



**KWAZULU-NATAL PROVINCE**  
**EDUCATION**  
REPUBLIC OF SOUTH AFRICA

**NATIONAL  
SENIOR CERTIFICATE**



**GRADE 12**

**MATHEMATICS P2**

**PREPARATORY EXAMINATION**

**SEPTEMBER 2022**

*Stanmorephysics.com*

**MARKS:** **150**

**TIME:** **3 hours**

This question paper consists of 13 pages, 1 information sheet and an answer book with 22 pages.

## **INSTRUCTIONS AND INFORMATION**

Read the following instructions carefully before answering the questions.

1. This question paper consists of 11 questions.
2. Read the questions carefully.
3. Answer ALL the questions.
4. Number your answers exactly as the questions are numbered.
5. Clearly show ALL calculations, diagrams, graphs, etc. which you have used in determining your answers.
6. Answers only will NOT necessarily be awarded full marks.
7. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
8. If necessary, round off answers correct to TWO decimal places, unless stated otherwise.
9. Diagrams are NOT necessarily drawn to scale.
10. An information sheet with formulae is included at the end of the question paper.
11. Write neatly and legibly.

**QUESTION 1**

The weight (in kg) of 20 boys in the soccer squad of school A are given below:

40	47	48	51	53	57	58	58	59	59
60	60	60	60	61	62	63	64	66	69

- 1.1 Calculate:
- 1.1.1 the mean weight of the boys in this soccer squad. (2)
  - 1.1.2 the standard deviation of this data. (1)
- 1.2 Determine the number of boys that have a weight within one standard deviation of the mean. (1)
- 1.3 The following information was obtained from the coach of the soccer squad of school B:
- $$\sum_{n=1}^{22} x_n = 1320$$
- 1.3.1 How many boys are in the school B squad? (1)
  - 1.3.2 Calculate the mean weight of a boy in the soccer squad of school B. (1)
- 1.4 Assume that the mean weight of the boys in the soccer squad at school B is 60 kg. Five boys of equal weight are added to the school A squad so that the means of both school squads are the same. Calculate the weight of each of these five boys. (4)

[11]

**QUESTION 2**

A survey was done on 250 people to determine the distances they travel to work daily.

The results are shown in the table below.

DISTANCE, $d$ (in km)	FREQUENCY	CUMULATIVE FREQUENCY
$0 < d \leq 5$	8	
$5 < d \leq 10$	41	
$10 < d \leq 15$	63	
$15 < d \leq 20$	52	
$20 < d \leq 25$	41	
$25 < d \leq 30$	38	
$30 < d \leq 35$	7	
<b>TOTAL</b>		

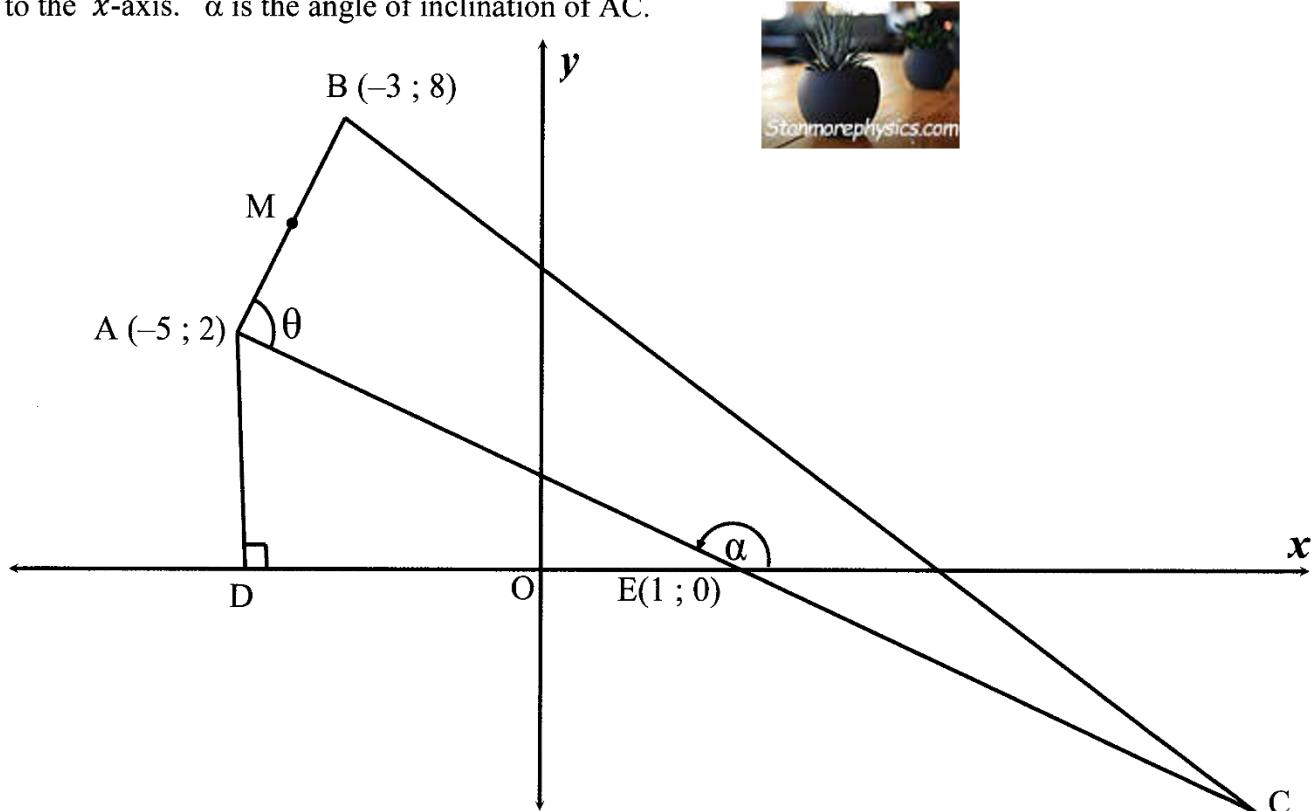
- 2.1 Complete the cumulative frequency column, on the attached ANSWER SHEET. (3)
- 2.2 Draw a cumulative frequency graph (ogive) for the given data on the grid provided on the attached ANSWER SHEET. (4)
- 2.3 Use the graph to determine the median distance travelled. Indicate on your graph the median distance. (2)  
[9]

Please turn over

### QUESTION 3

In the diagram below,  $A(-5 ; 2)$ ,  $B(-3 ; 8)$  and  $C$  are vertices of  $\Delta ABC$ .

$E(1 ; 0)$  is the midpoint of  $AC$ .  $D$  is a point on the  $x$ -axis such that  $AD$  is a line perpendicular to the  $x$ -axis.  $\alpha$  is the angle of inclination of  $AC$ .

