



GAUTENG PROVINCE

EDUCATION

REPUBLIC OF SOUTH AFRICA



GAUTENG DEPARTMENT OF EDUCATION
GRADE 12
JUNE EXAMINATION 2022

10612

MATHEMATICS PAPER 2

TIME: 3 hours
MARKS: 150
23 pages

Answer all questions on this question paper.

LEARNER'S NAME: _____

GRADE 12 _____

Marker/Merker			Moderator's Initials / Moderator se paraaf											
Question Vraag	Mark Punt	Initial Parafeer	Marks Punte		S M	Marks Punte		D M	Marks Punte		C M	Marks Punte		EM
1														
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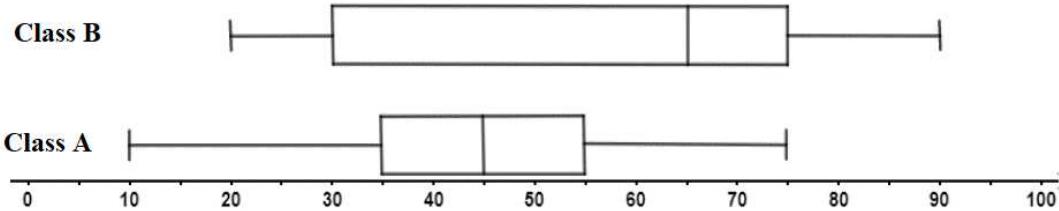
INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of ELEVEN questions. Answer ALL the questions on this question paper.
2. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answer.
3. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
4. Answers only will not necessarily be awarded full marks.
5. If necessary, round-off answers to TWO decimal places, unless stated otherwise.
6. Diagrams are NOT necessarily drawn to scale.
7. An information sheet with formulae is included at the end of the question paper.
8. Write neatly and legibly.

QUESTION 1

The box and whisker diagrams below show the Mathematics results of class A and class B in the June Examination. It is also given that class B has a Median of 65%.



- 1.1 Which class had the top learners?

(1)

- 1.2 Determine which class had the greatest Inter Quartile Range (IQR).

(1)

- 1.3 What percentage of class A scored lower than 60%?

(1)

- 1.4 If all the learners in class A were given an extra 5%, what would happen to the standard deviation of the marks in class A?

(1)

- 1.5 Determine the semi-interquartile range of class B.

(1)

[5]

QUESTION 2

A group of 30 pupils was asked to complete an obstacle course at their Grade 11 camp. The times (in seconds) taken by the pupils to complete the obstacle course are given in the table below.

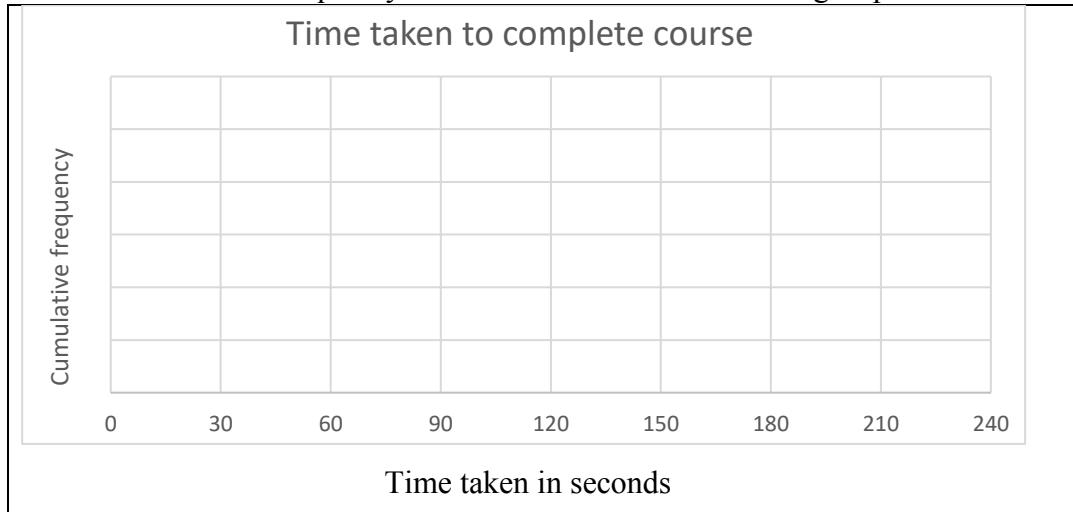
Time taken:	$60 \leq t < 90$	$90 \leq t < 120$	$120 \leq t < 150$	$150 \leq t < 180$	$180 \leq t < 210$
No. of pupils:	3	6	7	8	6

- 2.1 Complete the cumulative frequency table for the above data.

Time taken	No. of pupils	Cumulative frequency
$60 \leq t \leq 90$	3	
$90 \leq t \leq 120$	6	
$120 \leq t \leq 150$	7	
$150 \leq t \leq 180$	8	
$180 \leq t \leq 210$	6	

(1)

- 2.2 Draw a cumulative frequency curve for the above data on the grid provided.



(4)

- 2.3 Indicate on your graph where you would read off ...

- 2.3.1 the number of pupils that took 135 seconds to complete the course.
(Use the letter A)

(1)

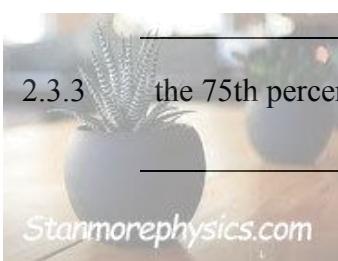
- 2.3.2 the value of t if 60% of the pupils took less than t seconds to complete the obstacle course. (Use the letter B)

(1)

- 2.3.3 the 75th percentile. (Use the letter C)

(1)

[8]



QUESTION 3

Consider the following set of four positive whole numbers and their frequency.

Scores	$x + 3$	$2x$	$x - 1$	6
Frequency	4	3	2	2

- 3.1 Determine the median score.

(1)

- 3.2 Determine the mean in terms of x .

(3)

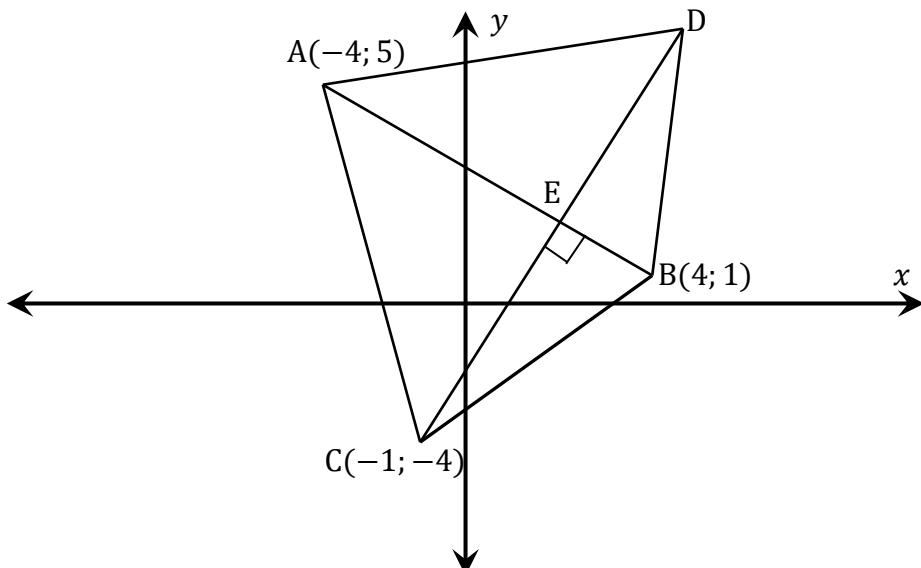
- 3.3 If only the scores are taken into consideration (without the frequency), determine the standard deviation if it is given that $x = 5$.

