{B} = x$ [tan chord theorem / *raaklyn koordst.*] $\hat{D}_2 = \hat{B} = x$ [∠s opp equal sides / ∠ *teenoor gelyke sye*] $\therefore \hat{D}_1 = \hat{D}_2 = x$ $\hat{E}_1 = \hat{C}_3$ [3rd ∠ of Δ / ∠*e* Δ] ∴ ΔECD ||| ΔCBD	✓ S / R ✓ S ✓ R (3) ✓ S / R ✓ S ✓ S (3)

10.2.1	$\frac{EC}{BC}=\frac{CD}{BD}=\frac{ED}{CD}$ [$\Delta ECD \parallel\!| \Delta CBD$] $\frac{CD}{BD}=\frac{ED}{CD}$ $CD^2 = ED.BD$ $ED = CE$ $\therefore CD^2 = CE.BD$	✓ S ✓ $CD^2 = ED.BD$ ✓ $ED = CE$ (3)
10.2.2	$\hat{C}_2 = \hat{D}_2 = x$ [proven / *reeds bewys*] $BD \parallel CE$ [alt ∠s = / *verwisselende ∠ gelyk*] $\therefore \frac{FE}{DE}=\frac{FC}{CB}$ [line ∥ one side of Δ / *lyn* ∥ *een sy van* Δ] $\therefore \frac{CF^2}{EF^2}=\frac{CB^2}{DE^2}$ $\therefore \frac{CF^2}{EF^2}=\frac{DE.BD}{DE^2}$ [CB = CD] $\therefore \frac{CF^2}{EF^2}=\frac{BD}{DE}$	✓ S ✓ R ✓ S ✓ R ✓ squaring ✓ subst $CD^2 = ED.BD$ (6)
		[17]

TOTAL/*TOTAAL:* 150