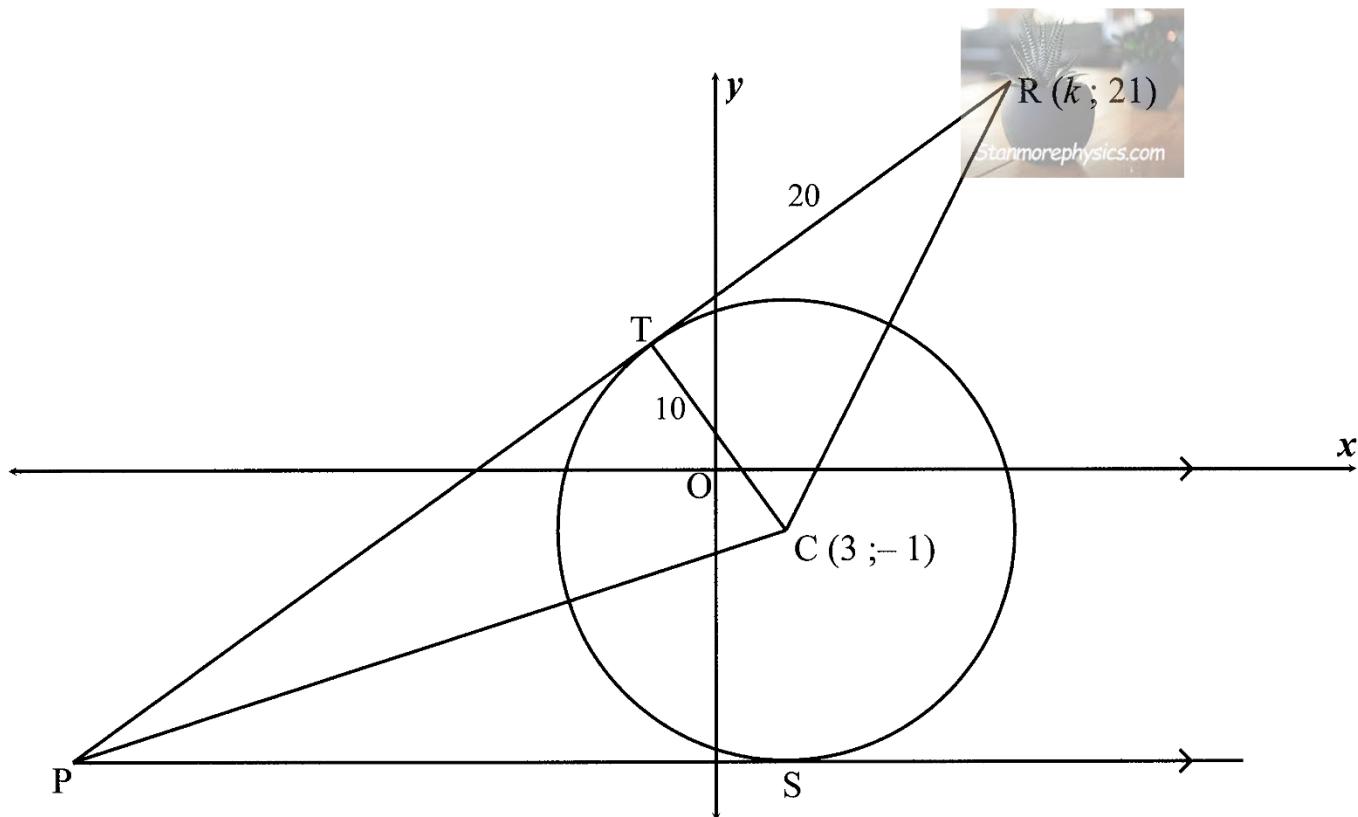


- 3.1 Determine the coordinates of  $M$ , the midpoint of  $AB$ . (2)
- 3.2 Write down the coordinates of point  $D$ . (1)
- 3.3 Show that the coordinates of  $C$  are  $(7 ; -2)$  (2)
- 3.4 Calculate the length of line  $AC$ . (Leave answer in simplest surd form) (2)
- 3.5 Determine the coordinates of  $F$ , if  $F$  lies in the first quadrant and  $CABF$  is a parallelogram. (2)
- 3.6 Determine the equation of the perpendicular bisector of  $AB$ . (4)
- 3.7 Calculate the size of  $\alpha$ , the angle of inclination of line  $AC$ . (3)
- 3.8 Determine the equation of the line parallel to  $AB$  passing through  $E$ . (2)
- 3.9 Calculate the size of angle  $\theta$ . (2)
- 3.10 Calculate the area of  $\Delta ABC$ . (4)

[24]

**QUESTION 4**

In the diagram, the circle TS centred at  $C(3; -1)$  has a radius CT of 10 units. PTR, where  $R(k; 21)$ , is a tangent to the circle at T. PS is a tangent to the circle at S and  $PS \parallel x\text{-axis}$ . PC, TC and CR are drawn.  $TR = 20$  units.



- 4.1 Give a reason why  $CT \perp TR$ . (1)
  - 4.2 Calculate the value of  $k$ , where  $R$  is in the first quadrant. (4)
  - 4.3 Write down the equation of the given circle. (2)
  - 4.4 Write down the equation of PS. (1)
  - 4.5 The equation of tangent PTR is  $3y = 4x + 35$ .
    - 4.5.1 Calculate the coordinates of P. (2)
    - 4.5.2 Calculate the length of PT. (3)
- [13]

**QUESTION 5**

5.1 If  $5\cos A = 2\sqrt{6}$  where  $A \in [90^\circ; 360^\circ]$ , calculate, **without using a calculator** and with the aid of a diagram, the values in simplest form of:

5.1.1  $-\sqrt{6} \cdot \tan A$  (4)

5.1.2  $\sin 2A$  (4)

5.2 Given:  $\sin 18^\circ = p$

**Without using a calculator**, determine each of the following in terms of  $p$ .

5.2.1  $\cos 18^\circ$  (2)

5.2.2  $\cos 48^\circ$  (5)

5.2.3  $\sin 9^\circ$  (3)

[18]

**QUESTION 6**

6.1 **Without using a calculator**, simplify the following expression fully:

$$\frac{\sin(180^\circ - x) \cdot \tan(x - 180^\circ) \cdot \cos(360^\circ + x)}{\sin^2(180^\circ + x) + \sin^2(90^\circ - x)} \quad (6)$$

6.2 **Without using a calculator**, determine the value of:

$$\frac{\cos 330^\circ \cdot \tan 150^\circ \cdot \sin 12^\circ}{\tan 675^\circ \cdot \cos 258^\circ} \quad (7)$$

6.3 Given the identity:  $\frac{\cos \alpha + \cos 2\alpha}{\sin 2\alpha - \sin \alpha} = \frac{\cos \alpha + 1}{\sin \alpha}$

6.3.1 Prove the identity. (4)

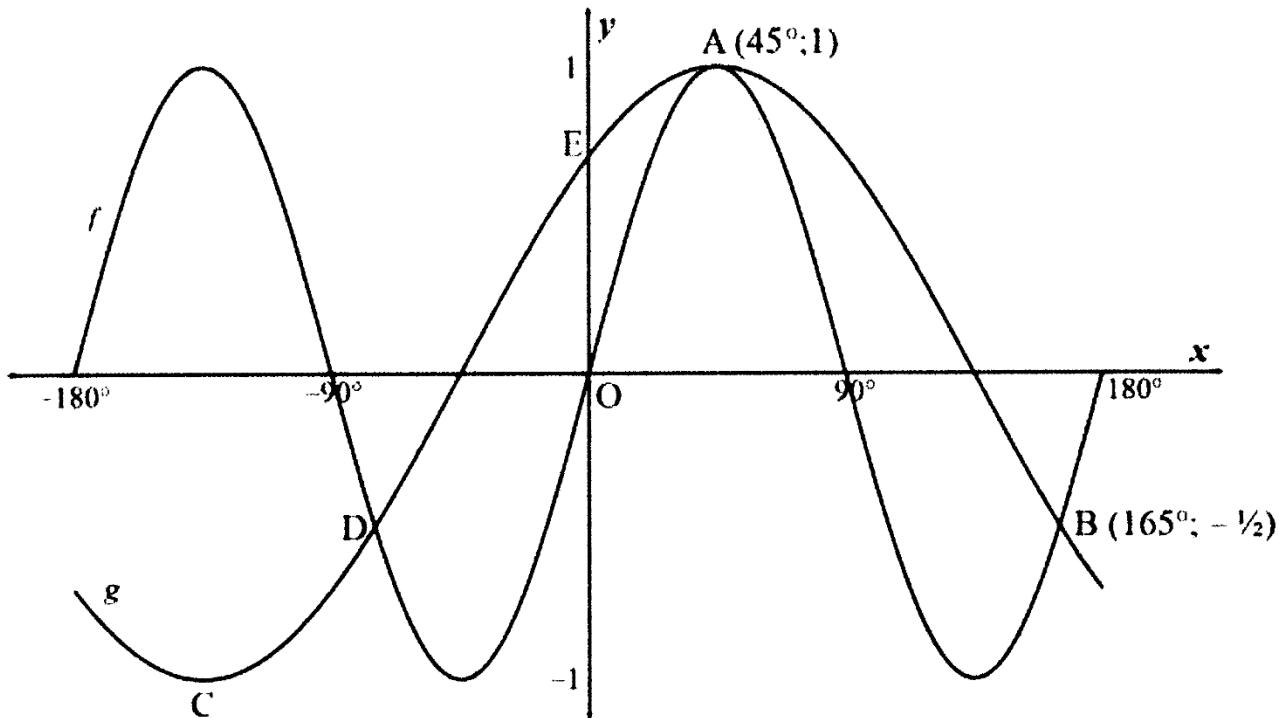
6.3.2 For which other values of  $\alpha$  is the identity undefined? (5)

[22]

**QUESTION 7**

Given:  $f(x) = \sin 2x$  and  $g(x) = \cos(x+a)$  where  $x \in [-180^\circ; 180^\circ]$

The graphs of  $f$  and  $g$  intersect at B and D. E is the  $y$ -intercept of  $g$ , and C is a turning point of  $g$ . A is a turning point of both  $f$  and  $g$ .



7.1 Write down the value of  $a$ . (1)

7.2 State the period of  $f$ . (1)

7.3 Determine the coordinates of C and E. (3)

7.4 Write down the amplitude of  $h$  if  $h(x) = 3f(x)$ . (1)

7.5 Determine for which value(s) of  $x$ , if  $x \in [0^\circ; 180^\circ]$ , will:

7.5.1  $g(x) > f(x)$  (2)

7.5.2  $g'(x) \cdot f'(x) \geq 0$  (2)

7.6 **Without solving the equation**, use the above graphs to show how you would solve the following equation:

$$\sqrt{2} \sin 2x = \cos x + \sin x \quad (3)$$

[13]

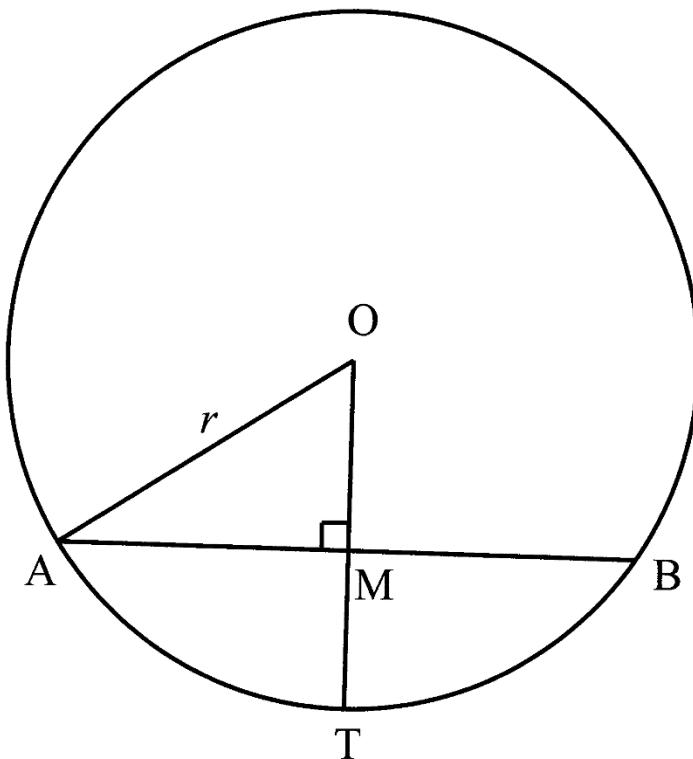
**QUESTION 8**

- 8.1 Complete the following statement:

The line drawn from the centre of the circle perpendicular to the chord ..... (1)

- 8.2 The circle below with centre O has chord AB = 8 cm.