(2)

[6]

QUESTION 4

In the diagram below, the coordinates of $A(-4; 5)$, $C(-1; -4)$ and $B(4; 1)$ are the vertices of a triangle in a Cartesian plane. $CE \perp AB$ with E on AB . E is the midpoint of straight line CD .



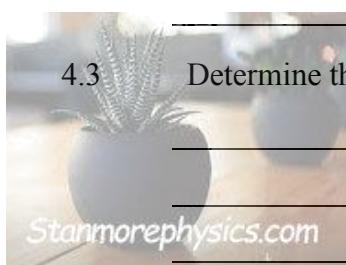
- 4.1 Determine the gradient of AB .

(2)

- 4.2 Determine the equation of CD .

(4)

- 4.3 Determine the coordinates of E .



(6)

- 4.4 Determine the coordinates of D.

(2)

- 4.5 Determine the equation of the straight line passing through point D and parallel to AC.

(4)

- 4.6 Determine, by showing all calculations, whether the x -intercept of the straight line CD also lies on the altitude (perpendicular height) from A to BC.

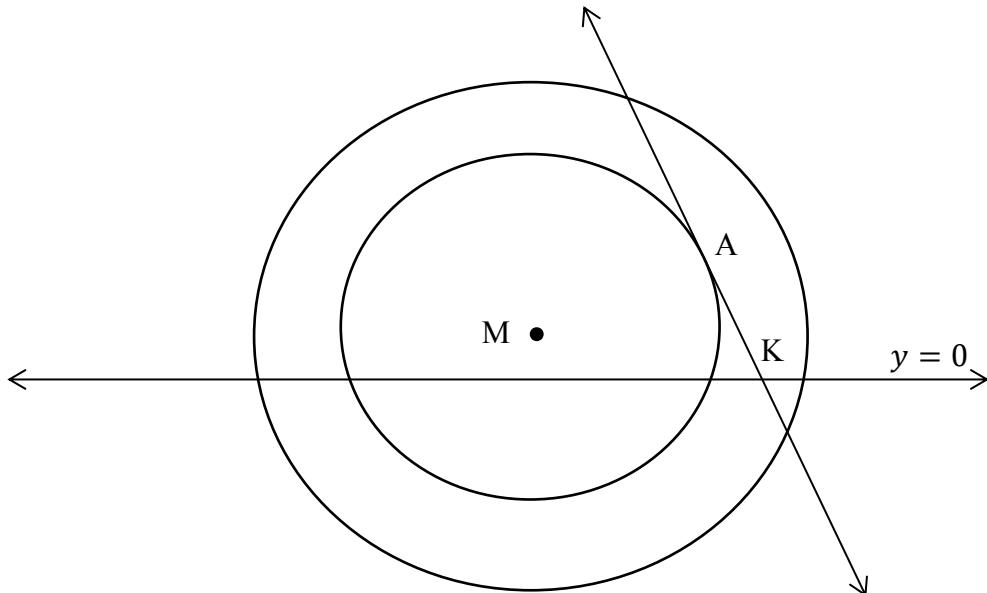


(6)

[24]

QUESTION 5

In the figure below, M is the common centre of two circles. The larger circle has equation $x^2 + y^2 = 4y - 2x + 44$. The smaller circle touches the straight line $y = -x + 5$ at point A. The straight line $y = 0$ cuts both circles.



- 5.1 Determine the coordinates of M.

(4)

- 5.2 Determine the coordinates of A.

(5)

- 5.3 Determine the equation of the smaller circle.

(3)

- 5.4 Write down the coordinates of K.

(1)

- 5.5 The straight line $y = -x + 5$ meets the straight line $y = 0$ at point K.
Determine the area of ΔAMK .

(3)
[16]

QUESTION 6

- 6.1 If $\cos 26^\circ = \frac{1}{p}$. Determine the following in terms of p .

Draw your sketch here.	Do your calculations here.
------------------------	----------------------------

- 6.1.1 $\sin 26^\circ$

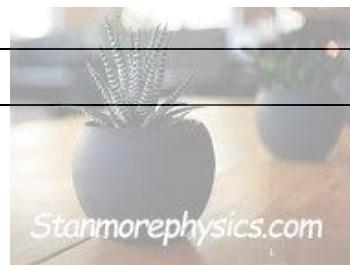
(3)

- 6.1.2 $\cos 52^\circ$

(3)

- 6.1.3 $\tan^2 64^\circ \times (p + 1)$

(4)



6.2 Simplify: $\frac{\sin(-\beta) + \sin(360^\circ - \beta)}{\sin(180^\circ - \beta) + \sin 180^\circ}$

(5)

6.3 Determine the value of p , correct to two decimal places, if:

$$\theta = 82^\circ \text{ and } 2p \tan\left(\frac{\theta}{2}\right) = \sin(2\theta)$$

(3)

6.4 Prove the identity: $4 \sin \theta \cdot \cos^3 \theta - 4 \cos \theta \cdot \sin^3 \theta = \sin 4\theta$

(6)

[24]

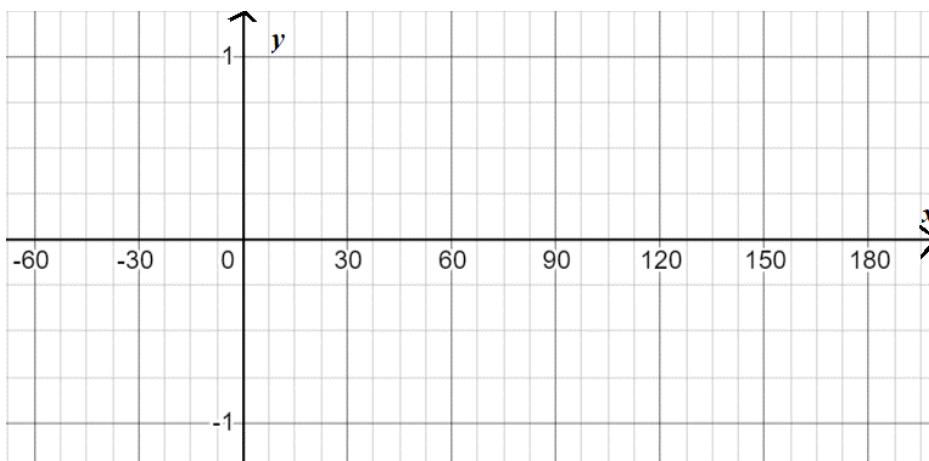
QUESTION 7

Given: $f(x) = \sin(x - 30^\circ)$ and $g(x) = \cos 3x$.

- 7.1 Solve for x : $\cos 3x = \sin(x - 30^\circ)$ for $x \in [-60^\circ; 180^\circ]$.

(7)

- 7.2 Draw the graphs of f and g for $x \in [-60^\circ; 180^\circ]$ on the grid provided.



(6)

- 7.3 Use your graph and your answers to Question 7.1 to answer the following question.

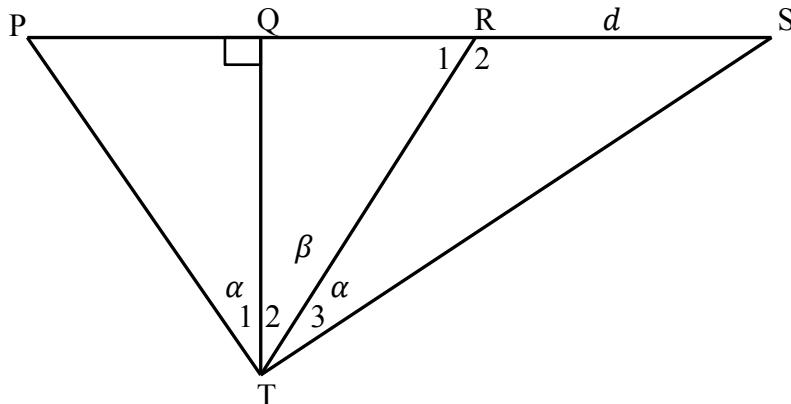
For which value(s) of x is $f(x) \times g(x) < 0$?