OMT  $\perp$  AB with MT = 2 cm. The radius of the circle is  $r$  cm.



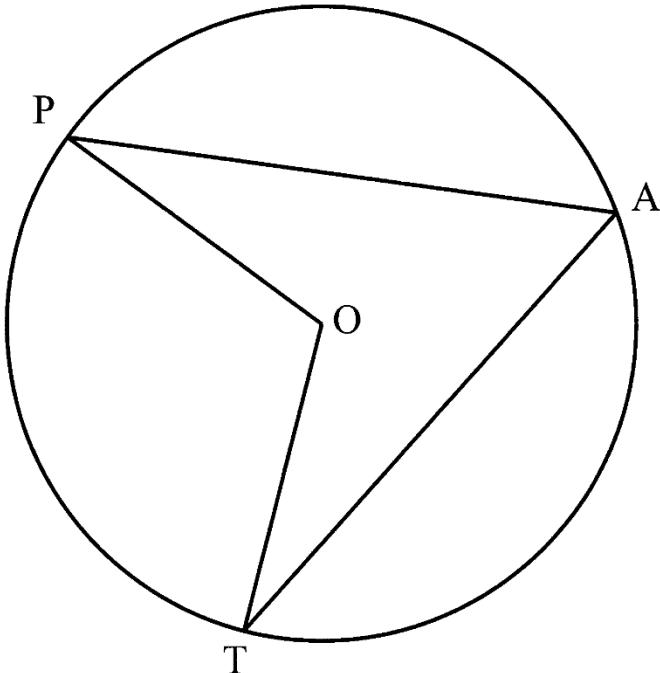
- 8.2.1 Write down, with a reason, the value of AM. (2)

- 8.2.2 Calculate the length of the radius of the circle. (4)

[7]

**QUESTION 9**

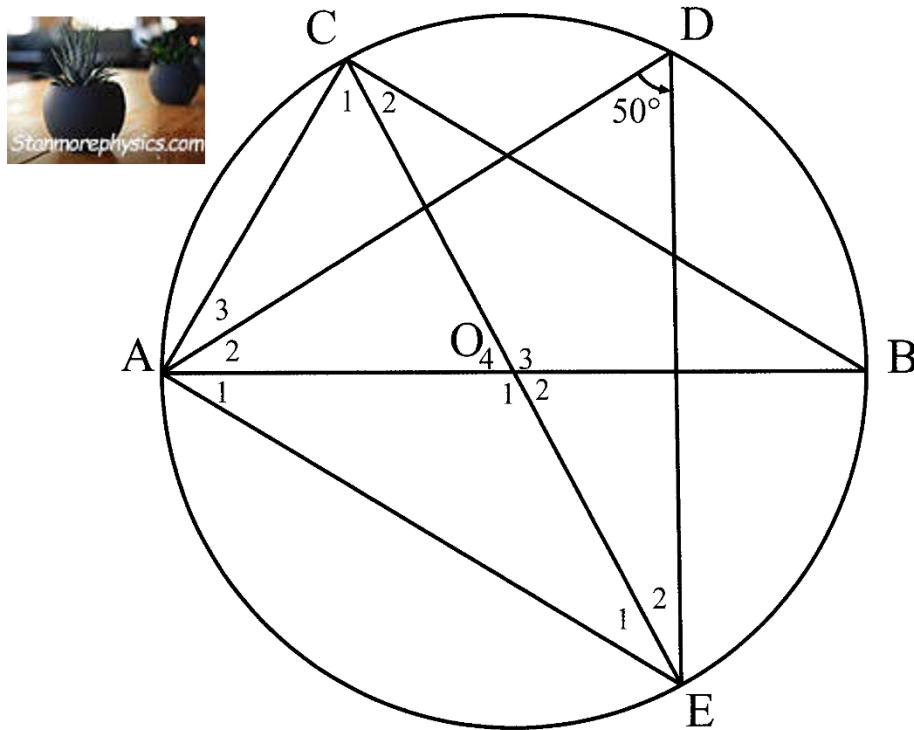
- 9.1 In the diagram below, O is the centre of the circle. P, A and T are points on the circumference of the circle. PA, TA, PO and TO are drawn.



Prove the theorem which states that  $\hat{POT} = 2\hat{PAT}$ .

(5)

- 9.2 AOB and COB are diameters of circle ACDBE with centre O.  
Chords AC, CB, AE, AD and DE are drawn.  $\hat{D} = 50^\circ$ .



- 9.2.1 Calculate, with reasons, the size of the following angles:

(a)  $\hat{O}_1$  (2)

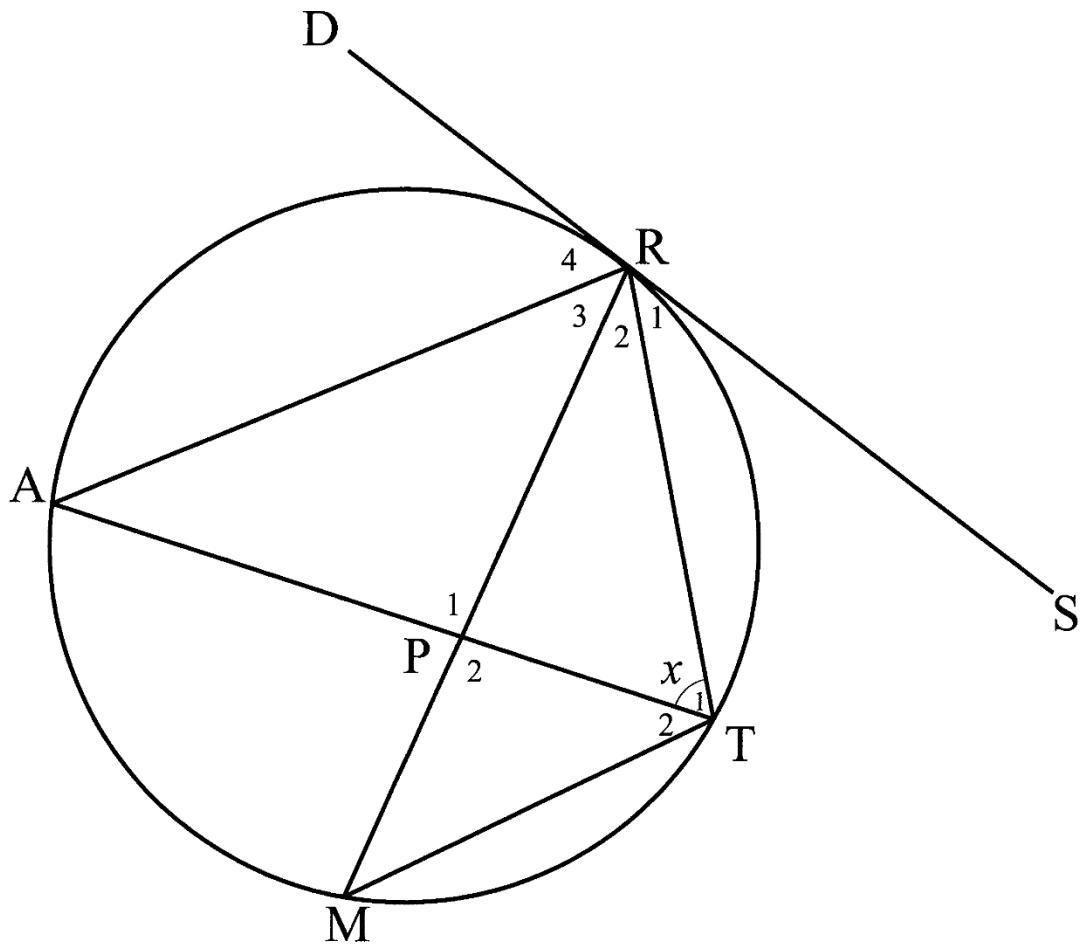
(b)  $\hat{E}_1$  (3)

- 9.2.2 Prove that  $AE \parallel CB$ . (4)

[14]

**QUESTION 10**

In the diagram DRS is a tangent to the circle TMAR at R. AT bisects  $\hat{MTR}$ . AT intersects MR at P. AR is drawn.  $\hat{T}_1 = x$ .