(4)

QUESTION 8

In the figure below, PQRS forms a straight road. TQ, another road, is perpendicular to road PQRS. The distance RS = d kilometres.

$$\hat{T}_1 = \hat{T}_3 = \alpha \text{ and } Q\hat{T}S = \beta$$



- 8.1 Write down the size of $Q\hat{T}R$ in terms of α and β .

(1)

- 8.2 In ΔSQT , write down the size of \hat{S} .

(1)

- 8.3 In ΔPQT , write down the size of \hat{P} .

(1)

- 8.4 Determine the length of RT in terms of α and β .

(3)

8.5 Hence, or otherwise, show that:

$$PR = \frac{d \cos \beta \sin \beta}{\sin \alpha \cdot \cos \alpha}$$

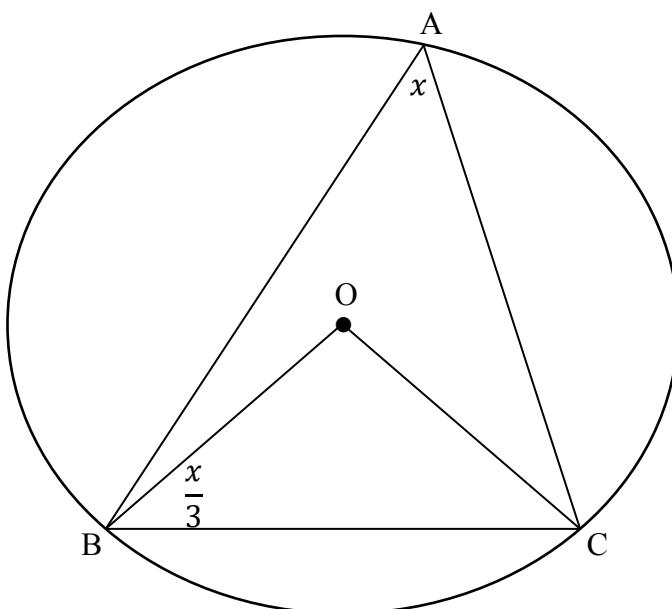
(3)
[9]

QUESTION 9

- 9.1 Complete the statement: The angle at the ... is equal to two times the angle at the circumference of the circle.

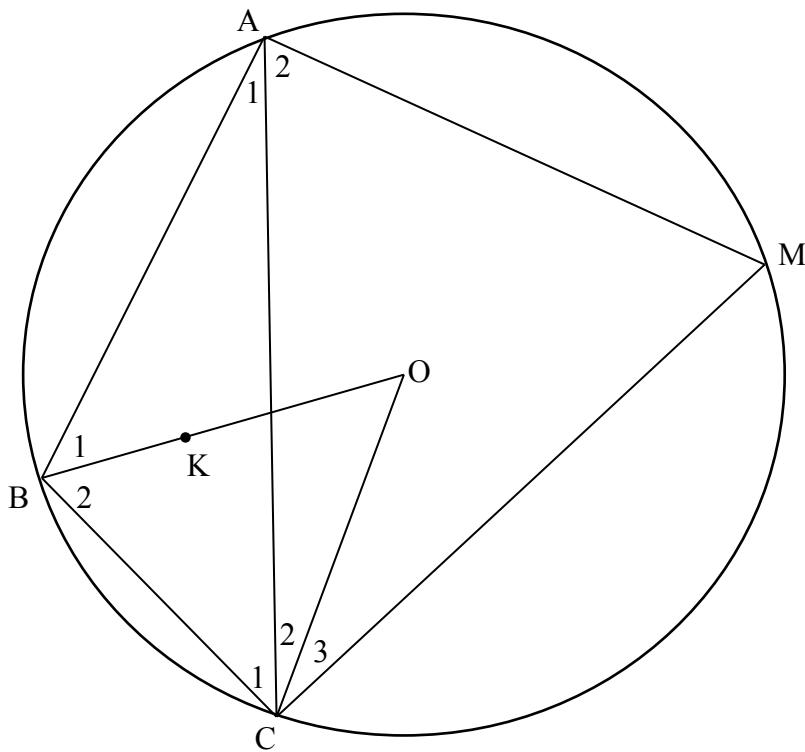
(1)

- 9.2 In the diagram below, O is the centre of the circle with points A, B and C on the circumference of the circle. $\widehat{BAC} = x$ and $\widehat{OBC} = \frac{x}{3}$. Determine, with reasons, the value of x .



(6)

- 9.3 In the diagram below, O is the centre of the circle passing through A, B, C and M. K is the centre of the circle (not drawn) passing through points A, B and C of $\triangle ABC$ such that K lies on radius BO. $\hat{A}_1 = 30^\circ$. BO bisects \hat{ABC} .



- 9.3.1 Determine the size of \hat{B}_1 . (Give reasons for your answer.)

(5)

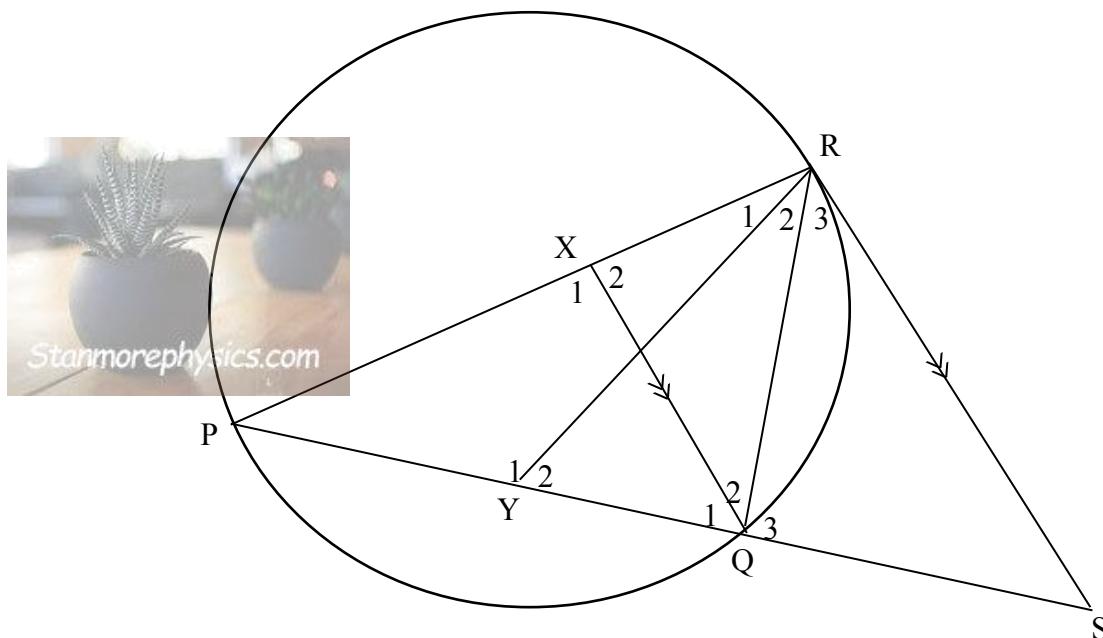
9.3.2 Prove that $\hat{M} = 2\hat{A}_1$

(3)

[15]

QUESTION 10

In the diagram below, P, Q and R are points on a circle. YR bisects \widehat{PRQ} with Y on PQ. PQ is produced to meet RS at S such that $SR = SY$. $QX \parallel SR$.



- 10.1 Prove that SR is a tangent to the circle at R.

(6)

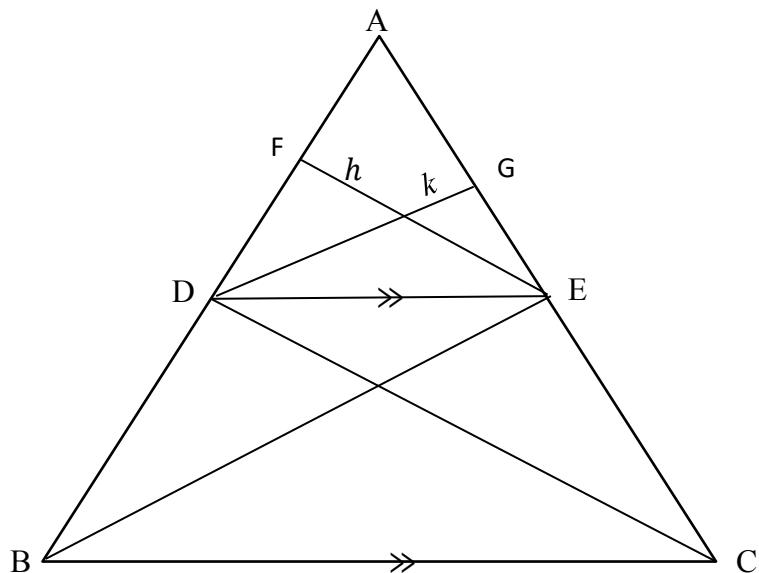
- 10.2 Prove that QR is a tangent to the circle passing through Q, X and P.

(3)
[9]

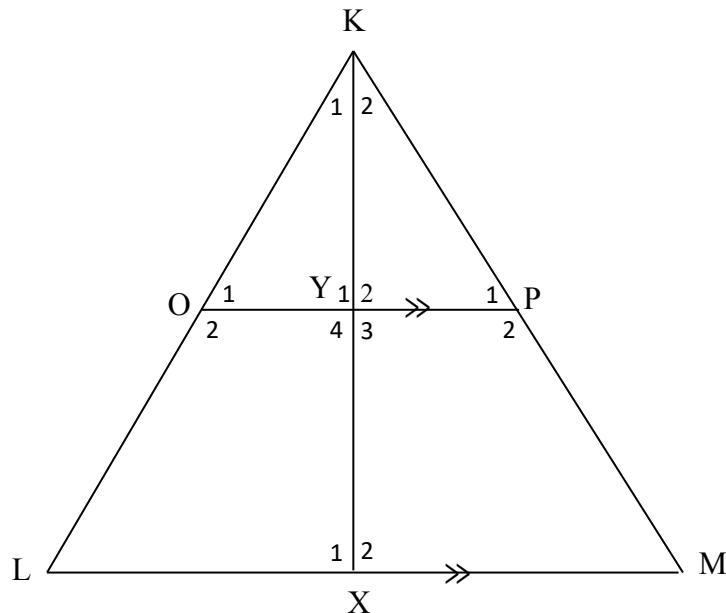
QUESTION 11

- 11.1 In the diagram below D and E are points on sides AB and AC of $\triangle ABC$ such that $DE \parallel BC$.

Use the diagram to prove the theorem which states that $\frac{AD}{DB} = \frac{AE}{EC}$.



- 11.2 In the diagram below, $OP \parallel LM$ such that the area of $\Delta KOP =$ area of quadrilateral $OLMP$. KYX is perpendicular to OP and LM at Y and X respectively.



Prove that:

11.2.1 $\Delta KOP \sim \Delta KLM$

(3)

11.2.2 $\frac{KY}{KX} = \frac{OP}{LM}$

(2)

11.2.3 $\frac{KO}{KL} = \frac{1}{\sqrt{2}}$

(6)

[17]

TOTAL:

150

Additional Space

Additional Space

END

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1+ni) \quad A = P(1-ni) \quad A = P(1-i)^n \quad A = P(1+i)^n$$

$$F = \frac{x[(1+i)^n - 1]}{i} \quad P = \frac{x[1 - (1+i)^{-n}]}{i}$$

$$T_n = a + (n-1)d \quad S_n = \frac{n}{2}(2a + (n-1)d)$$

$$T_n = ar^{n-1} \quad S_n = \frac{a(r^n - 1)}{r - 1}; \quad r \neq 1 \quad S_\infty = \frac{a}{1-r}; \quad -1 < r < 1$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \quad y - y_1 = m(x - x_1) \quad m = \frac{y_2 - y_1}{x_2 - x_1} \quad m = \tan \theta$$

$$(x-a)^2 + (y-b)^2 = r^2$$

$$\text{In } \Delta ABC: \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad a^2 = b^2 + c^2 - 2bc \cos A \quad \text{area } \Delta ABC = \frac{1}{2} ab \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta \quad \sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta \quad \cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases} \quad \sin 2\alpha = 2\sin \alpha \cos \alpha$$

$$\bar{x} = \frac{\sum x}{n} \quad \sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n} \quad P(A) = \frac{n(A)}{n(S)} \quad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx \quad b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$



**GAUTENG DEPARTMENT OF EDUCATION/
GAUTENGSE DEPARTEMENT VAN ONDERWYS**

**JUNE EXAMINATION/
JUNIE EKSAMEN 2022**

**MARKING GUIDELINES/
NASIENRIGLIYNE**

**MATHEMATICS/WISKUNDE
(PAPER/VRAESTEL 2) (10612)**

BILINGUAL/TWEETALIG

15 pages/ bladsye

Stanmorephysics.com

QUESTION/VRAAG 1

1.1	B	✓ answer/antwoord	(1)
1.2	B	✓ answer/antwoord	(1)
1.3	75%	✓ answer/antwoord	(1)
1.4	Nothing. It remains the same. No change in standard deviation. <i>Niks. Dit bly dieselfde. Geen verandering in standaardafwyking</i>	✓ reason/rede	(1)
1.5	$\text{Semi - IQR: IKV} = \frac{75 - 30}{2}$ $\text{Semi - IQR: IKV} = 22.5$	✓ answer/antwoord	(1)
			[5]