10.1 Prove, giving reasons, that:

10.1.1  $\hat{R}_3 = \hat{R}_4$ . (4)

10.1.2  $\Delta APR \parallel \Delta MPT$ . (3)

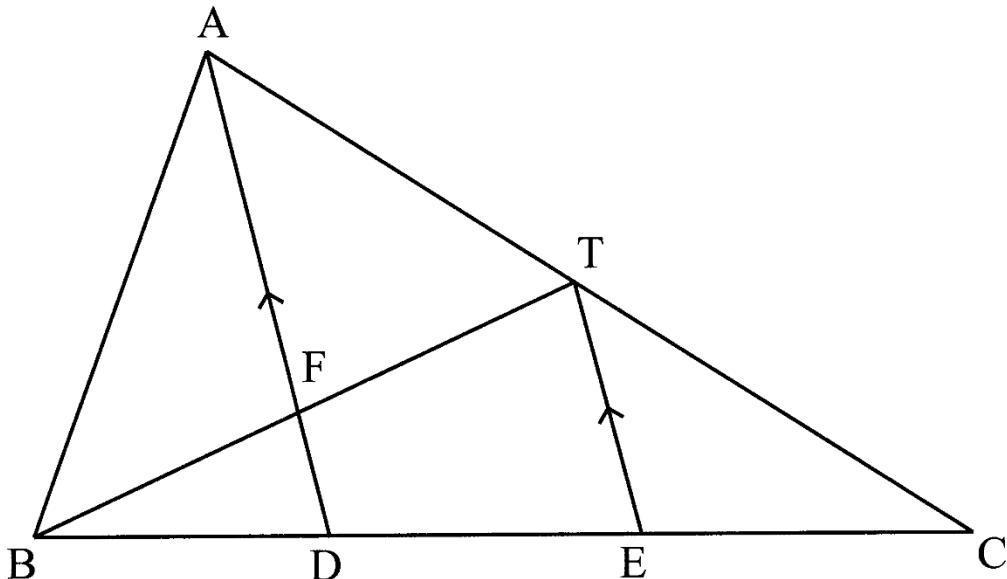
10.2 If  $AR = \frac{3}{2}MT$ , then calculate the value of  $\frac{PT}{PR}$ . (3)

[10]

**QUESTION 11**

In the diagram below,  $\triangle ABC$  has D and E on BC.  $BD = 6 \text{ cm}$  and  $DC = 9 \text{ cm}$ .

$AT : TC = 2 : 1$  and  $AD \parallel TE$ .



11.1 Write down the numerical value of  $\frac{CE}{ED}$ . (1)

11.2 Show that D is the midpoint of BE. (1)

11.3 If  $FD = 2 \text{ cm}$ , calculate the length of TE. (2)

11.4 Calculate the numerical value of :

11.4.1 
$$\frac{\text{Area of } \triangle ADC}{\text{Area of } \triangle ABD} \quad (2)$$

11.4.2 
$$\frac{\text{Area of } \triangle TEC}{\text{Area of } \triangle ABC} \quad (3)$$

[9]

**TOTAL: 150**

**INFORMATION SHEET: MATHEMATICS**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1+ni)$$

$$A = P(1-ni)$$

$$A = P(1-i)^n$$

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2} \{2a + (n-1)d\}$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r-1}; r \neq 1$$

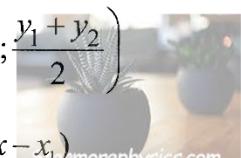
$$S_{\infty} = \frac{a}{1-r}; -1 < r < 1$$

$$F = \frac{x[(1+i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$


$$m = \tan \theta$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$


$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$(x-a)^2 + (y-b)^2 = r^2$$

$$\text{In } \Delta ABC: \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{area } \Delta ABC = \frac{1}{2} ab \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2 \sin^2 \alpha \\ 2 \cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha$$

$$\bar{x} = \frac{\sum fx}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$



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SEPTEMBER 2022

#### MARKING GUIDELINE

Stanmorephysics.com

MARKS: 150

TIME: 3 hours

These marking guideline consists of 15 pages.

NOTE:

- If a candidate answered a QUESTION TWICE, mark only the FIRST attempt.
- If a candidate crossed out an answer and did not redo it, mark the crossed-out answer.
- Consistent accuracy applies to ALL aspects of the marking guidelines.
- Assuming values/answer in order to solve a problem is unacceptable.

GEOMETRY	
S	A mark for a correct statement (A statement mark is independent of a reason.)
R	A mark for a correct reason (A reason mark may only be awarded if the statement is correct.)
S/R	Award a mark if the statement AND reason are both correct.

**QUESTION 1**

40	47	48	51	53	57	58	58	59	59
60	60	60	60	61	62	63	64	66	69

1.1.1	$\bar{x} = \frac{1155}{20}$ $\bar{x} = 57,75 \text{ kg}$	Answer only: full marks	A✓ 1155 CA✓ answer Penalty 1 mark for rounding here for the entire paper	(2)
1.1.2	$\sigma = 6,73702 \approx 6,74 \text{ kg}$		A✓ answer	(1)
1.2	$(\bar{x} - \sigma ; \bar{x} + \sigma)$ $(57,75 - 6,74 ; 57,75 + 6,74)$ limit = $(51,01 ; 64,49)$ $\therefore 14$ boys		CA✓ interval CA✓ answer	(2)
1.3.1	22		A✓ answer	(1)
1.3.2	$\bar{x} = \frac{1320}{22}$ $\bar{x} = 60 \text{ kg}$		CA✓ based on 1.3.1	(1)
1.4	$\bar{x} = \frac{5x+1155}{25} = 60$ $5x+1155=1500$ $5x=345$ $x=69 \text{ kg}$		CA✓ $\frac{5x+1155}{25}$ CA✓ equation CA✓ simplification CA✓ answer	(4)
				[11]

**QUESTION 2**

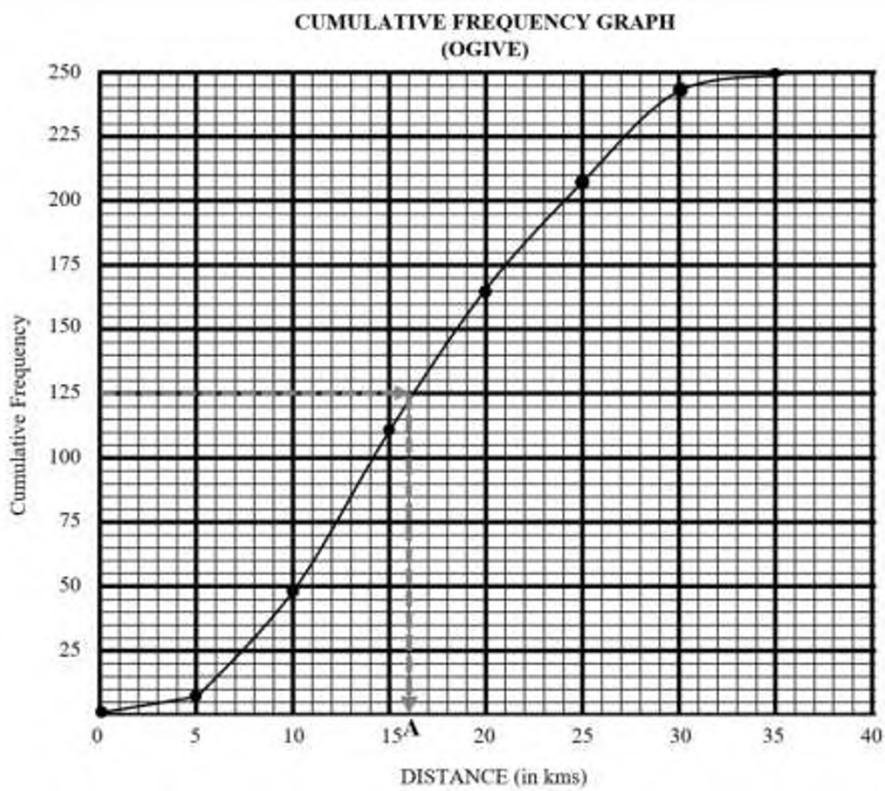
2.1

DISTANCE, $d$ (in km)	FREQUENCY	CUMULATIVE FREQUENCY
$0 < d \leq 5$	8	8
$5 < d \leq 10$	41	49
$10 < d \leq 15$	63	112
$15 < d \leq 20$	52	164
$20 < d \leq 25$	41	205
$25 < d \leq 30$	38	243
$30 < d \leq 35$	7	250
<b>TOTAL</b>	<b>250</b>	

- A ✓ 8 and 49  
 CA ✓ 112 and 164  
 CA ✓ 205, 243 and 250

(3)

2.2



- A ✓ grounded at (0 ; 0)  
 CA ✓ cumulative frequencies for y-coords  
 CA ✓ 5 other points correct  
 A ✓ smooth shape



(4)

2.3

See ogive for dotted lines and point marked A.