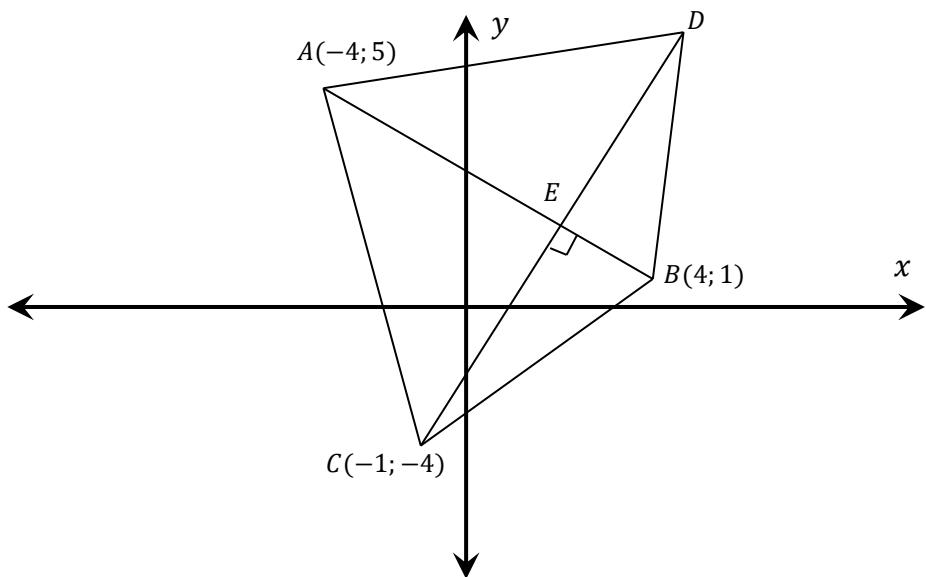
QUESTION 2/VRAAG 2

2.1	<table border="1"> <thead> <tr> <th>Time taken Tyd geneem</th><th>No. of pupils Aantal leerlinge</th><th>Cumulative frequency Kummulatiewe frekwensie</th></tr> </thead> <tbody> <tr> <td>$60 \leq t \leq 90$</td><td>3</td><td>3</td></tr> <tr> <td>$90 \leq t \leq 120$</td><td>6</td><td>9</td></tr> <tr> <td>$120 \leq t \leq 150$</td><td>7</td><td>16</td></tr> <tr> <td>$150 \leq t \leq 180$</td><td>8</td><td>24</td></tr> <tr> <td>$180 \leq t \leq 210$</td><td>6</td><td>30</td></tr> </tbody> </table>	Time taken Tyd geneem	No. of pupils Aantal leerlinge	Cumulative frequency Kummulatiewe frekwensie	$60 \leq t \leq 90$	3	3	$90 \leq t \leq 120$	6	9	$120 \leq t \leq 150$	7	16	$150 \leq t \leq 180$	8	24	$180 \leq t \leq 210$	6	30	✓ for values vir waardes	(1)
Time taken Tyd geneem	No. of pupils Aantal leerlinge	Cumulative frequency Kummulatiewe frekwensie																			
$60 \leq t \leq 90$	3	3																			
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$120 \leq t \leq 150$	7	16																			
$150 \leq t \leq 180$	8	24																			
$180 \leq t \leq 210$	6	30																			
2.2	<p style="text-align: center;">Time taken to complete course.</p>	✓ anchor point/ ankerpunt (60;0) ✓ (120; 9) ✓ (150; 16) ✓ (210; 30)	(4)																		
2.3	<p>2.3.1 See diagram above / Sien diagram hierbo</p> <p>2.3.2 See diagram above / Sien diagram hierbo</p> <p>2.3.3 See diagram above / Sien diagram hierbo</p>	✓ A ✓ B ✓ C	(1) (1) (1) [8]																		

QUESTION/VRAAG 3

3.1	Median score / Mediaan telling = $2x$	✓ answer/antwoord	(1)
3.2	$\text{Mean/Gemiddelde} = \frac{\sum x}{n}$ $= \frac{4(x + 3) + 3(2x) + 2(x - 1) + 2(6)}{11}$ $= \frac{12x + 22}{11}$	✓ substitution/vervanging ✓ simplification/vereenvoudiging ✓ answer/antwoord	(3)
3.3	Use of a calculator where the four values are as follows: <i>Gebruik van 'n sakrekenaar waar die vier waardes soos volg is:</i> $8 ; 10 ; 4$ and/ <i>en</i> 6 $sd(\sigma) = \sqrt{5}$	✓ four values/vier waardes ✓ answer/antwoord	(2)
			[6]

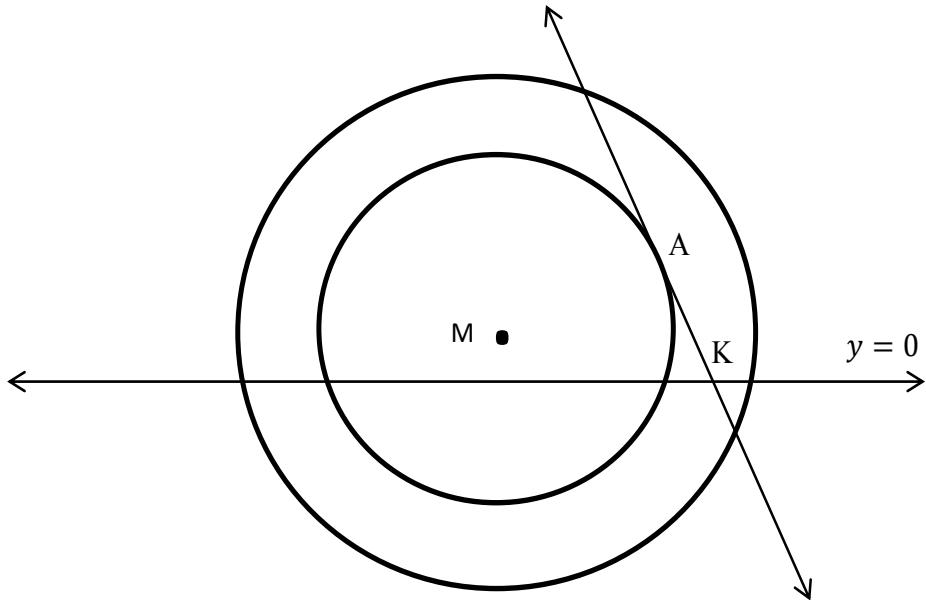
QUESTION/VRAAG 4



4.1	$\begin{aligned} m_{AB} &= \frac{5-1}{-4-4} \\ &= \frac{4}{-8} \\ &= -\frac{1}{2} \\ m_{AB} &= -\frac{1}{2} \\ \therefore m_{CD} &= 2 \end{aligned}$	\checkmark subst into gradient formula. <i>verv. in gradiënt formule</i> $\checkmark m_{AB} = -\frac{1}{2}$ (2)
4.2	$\begin{aligned} \therefore m_{CD} &= 2 \\ \overline{CD} : \quad y &= 2x + c \\ -4 &= 2(-1) + c \\ -2 &= c \\ \overline{CD} : \quad y &= 2x - 2 \end{aligned}$	$\checkmark m_{CD} = 2$ \checkmark sub of point/ <i>verv. van punt</i> $(-1; -4)$ $\checkmark -2 = c$ \checkmark equation of CD / <i>vergelyking van CD</i> (4)
4.3	$\begin{aligned} \overline{AB} : \quad y &= -\frac{1}{2}x + c \\ 1 &= -\frac{1}{2}(4) + c \\ 3 &= c \\ y &= -\frac{1}{2}x + 3 \\ -\frac{1}{2}x + 3 &= 2x - 2 \\ -x + 6 &= 4x - 4 \\ 5x &= 10 \\ x &= 2 \\ y &= 2(2) - 2 \\ y &= 2 \\ E(2; 2) \end{aligned}$	\checkmark substitution of point (4; 1) <i>vervanging van punt (4;1)</i> \checkmark equation of \overline{AB} <i>vergelyking van \overline{AB}</i> \checkmark equating of AB and CD <i>gelykstel van AB en CD</i> $\checkmark x$ -value/ <i>waarde</i> \checkmark substitution of / <i>vervanging van</i> x -value/ <i>waarde</i> $\checkmark y$ -value/ <i>waarde</i> (6)

4.4	$D(x; y)$ $\frac{x-1}{2} = 2$ $x - 1 = 4$ $x = 5$ $D(5; 8)$	$\frac{y-4}{2} = 2$ $y - 4 = 4$ $y = 8$	✓ $x = 5$ ✓ $y = 8$	(2)
4.5	$m_{AC} = \frac{5 + 4}{-4 + 1}$ $m_{AC} = -3$ Line parallel to AC has same gradient. <i>Lyn ewewydig aan AC het dieselfde gradiënt</i> $y = -3x + c$ $8 = -3(5) + c$ $c = 23$ $y = -3x + 23$		✓ substitution into gradient formula. <i>vervanging in gradiënt formule</i> ✓ $m_{AC} = -3$ ✓ $c = 23$ ✓ equation of line. <i>vergelyking van lyn</i>	(4)
4.6	x -intercept of CD : x – <i>afsnit van</i> CD $2x - 2 = 0$ $x = 1$ $m_{BC} = 1$ Equation of Altitude/ <i>Vergelyking van Hoogtelyn</i> $y = -x + c$ $5 = -(-4) + c$ $c = 1$ $\therefore y = -x + 1$ x -intercept of Altitude / <i>x-afsnit van hoogtelyn</i> $x = 1$ x -intercept of CD = x -intercept of Altitude x - <i>afsnit van</i> CD = x - <i>afsnit van</i> hoogtelyn		✓ $x = 1$ ✓ $m_{BC} = 1$ ✓ gradient of Altitude -1 <i>gradiënt van hoogtelyn</i> -1 ✓ substitution of point <i>vervanging van punt</i> ✓ equation of altitude <i>vergelyking van hoogtelyn</i> ✓ $x = 1$	(6)

QUESTION/VRAAG 5

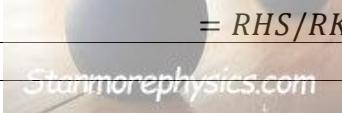


5.1	$\begin{aligned}x^2 + y^2 &= 4y - 2x + 44 \\x^2 + 2x + 1 + y^2 - 4y + 4 &= 44 + 1 + 4 \\(x + 1)^2 + (y - 2)^2 &= 49 \\\therefore M(-1; 2)\end{aligned}$	<ul style="list-style-type: none"> ✓✓ completing the square <i>voltooiing van die vierkant</i> ✓ factorizing / <i>faktorisering</i> ✓ $M(-1; 2)$ 	(4)
5.2	$\begin{aligned}m_{MA} &= \frac{y - 2}{x + 1} \\ \frac{y - 2}{x + 1} &= 1 \\ y - 2 &= x + 1 \\ y &= x + 3 \\ x + 3 &= -x + 5 \\ 2x &= 2 \\ x &= 1 \\ \therefore y &= 4 \\ A(1; 4)\end{aligned}$	<ul style="list-style-type: none"> ✓ gradient of MA/<i>gradiënt van MA</i> ✓ equating it to 1/<i>stel dit gelyk aan 1</i> ✓ making y or x the subject. <i>maak y of x die onderwerp</i> ✓ equating the two linear functions. <i>gelykstel van twee lineêre funksies</i> ✓ coordinates of $A(1; 4)$ <i>koördinate van A(1; 4)</i> 	(5)
5.3	$\begin{aligned}(x + 1)^2 + (y - 2)^2 &= r^2 \\(1 + 1)^2 + (4 - 2)^2 &= r^2 \\8 &= r^2 \\(x + 1)^2 + (y - 2)^2 &= 8\end{aligned}$	<ul style="list-style-type: none"> ✓ substitution of A/<i>vervanging van A</i> ✓ $8 = r^2$ ✓ equation of the circle/<i>vergelyking van die sirkel</i> 	(3)

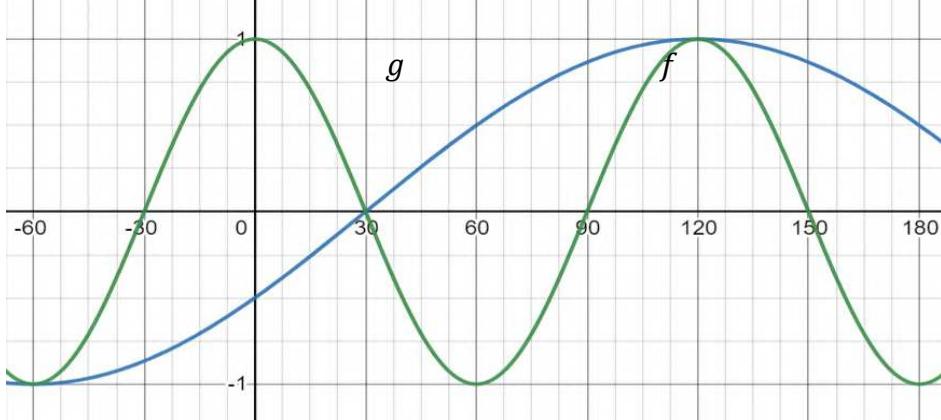
5.4	$K(5; 0)$	$\checkmark K(5; 0)$	(1)
5.5	$AK = \sqrt{32}$ Area of/van $\Delta AMK = \frac{1}{2} AK \times AM$ Area of/van $\Delta AMK = \frac{1}{2} \sqrt{32} \times \sqrt{8}$ Area of/van $\Delta AMK = 8 \text{ units}^2/\text{eenhede}^2$	$\checkmark AK = \sqrt{32}$ $\checkmark AM = \sqrt{8}$ $\checkmark 8 \text{ units}^2/\text{eenhede}^2$	(3)
			[16]

QUESTION/VRAAG 6

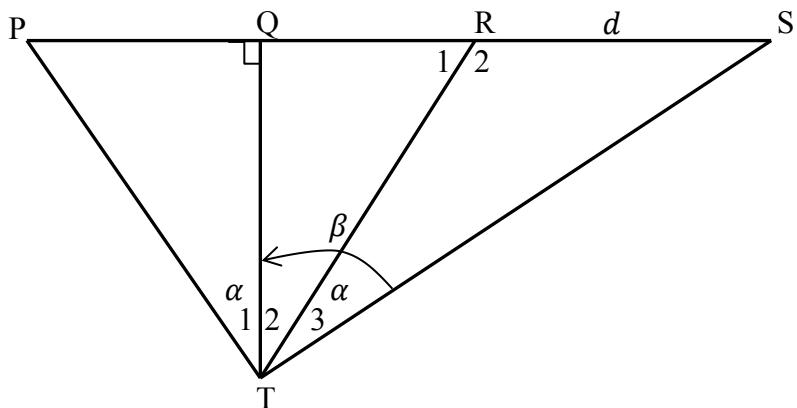
6.1			
6.1.1	$\sin 26^\circ = \frac{\sqrt{p^2 - 1}}{p}$	\checkmark sketch in Quadrant 1 <i>skets in kwadrant 1</i> \checkmark $\sqrt{p^2 - 1}$ \checkmark answer/antwoord	(3)
6.1.2	$\begin{aligned} \cos 52^\circ &= \cos 2(26^\circ) \\ &= 2 \cos^2 26^\circ - 1 \\ &= 2 \left(\frac{1}{p}\right)^2 - 1 \\ &= \frac{2}{p^2} - 1 \end{aligned}$	$\checkmark \cos 2(26^\circ)$ $\checkmark 2 \cos^2 26^\circ - 1$ \checkmark answer/antwoord	(3)
6.1.3	$\begin{aligned} \tan^2 64^\circ &\times (p + 1) \\ &= \left(\frac{1}{\sqrt{p^2 - 1}}\right)^2 \times (p + 1) \\ &= \frac{1}{p^2 - 1} \times (p + 1) \\ &= \frac{1}{(p - 1)(p + 1)} \times (p + 1) \\ &= \frac{1}{p - 1} \end{aligned}$	$\checkmark \left(\frac{1}{\sqrt{p^2 - 1}}\right)^2$ $\checkmark \frac{1}{p^2 - 1}$ $\checkmark (p - 1)(p + 1)$ \checkmark answer/antwoord	(4)