Median = 16 km

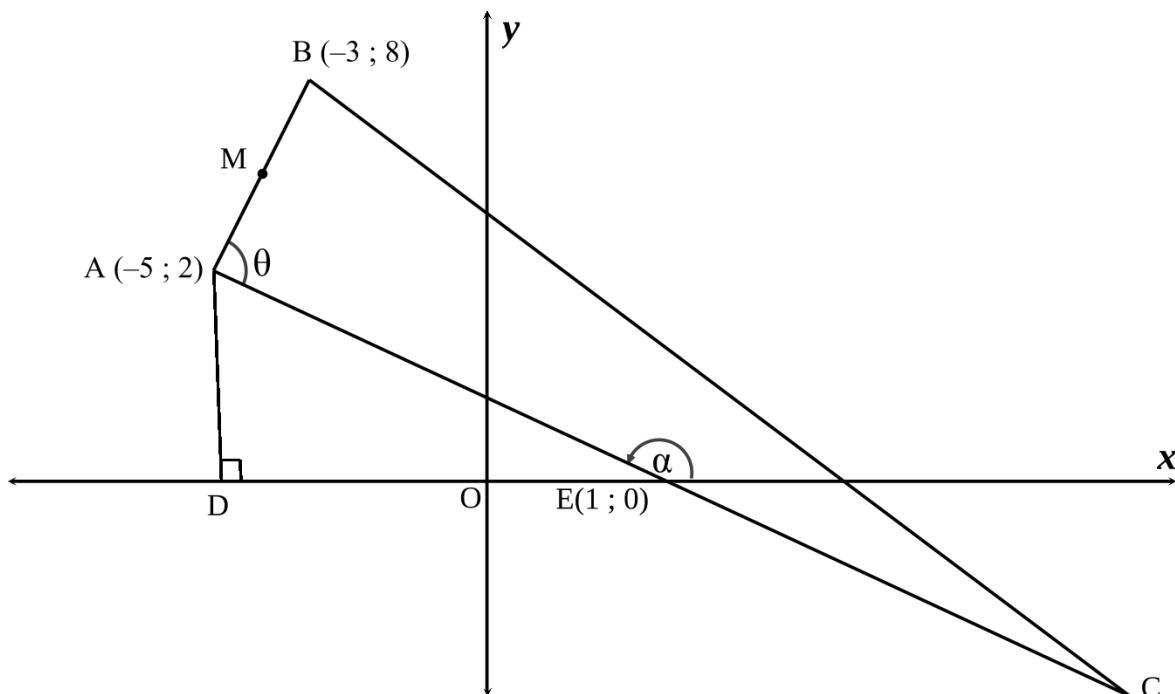
OR

Answer only: Full marks (accept 15,16,17)

- CA ✓ indication on graph  
 CA ✓ approx. median

(2)

[9]

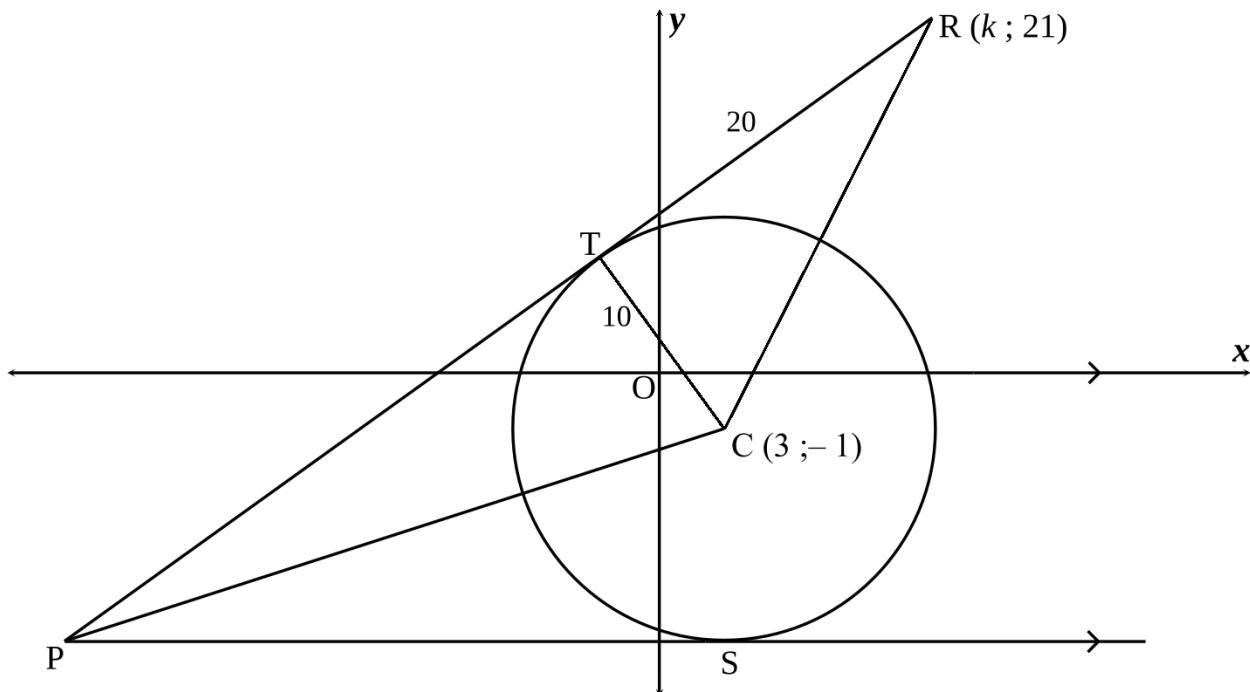
**QUESTION 3**

3.1	$M\left(\frac{x_2 + x_1}{2}; \frac{y_2 + y_1}{2}\right)$ $M\left(\frac{-5+3}{2}; \frac{2+8}{2}\right)$ $M(-4; 5)$ <div style="border: 1px solid black; padding: 5px; margin-top: 10px;">Answer only: full marks</div>	A✓ Substitution of A and B into midpoint formula CA✓ answer	(2)
3.2	D(-5; 0)	A✓ answer	(1)
3.3	$\frac{-5+x_C}{2} = 1$ $\therefore x_C = 7$ $C(7; -2)$ <p><b>OR</b></p> $C(1+6; 0-2)$ $C(7; -2)$ <div style="text-align: center; margin-top: 20px;">  </div>	A✓ midpoint ITO x A✓ midpoint ITO y	(2)
3.4	$AC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $AC = \sqrt{(-5-7)^2 + (2+2)^2}$ $AC = 4\sqrt{10}$	A✓ substitute A and C into distance formula CA✓ answer	(2)

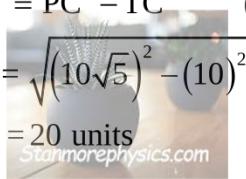
3.5	<p>Method: translation  <math>A \rightarrow B: (x; y) \rightarrow (x + 2; y + 6)</math></p> <p><math>\therefore</math> by symmetry: <math>C \rightarrow F: C(7; -2) \rightarrow F(7 + 2; -2 + 6)</math>  <math>\therefore F(9; 4)</math></p> <p><b>OR</b>          Midpoint of intersection of diagonals = <math>T(2; 3)</math>          Let coordinates of F be <math>(a; b)</math></p> $\frac{a - 5}{2} = 2 \quad \text{and} \quad \frac{b + 2}{2} = 3$ $a = 9 \quad \text{and} \quad b = 4$ $\therefore F(9; 4)$	<p>A✓ <math>x</math>-coordinate          A✓ <math>y</math>-coordinate  <b>OR</b></p> <p>A✓ <math>x</math>-coordinate          A✓ <math>y</math>-coordinate</p>	(2)
3.6	$m_{AB} = \frac{8 - 2}{-3 + 5} = 3$ <p><math>\therefore</math> gradient of perpendicular bisector: <math>-\frac{1}{3}</math></p> $\therefore y = -\frac{1}{3}x + c$ <p>sub <math>(-4; 5)</math>: <math>5 = -\frac{1}{3}(-4) + c</math>  <math>c = \frac{11}{3}</math></p> <p><math>\therefore</math> equation of perpendicular bisector: <math>y = -\frac{1}{3}x + \frac{11}{3}</math></p>	<p>A✓ <math>m_{AB} = 3</math></p> <p>CA✓ <math>m_{\text{perp bisector}} = -\frac{1}{3}</math></p> <p>CA✓ substitution</p> <p>CA✓ equation</p>	(4)
3.7	$m_{AC} = \frac{2 + 2}{-5 - 7} = -\frac{1}{3}$ $\tan \alpha = -\frac{1}{3}$ $\alpha = \tan^{-1}\left(-\frac{1}{3}\right) + 180^\circ$ $\alpha = 161,57^\circ$ $\therefore \alpha \approx 162^\circ$ <p><b>OR</b>          Accept method using Cosine Rule</p>	<p>A✓ <math>m_{AC}</math></p> <p>CA✓ <math>\tan \alpha = -\frac{1}{3}</math></p> <p>CA✓ answer</p>	(3)
3.8	$m_{AB} = 3$ <p><math>\therefore</math> gradient of new line = 3      (<math>\parallel</math> lines = gradients )</p> $y = 3x + c$ <p>sub <math>E(1; 0)</math>: <math>0 = 3(1) + c</math>  <math>= -3</math></p> $y = 3x - 3$	<p>CA✓ equal gradients</p> <p>CA✓ answer</p>	(2)

3.9	$m_{AB} \times m_{AC} = 3 \times -\frac{1}{3}$ $\therefore m_{AB} \times m_{AC} = -1$ $\therefore AB \perp AC$ $\therefore \theta = 90^\circ$ Accept methods using Cosine Rule & Trig ratios	A✓ $m_{AB} \times m_{AC} = -1$  A✓ answer	(2)
3.10	$AC = 4\sqrt{10}$ units $AB = \sqrt{(-3+5)^2 + (8-2)^2} = 2\sqrt{10}$ units  $\therefore \text{Area of } \Delta ABC = \frac{1}{2} AC \cdot AB$ $\therefore \text{Area of } \Delta ABC = \frac{1}{2} (4\sqrt{10})(2\sqrt{10})$ $\therefore \text{Area of } \Delta ABC = 40 \text{ units}^2$	A✓ substitute A and B into distance formula CA✓ length of AB  CA✓ substitution  CA✓ answer	(4)
			<b>[24]</b>

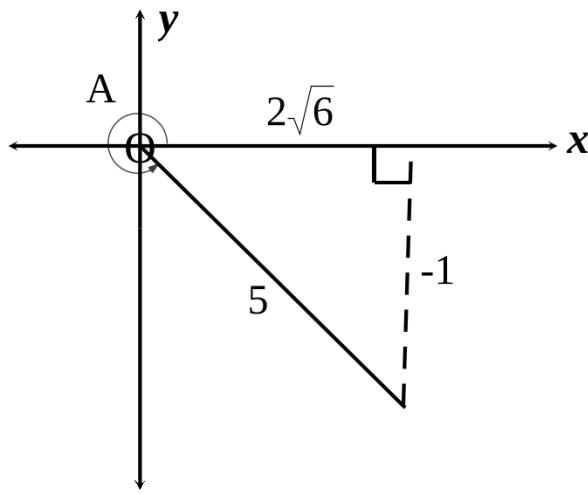
**QUESTION 4**

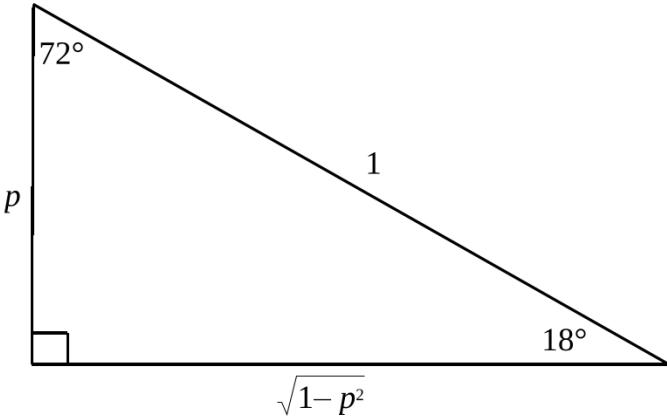


4.1	Radius is perpendicular to tangent	A✓ reason	(1)
4.2	$CR = \sqrt{10^2 + 20^2} = \sqrt{500}$ (pythag) $CR = \sqrt{(k-3)^2 + (21+1)^2} = \sqrt{(k-3)^2 + 484}$ $\therefore \sqrt{(k-3)^2 + 484} = \sqrt{500}$ $\therefore (k-3)^2 + 484 = 500$ $\therefore (k-3)^2 = 16$ $\therefore k-3 = \pm 4$ $\therefore k = -1 \text{ or } k = 7$ but $k \neq -1$ (given that R is the first quadrant) $k = 7 \text{ only}$	A✓ 500 A✓ equate CA✓ simplification CA✓ both values and rejection of $k = -1$	(4)
4.3	$(x-3)^2 + (y+1)^2 = 100$	A✓ $(x-3)^2 + (y+1)^2$ A✓ 100	(2)
4.4	$S(3;-11)$ $\therefore y = -11$	A✓ answer	(1)
4.5.1	$y = -11$ Eq 1 $3y = 4x + 35$ Eq 2 Sub Eq 1 into Eq 2: $3(-11) = 4x + 35$ $4x = -68$ $x = -17$ $\therefore P(-17;-11)$	CA✓ substitution CA✓ $x$ -value	(2)

<p>4.5.2</p> <p><math>PC = \sqrt{(3+17)^2 + (-1+11)^2} = 10\sqrt{5}</math></p> <p><math>TC = 10</math></p> <p><math>PT^2 = PC^2 - TC^2</math> (pythag)</p> <p><math>PT = \sqrt{(10\sqrt{5})^2 - (10)^2}</math></p> <p><math>PT = 20</math> units</p> <p></p> <p><b>OR</b></p> <p><math>PT = PS</math> (tangents from common point)</p> <p><math>PS = 3 - (-17) = 20</math> units</p> <p><math>\therefore PT = 20</math> units</p>	<p>CA✓ <math>PC^2 = 500</math> <math>PC = 10\sqrt{5}</math></p> <p>CA✓ Pythagoras</p> <p>CA✓ answer</p> <p>A✓ S/R CA✓ Substitution CA✓ answer</p>	<p>(3)</p>
<b>[13]</b>		

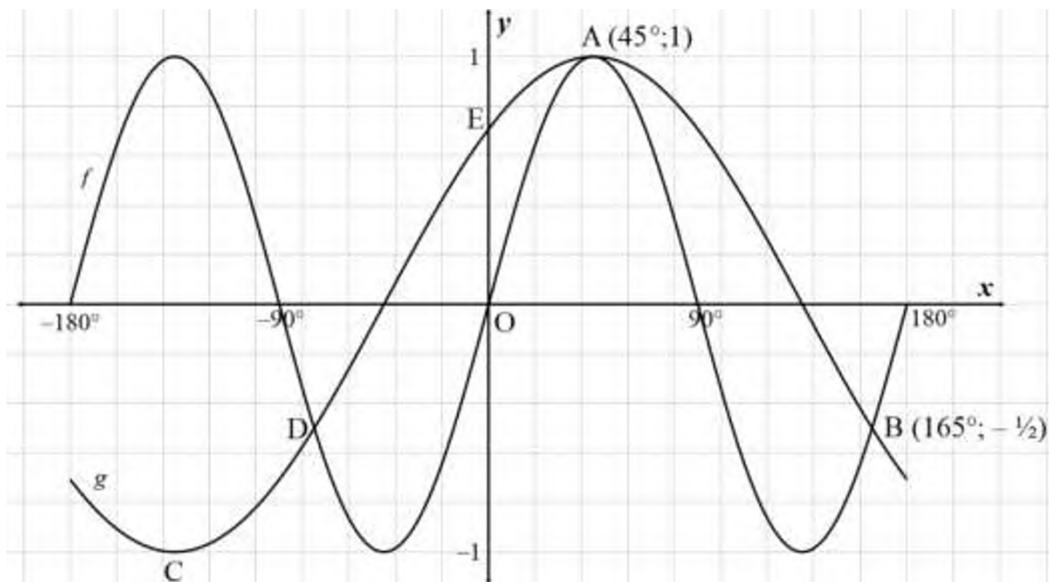
### QUESTION 5

<p>5.1.1</p>  <p><math>y = \sqrt{r^2 - x^2}</math></p> <p><math>y = \sqrt{(5)^2 - (2\sqrt{6})^2} = 1</math></p> <p><math>\therefore y = -1</math> (quadrant 4)</p> <p><math>\therefore -\sqrt{6} \cdot \tan A</math></p> <p><math>= -\sqrt{6} \times \left( \frac{-1}{2\sqrt{6}} \right)</math></p> <p><math>= \frac{1}{2}</math></p>	<p>A✓ diagram</p> <p>A✓ <math>y = -1</math></p> <p>CA✓ <math>\tan A = \frac{2\sqrt{6}}{-1}</math></p> <p>CA✓ answer</p>	<p>(4)</p>
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5.1.2	$\begin{aligned}\sin 2A &= 2 \sin A \cos A \\ &= 2\left(\frac{-1}{5}\right)\left(\frac{2\sqrt{6}}{5}\right) \\ &= -\frac{4\sqrt{6}}{25}\end{aligned}$	A✓ double angle identity CA✓ $\sin A = \frac{-1}{5}$ CA✓ $\cos A = \frac{2\sqrt{6}}{5}$ CA✓ answer	(4)
5.2			
5.2.1	$\cos 18^\circ = \sqrt{1-p^2}$	A✓ diagram A✓ answer Answer only full marks	(2)
5.2.2	$\begin{aligned}\cos 48^\circ &= \cos(30^\circ + 18^\circ) \\ &= \cos 30^\circ \cos 18^\circ - \sin 30^\circ \sin 18^\circ \\ &= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{1-p^2}}{1}\right) - \left(\frac{1}{2}\right)\left(\frac{p}{1}\right) \\ &= \frac{\sqrt{3}\sqrt{1-p^2} - p}{2} \\ &= \frac{\sqrt{3-3p^2} - p}{2}\end{aligned}$	A✓ $(30^\circ + 18^\circ)$ A✓ expansion A✓ special angle substitution CACA✓ substitution of $\sin 18^\circ$ and $\cos 18^\circ$	(5)
5.2.3	$\begin{aligned}\cos 18^\circ &= 1 - 2 \sin^2 9^\circ \\ -2 \sin^2 9^\circ &= \cos 18^\circ - 1 \\ \sin^2 9^\circ &= \frac{\cos 18^\circ - 1}{-2} \\ \sin 9^\circ &= \sqrt{\frac{\cos 18^\circ - 1}{-2}} \\ \sin 9^\circ &= \sqrt{\frac{\sqrt{1-p^2} - 1}{-2}} = \sqrt{\frac{1-\sqrt{1-p^2}}{2}}\end{aligned}$	A✓ double angle expansion  A✓ making $\sin 9^\circ$ the subject  CA✓ answer	(3)
			[18]