6.2	$\begin{aligned} & \frac{\sin(-\beta) + \sin(360^\circ - \beta)}{\sin(180^\circ - \beta) + \sin 180^\circ} \\ &= \frac{-\sin \beta + (-\sin \beta)}{\sin \beta + 0} \\ &= \frac{-2 \sin \beta}{\sin \beta} \\ &= -2 \end{aligned}$	<ul style="list-style-type: none"> ✓ $-\sin \beta$ ✓ $-\sin \beta$ ✓ $\sin \beta$ ✓ simplification <i>vereenvoudiging</i> ✓ answer/antwoord 	(5)
6.3	$\begin{aligned} 2p \tan\left(\frac{\theta}{2}\right) &= \sin(2\theta) \\ 2p \tan\left(\frac{82^\circ}{2}\right) &= \sin(2 \times 82^\circ) \\ p &= \frac{\sin 162^\circ}{2 \tan 41^\circ} \\ p &= 0,16 \end{aligned}$	<ul style="list-style-type: none"> ✓ substitution/<i>vervanging</i> ✓ simplification/<i>vereenvoudiging</i> ✓ answer/antwoord 	(3)
6.4	$\begin{aligned} 4 \sin \theta \cdot \cos^3 \theta - 4 \cos \theta \cdot \sin^3 \theta &= \sin 4\theta \\ LHS/LK = & 4 \sin \theta \cdot \cos^3 \theta \\ &- 4 \cos \theta \cdot \sin^3 \theta \\ &= 4 \sin \theta \cdot \cos \theta (\cos^2 \theta - \sin^2 \theta) \\ &= 2 \times 2 \sin \theta \cos \theta (\cos 2\theta) \\ &= 2 \cdot \sin 2\theta \cdot \cos 2\theta \\ &= \sin 4\theta \\ &= RHS/RK \end{aligned}$	<ul style="list-style-type: none"> ✓ common factor/<i>gemene faktor</i> ✓ $2 \times 2 \sin \theta \cos \theta$ ✓ $(\cos 2\theta)$ ✓ $\sin 2\theta$. ✓ $2 \cdot \sin 2\theta \cdot \cos 2\theta$ ✓ answer/antwoord 	(6)
			[24]

QUESTION/VRAAG 7

7.1	$\cos 3x = \sin(x - 30^\circ)$ $\cos 3x = \cos[90^\circ - (x - 30^\circ)]$ $\cos 3x = \cos[120 - x]$ $3x = 120 - x$ (ref angle) $3x = 120^\circ - x + k \cdot 360^\circ$ OR/OF $3x = 360^\circ - [120^\circ - x]k \cdot 360^\circ$ $4x = 120^\circ + k \cdot 360^\circ$ $x = 30^\circ + k \cdot 90^\circ$ $x = 30^\circ; -60^\circ; 120^\circ$	✓ Co-ratio <i>Ko-verhoud.</i> ✓ ref angle <i>verwys. ∠</i> ✓ quadrant 1 <i>kwadrant 1</i> ✓ quadrant 4 <i>kwadrant 4</i> ✓ 30° ✓ -60° ✓ 120°	(7)
7.2		✓ Shape of f . ✓ x -intercepts ✓ start and end points <i>Vorm van f</i> <i>x-afsnitte</i> <i>begin en eindpunte</i> ✓ Shape of g . ✓ x -intercepts ✓ start and end points <i>Vorm van g</i> <i>x-afsnitte</i> <i>begin en eindpunte</i>	(6)
7.3	$-30^\circ < x < 30^\circ$ OR/OF $30^\circ < x < 90^\circ$ OR/OF $150^\circ < x < 180^\circ$	✓✓ $-30^\circ < x < 30^\circ$ ✓ $30^\circ < x < 90^\circ$ ✓ $150^\circ < x < 180^\circ$	(4)
			[17]

QUESTION/VRAAG 8



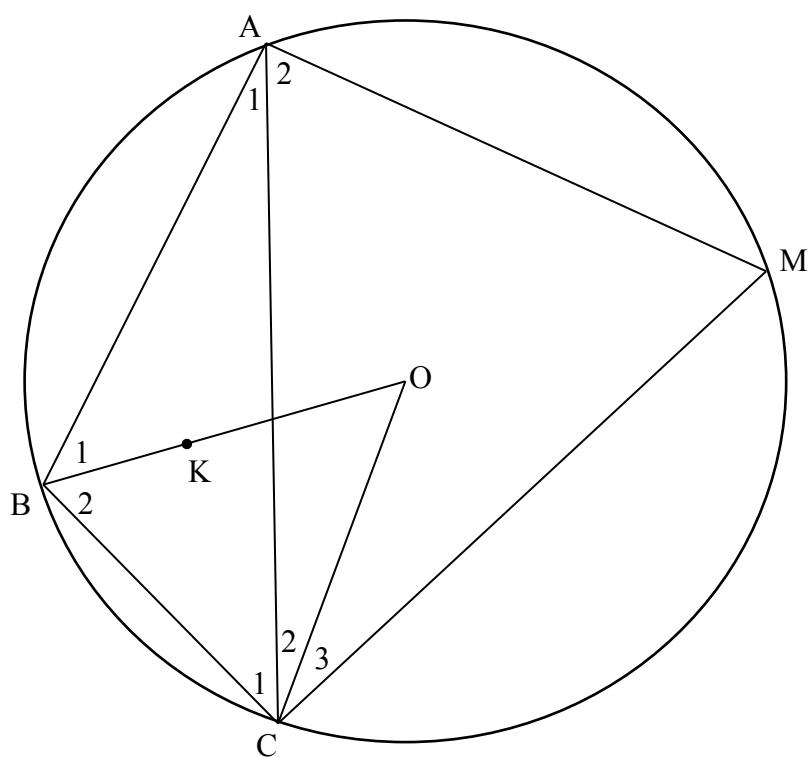
8.1	$Q\hat{T}R = \beta - \alpha$	✓ answer/antwoord	(1)
8.2	$\hat{S} = 90^\circ - \beta$	✓ answer/antwoord	(1)
8.3	$\hat{S} = 90^\circ - \alpha$	✓ answer/antwoord	(1)
8.4	In ΔRST $\frac{d}{\sin \alpha} = \frac{RT}{\sin(90^\circ - \beta)}$ $RT = \frac{d \cos \beta}{\sin \alpha}$	✓ use of sine rule <i>gebruik van sinusreël</i> ✓ use of co- function <i>gebruik van ko-funksie</i> ✓ answer/antwoord	(3)
8.5	$\frac{PR}{\sin \beta} = \frac{RT}{\sin(90^\circ - \alpha)}$ $PR = \frac{RT \sin \beta}{\cos \alpha}$ $PR = \frac{d \cos \beta \sin \beta}{\sin \alpha \cos \alpha}$	✓ use of sine rule <i>gebruik van sinusreël</i> ✓ sub of RT <i>vervanging van RT</i> ✓ answer/antwoord	(3)
			[9]



QUESTION/VRAAG 9

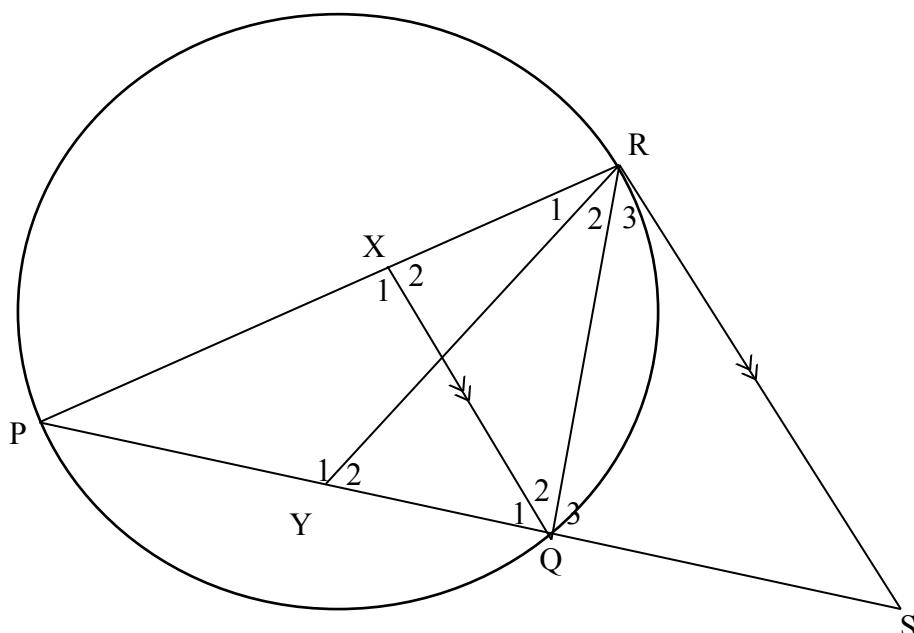
9.1	centre / middelpuntshoek	✓ answer/antwoord	(1)
9.2	$B\hat{O}C = 2x$ [angle at centre = $2 \times$ angle at circum] [Middelpuntshoek = $2 \times$ Omtrekshoek] $B\hat{C}O = \frac{x}{3}$ [angles opp = sides ; OB = OC] [hoeke teenoor = sye ; OB = OC] $\therefore \frac{x}{3} + \frac{x}{3} + 2x = 180^{\circ}$ [sum of angles of Δ] [som van hoeke van Δ] $8x = 540^{\circ}$ $x = 67,5^{\circ}$	✓ statement / stelling (S) ✓ reason/rede (R) ✓ statement / stelling (S) ✓ reason/rede (R) ✓ S/R ✓ answer/antwoord	(6)