## QUESTION 6

6.1	$\begin{aligned} & \frac{\sin(180^\circ - x) \cdot \tan(x - 180^\circ) \cdot \cos(360^\circ + x)}{\sin^2(180^\circ + x) + \sin^2(90^\circ - x)} \\ &= \frac{\sin x \cdot \tan x \cdot \cos x}{\sin^2 x + \cos^2 x} \\ &= \sin x \cdot \frac{\sin x}{\cos x} \cdot \cos x \\ &= \sin^2 x \end{aligned}$	A✓ sin x A✓ tan x A✓ $\sin^2 x$ A✓ $\cos^2 x$ A✓ $\tan x = \frac{\sin x}{\cos x}$ CA✓ answer	(6)
6.2	$\begin{aligned} & \frac{\cos 330^\circ \cdot \tan 150^\circ \cdot \sin 12^\circ}{\tan 675^\circ \cdot \cos 258^\circ} \\ &= \frac{(\cos 30^\circ) \cdot (-\tan 30^\circ) \cdot (\sin 12^\circ)}{(-\tan 45^\circ) \cdot (-\cos 78^\circ)} \\ &= -\frac{(\cos 30^\circ) \cdot (\tan 30^\circ) \cdot (\sin 12^\circ)}{(\tan 45^\circ) \cdot (\sin 12^\circ)} \\ &= -\frac{\left(\frac{\sqrt{3}}{2}\right) \left(\frac{\sqrt{3}}{3}\right)}{1} \\ &= -\frac{1}{2} \end{aligned}$	A✓ $\cos 30^\circ$ A✓ $-\tan 30^\circ$ A✓ co ratio A✓ $-\cos 78^\circ$ A✓ $\tan 45^\circ$  A✓ special angles CA✓ answer	(7)
6.3.1	$\begin{aligned} \text{LHS: } & \frac{\cos \alpha + \cos 2\alpha}{\sin 2\alpha - \sin \alpha} \\ &= \frac{\cos \alpha + (2\cos^2 \alpha - 1)}{(2\sin \alpha \cos \alpha) - \sin \alpha} \\ &= \frac{2\cos^2 \alpha + \cos \alpha - 1}{2\sin \alpha \cos \alpha - \sin \alpha} \\ &= \frac{(2\cos \alpha - 1)(\cos \alpha + 1)}{\sin \alpha (2\cos \alpha - 1)} \\ &= \frac{(\cos \alpha + 1)}{\sin \alpha} \\ &= \text{RHS} \end{aligned}$	A✓ cos double angle A✓ sin double angle  A✓ numerator factors A✓ denominator factors	(4)
6.3.2	$\begin{aligned} & \sin 2\alpha - \sin \alpha = 0 \\ & 2\sin \alpha \cos \alpha - \sin \alpha = 0 \\ & \sin \alpha (2\cos \alpha - 1) = 0 \\ & \therefore \sin \alpha = 0 \text{ or } 2\cos \alpha - 1 = 0 \\ & \qquad \qquad \qquad \cos \alpha = \frac{1}{2} \\ & \alpha = 0^\circ + k \cdot 180^\circ; k \in \mathbb{Z} \\ & \alpha = \pm 60^\circ + k \cdot 360^\circ; k \in \mathbb{Z} \\ \\ & \text{N.B. If } \sin \alpha = 0 \text{ is solved only} - \text{max. 3/5 marks} \end{aligned}$	A✓ equating to 0  A✓ factors CA✓ $0^\circ + k \cdot 180^\circ$ CA✓ $\pm 60^\circ + k \cdot 360^\circ$  A✓ $k \in \mathbb{Z}$	(5)
			[22]

**QUESTION 7**

7.1	$a = -45^\circ$	A✓ answer	(1)
7.2	$180^\circ$	A✓ answer	(1)
7.3	$C(-135^\circ; -1)$ E : $y = \cos(0^\circ + 45^\circ)$ $y = \frac{\sqrt{2}}{2}$ $E\left(0; \frac{\sqrt{2}}{2}\right)$	A✓ $C(-135^\circ; -1)$ A✓ solving for $y$ A✓ $E\left(0; \frac{\sqrt{2}}{2}\right)$	(3)
7.4	Amplitude = 3	A✓ answer	(1)
7.5.1	$x \in [0^\circ; 165^\circ); x \neq 45^\circ$ OR $x \in [0^\circ; 45^\circ) \cup (45^\circ; 165^\circ)$ OR $0^\circ \leq x < 165^\circ; x \neq 45^\circ$	A✓ $[0^\circ; 165^\circ)$ A✓ $x \neq 45^\circ$ A✓ critical values A✓ notation A✓ $tanmo 0^\circ \leq x < 165^\circ$ A✓ $x \neq 45^\circ$	(2)
7.5.2	$0^\circ \leq x \leq 135^\circ$	A✓ critical values A✓ notation	(2)
7.6	$\sqrt{2} \sin 2x = \cos x + \sin x$ $\sin 2x = \frac{\cos x}{\sqrt{2}} + \frac{\sin x}{\sqrt{2}}$ $\sin 2x = \cos x \frac{1}{\sqrt{2}} + \sin x \frac{1}{\sqrt{2}}$ $\sin 2x = \cos x \cos 45^\circ + \sin x \sin 45^\circ$ $\sin 2x = \cos(x - 45^\circ)$	A✓ division of $\sqrt{2}$ A✓ identifying compound $\angle$ $\cos(x - 45^\circ)$	

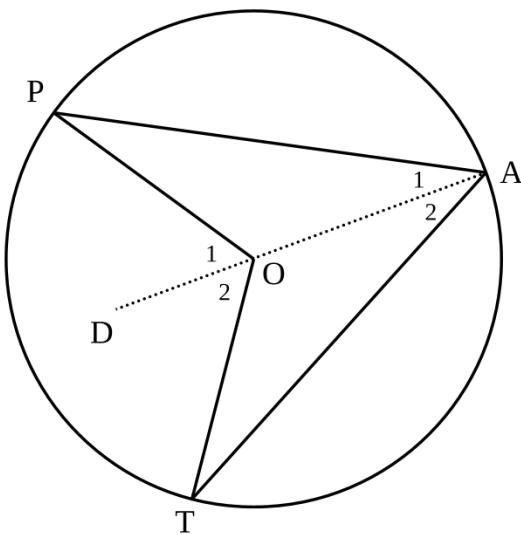
Equating the 2 functions gives the points of intersection of the 2 graphs.	A✓ conclusion	(3)
		[13]



### QUESTION 8

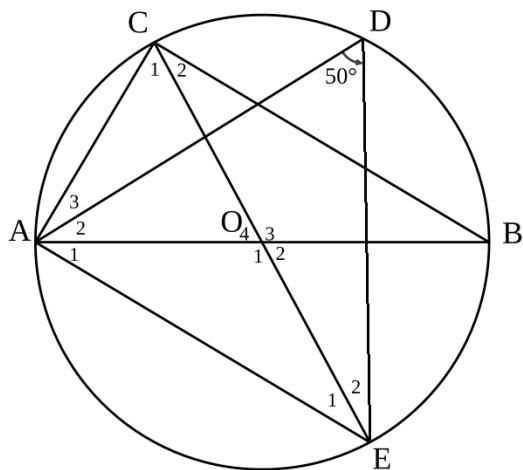
8.1	bisects the chord.	A✓ answer	(1)
8.2.1	$AM = 4\text{cm}$ (line from centre $\perp$ chord)	A✓ S A✓R	(2)
8.2.2	$OM = (r - 2)$ $\therefore r^2 = (r - 2)^2 + 4^2$ (pythag) $r^2 = r^2 - 4r + 4 + 16$ $4r = 20$ $r = 5 \text{ cm}$	A✓ $OM = (r - 2)$ CA✓ $r^2 = (r - 2)^2 + 4^2$ CA✓ simplification CA✓ answer	(4)
			[7]

### QUESTION 9



9.1	Constr: Draw line AO and extend to D. Proof: $OP = OT$ (radii) $\therefore \hat{A}_1 = \hat{P}$ ( $\angle$ s opp = sides) but $\hat{O}_1 = \hat{A}_1 + \hat{P}$ (ext $\angle$ of $\Delta$ ) $\therefore \hat{O}_1 = 2\hat{A}_1$ Similarly $\hat{O}_2 = 2\hat{A}_2$ $\therefore \hat{O}_1 + \hat{O}_2 = 2(\hat{A}_1 + \hat{A}_2)$ $\therefore \hat{POT} = 2\hat{PAT}$	A✓ construction A✓ S/R A✓ S/R A✓ S A✓ S	(5)
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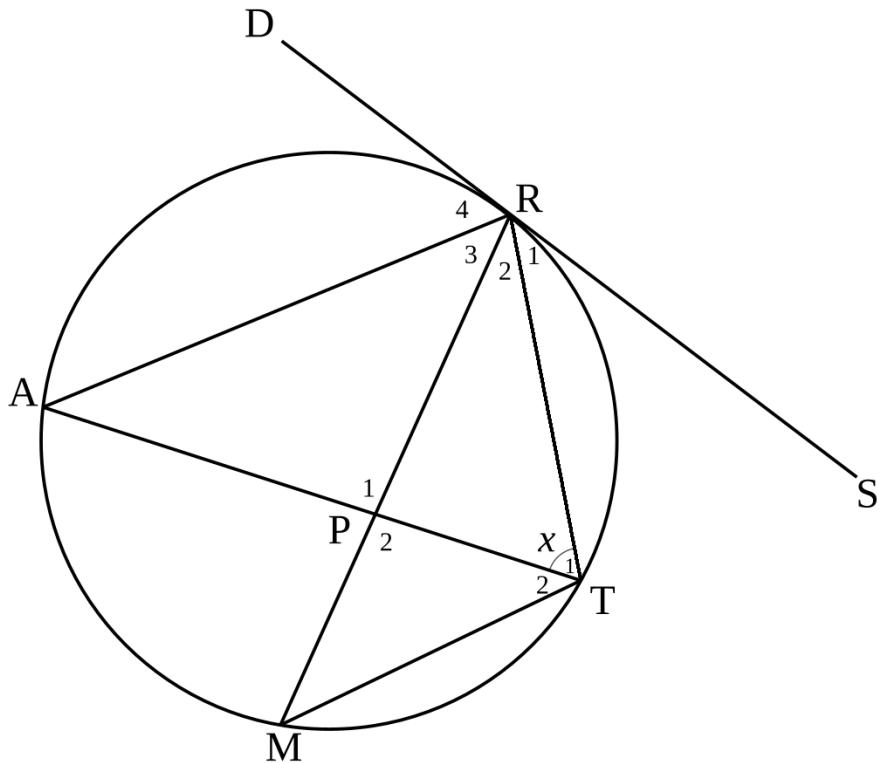
**9.2 Do not mark this question!!! (Maximum for Paper 2 – 141)**



9.2.1a	$\hat{O}_1 = 100^\circ$ ( $\angle$ at centre = $2 \times \angle$ at circ)	A✓ S A✓R	(2)
9.2.1b	$\hat{O}_1 = 100^\circ$ $\hat{A}_1 = \hat{E}_1$ ( $\angle$ s opp = radii) $\therefore \hat{E}_1 = \frac{180^\circ - 100^\circ}{2}$ (sum $\angle$ s $\Delta$ ) $\therefore \hat{E}_1 = 40^\circ$	A✓ S A✓R  CA✓ S/R	(3)
9.2.2	$\hat{A}_1 = \hat{E}_1$ ( $\angle$ s opp = radii) but $\hat{A}_1 = \hat{C}_2$ ( $\angle$ s in same segment) $\therefore \hat{E}_1 = \hat{C}_2$ $\therefore AE \parallel CB$ (alt $\angle$ s =)  OR  $\hat{A}_1 = \hat{E}_1$ ( $\angle$ s opp = radii) but $\hat{E}_1 = \hat{B}$ ( $\angle$ s in same segment) $\therefore \hat{A}_1 = \hat{B}$ $\therefore AE \parallel CB$ (alt $\angle$ s =)	A✓ S/R A✓ S/R A✓ S A✓ R  A✓ S/R A✓ S/R A✓ S A✓ R  A✓ S/R A✓ S/R	(4)

$\hat{C}AE = 90^\circ$ $\hat{A}CB = 90^\circ$ $\hat{C}AE + \hat{A}CB = 180^\circ$ $\therefore AE \perp CB$	$(\angle s \text{ in semi-arc})$ $(\angle s \text{ in semi-arc})$ $(\text{co-interior } \angle s)$	A✓ S A✓ R	(4)
		[14]	

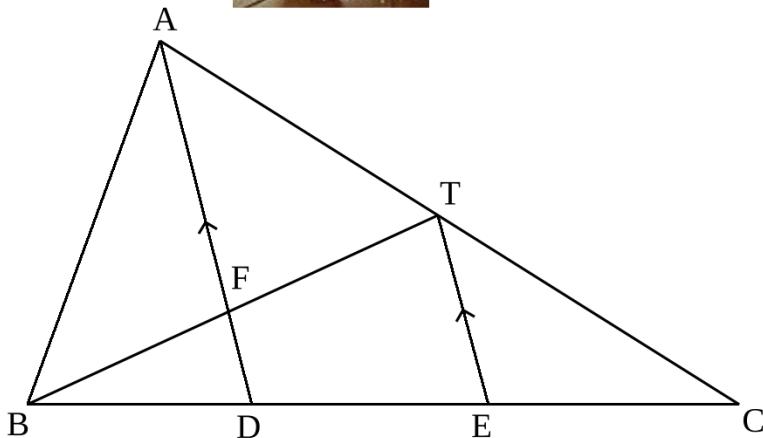
## **QUESTION 10**



10.1.1	$\hat{R}_3 = \hat{T}_2$ ( $\angle s$ in same segment) $\hat{R}_4 = \hat{T}_1$ (tan chord theorem) but $\hat{T}_1 = \hat{T}_2$ (given AT bisects MTR) $\therefore \hat{R}_3 = \hat{R}_4$	A✓ S A✓R	
10.1.2	In $\Delta APR$ and $\Delta MPT$ 1. $\hat{A} = \hat{M}$ ( $\angle s$ in same segment) 2. $\hat{P}_1 = \hat{P}_2$ (vert opp $\angle s$ ) 3. $\hat{R}_3 = \hat{T}_2$ (remaining $\angle s$ $\Delta$ ) $\therefore \Delta APR \parallel \Delta MPT$ ( $\angle \angle \angle$ )	A✓ S/R A✓ S/R A✓ R	(4)  (3)

10.2 $\frac{AR}{MT} = \frac{PR}{PT}$ $AR = \frac{3}{2} MT \quad (\text{given})$ $\therefore \frac{AR}{MT} = \frac{3}{2}$ $\therefore \frac{PT}{PR} = \frac{2}{3}$	$(\ \  \Delta s)$ $A\checkmark \quad S \quad A\checkmark R$ $A\checkmark \quad S$ $(3)$	<b>[10]</b>
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### QUESTION 11



11.1 $\frac{CE}{ED} = \frac{1}{2}$	$A\checkmark \quad S$ $(1)$
11.2 $DE = \frac{2}{3}(9\text{cm}) = 6\text{cm}$	$A\checkmark \quad S$ $(1)$
11.3 $\frac{TE}{BE} = \frac{2}{1} \quad (\text{/// triangles})$ $\therefore TE = 4\text{cm}$ <b>Accept solution using the Midpoint theorem</b>	$A\checkmark \quad S/R$ $A\checkmark \quad S$ $(2)$
11.4.1 $\frac{\text{Area of } \triangle ADC}{\text{Area of } \triangle ABD} = \frac{\frac{1}{2} DC \times \perp}{\frac{1}{2} BD \times \perp} = \frac{6\text{cm}}{6\text{cm}} = 1$	$A\checkmark \text{Areas}$ $A\checkmark \text{Answer}$ $(2)$

<b>11.4.2</b> <u>Area of <math>\Delta TEC</math></u> <u>Area of <math>\Delta ABC</math></u> $= \frac{\text{Area of } \Delta TEC}{\text{Area } \Delta TBC} \times \frac{\text{Area of } \Delta TBC}{\text{Area } \Delta ABC}$ $= \frac{EC}{BC} \times \frac{TC}{AC}$ $= \frac{1}{4} \times \frac{1}{3}$ $= \frac{1}{12}$  <b>OR</b> $\text{Area of } \Delta TEC = \frac{1}{4} (\text{Area of } \Delta TBC) \quad (\text{common vertex} = \text{altit.})$ $= \frac{1}{4} \left( \frac{1}{3} \text{Area of } \Delta ABC \right) \quad (\text{common vertex} = \text{altit.})$ $\frac{\text{Area of } \Delta TEC}{\text{Area of } \Delta ABC} = \frac{1}{12}$ <b>OR</b> <b>Accept area rule with the use of the angle of <math>\hat{C}</math>.</b>	A✓ S A✓ S A✓ Answer (3)  A✓ S A✓ S A✓ S (3)
<b>[9]</b>	<b>TOTAL:</b> <b>150</b>