9.3

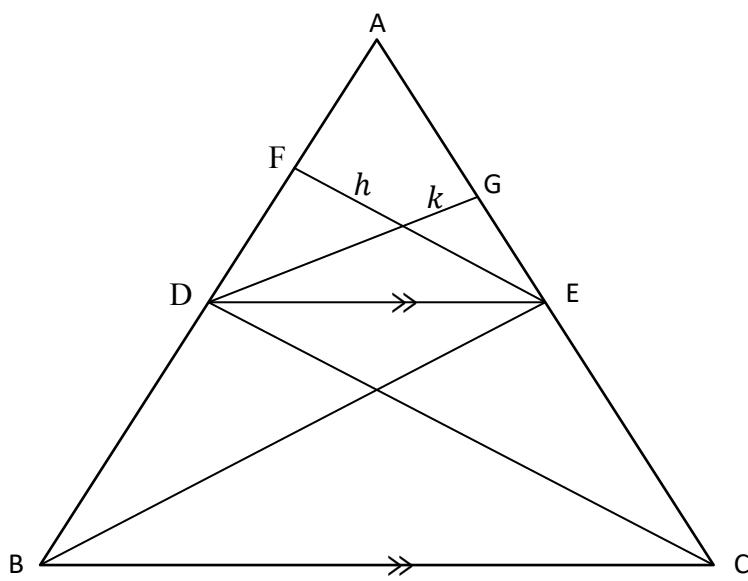


9.3.1	$\hat{A}_1 = 30^\circ$ $B\hat{O}C = 60^\circ$ [angle at centre = $2 \times$ angle at circum] $[middelpuntshoek = 2 \times omtrekshoek]$ $\hat{B}_2 = \hat{C}_1 + \hat{C}_2$ [angles opp = sides ; $OB = OC$] $[hoeke teenoor = sye ; OB = OC]$ $\therefore 2\hat{B}_2 = 180^\circ - 60^\circ$ [sum of angles of Δ] $[som van hoeke van \Delta]$ $\hat{B}_2 = 60^\circ$ $\hat{B}_1 = 60^\circ$ [BO bisects $A\hat{B}C$] / [BO halveer $A\hat{B}C$]	✓ S/R ✓ S/R ✓ S/R ✓ S ✓ S/R	(5)
9.3.2	$\hat{A}_1 = 30^\circ$ $\hat{B}_1 + \hat{B}_2 + \hat{M} = 180^\circ$ [opp angles of cyclic quad] $[teenoorst. hoeke van koordevierhoek]$ $\hat{M} = 180^\circ - 120^\circ$ $\hat{M} = 60^\circ$ $\therefore \hat{M} = 2 \times \hat{A}_1$	✓ S/R ✓ S ✓ answer/ antwoord	(3)
			[15]

QUESTION/VRAAG 10



10.1	$\widehat{R}_1 = \widehat{R}_2$ [YR bisects $P\widehat{R}Q$] / [YR halveer $P\widehat{R}Q$] $\widehat{R}_2 + \widehat{R}_3 = \widehat{Y}_2$ [angles opp = sides; RS = YS] $[hoeke teenoor = sye ; RS = YS]$ $\widehat{R}_1 + \widehat{P} = \widehat{Y}_2$ [ext angle of Δ]/[buitehoek van Δ] $\therefore \widehat{R}_3 = \widehat{P}$ $\therefore SR = \text{tangent/raaklyn}$ [converse tan – chord theorem] [omgekeerde raaklyn-koord stelling]	✓ S/R ✓ S ✓ R ✓ S ✓ R ✓ S	(6)
10.2	$\widehat{Q}_2 = \widehat{R}_3$ [alt angles = ; $SR \parallel QX$] $[verw. hoeke = ; SR \parallel QX]$ $\widehat{P} = \widehat{R}_3$ [proven] / [bewys] $\widehat{Q}_2 = \widehat{P}$ $\therefore QR = \text{tangent}$ [converse tan – chord theorem] $QR = \text{raaklyn}$ [omgekeerde raaklyn-koord stelling]	✓ S/R ✓ S ✓ S	(3)
			[9]



11.1	<p>Proof / Bewys:</p> $\frac{\text{Area of } \triangle ADE}{\text{Area of } \triangle BDE} = \frac{\frac{1}{2} AD \cdot h}{\frac{1}{2} BD \cdot h} = \frac{AD}{BD}$ $\frac{\text{Area of } \triangle ADE}{\text{Area of } \triangle CED} = \frac{\frac{1}{2} AE \cdot k}{\frac{1}{2} CE \cdot k} = \frac{AE}{CE}$ <p>But/Maar Area $\triangle BDE$ = Area $\triangle CED$ (same base DE, same height) <i>(dieselfde basis DE, dieselfde hoogte)</i> $DE \parallel BC$</p> $\frac{\text{Area of } \triangle ADE}{\text{Area of } \triangle BDE} = \frac{\text{Area of } \triangle ADE}{\text{Area of } \triangle CED}$ $\therefore \frac{AD}{BD} = \frac{AE}{CE}$	✓ S/R ✓S ✓ R ✓S ✓ R ✓ S	(6)
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11.2		
11.2.1	<p>In ΔKOP and /en ΔKLM</p> $\hat{K} = \hat{K}$ [common] / [gemeen] $\hat{O}_1 = \hat{L}$ [corresponding angles = ; $OP \parallel LM$] $\hat{P}_1 = \hat{M}$ [corresponding angles =, $OP \parallel LM$] $[ooreenkomsige hoeke = ; OP \parallel LM]$ $\therefore \Delta KOP \parallel\!\!\! \Delta KLM$ [A; A; A]	✓ S/R ✓ S/R ✓ R ✓ (3)
11.2.2	$\frac{KO}{KL} = \frac{OP}{LM}$ [similarity : gelykvormig] $\frac{KO}{KL} = \frac{KY}{KX}$ [line parallel to one side of Δ] $[lyn ewewydig aan een sy van \Delta]$ $\therefore \frac{KY}{KX} = \frac{OP}{LM}$	✓ S/R ✓ S/R ✓ (2)
11.2.3	<p>Area of ΔKOP = Area of Quad/Vierhoek OLMP \therefore Area of ΔKLM = $2 \times$ Area of ΔKOP</p> $\frac{1}{2} \times LM \times KX = 2 \times \frac{1}{2} \times OP \times KY$ $\frac{1}{2} = \frac{OP \cdot KY}{LM \cdot KX}$ but/maar $\frac{OP}{LM} = \frac{KY}{KX}$ $\frac{OP^2}{LM^2} = \frac{1}{2}$ $\frac{OP}{LM} = \frac{1}{\sqrt{2}}$ $\frac{OP}{LM} = \frac{KO}{KL}$ $\therefore \frac{KO}{KL} = \frac{1}{\sqrt{2}}$	✓ S ✓ S/R ✓ S/R ✓ (6)