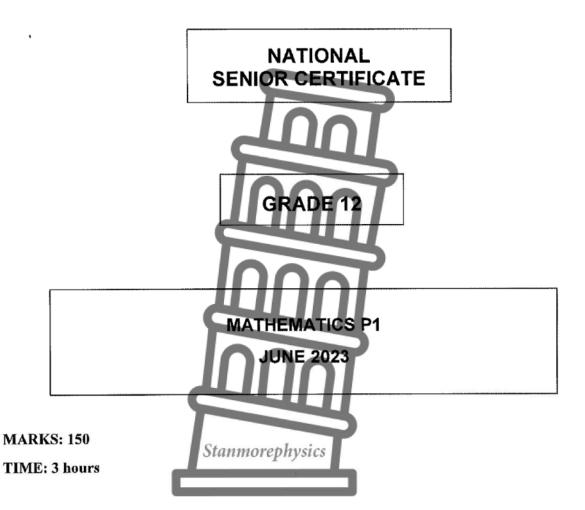
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This question paper consists of 8 pages and a one-page information sheet.

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#### INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

- This question paper consists of 10 questions.
   Answer All the questions.
- 2. Show clearly ALL the calculations, diagrams, graphs, etcetera, which you have used in determining the answers.
- 3. An approved scientific calculator (non-programmable and non-graphical) may be used, unless stated otherwise.
- 4. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.
- 5. Answers only will not necessarily be awarded full marks.
- Number the answers correctly according to the numbering system used in this question paper.
- 7. Diagrams and graphs are NOT necessarily drawn to scale.
- 8. An information sheet with formulae is included at the end of the question paper.
- 9. Write neatly and legibly.



1.1 Solve for x.

1.1.1 
$$(2+x)(-x+4) = 0$$
 (2)

1.1.2 
$$3x^2 = 2x + 4$$
 (Correct to 2 decimal places) (4)

1.1.2 
$$3x^2 = 2x + 4$$
 (Correct to 2 decimal places) (4)  
1.1.3  $x - 2\sqrt{x - 1} = 4$  (6)  
1.1.4  $x - 12 > -x^2$  (4)

$$1.1.4 + r - 12 > -r^2 \tag{4}$$

Solve the following equations simultaneously: 1.2

$$x^2 - xy - 5y^2 = -5$$
 and  $x + 2y = 1$  (6)

Determine the values of t for which the equation 1.3

$$5^x = 2 - t \text{ will have real solutions.}$$
 (3)

[25]

## **QUESTION 2**

Given the arithmetic series  $3 + 8 + 13 + \dots$  (to 253 terms)

- Write down the 4th term of the series. (1) 2.1
- Calculate the 253rd term of the series. (3) 2.2
- (2)Express the series in sigma notation. 2.3
- (2)2.4 Calculate the sum of the series.
- (4) How many terms of the series are divisible by 4? 2.5

[12]

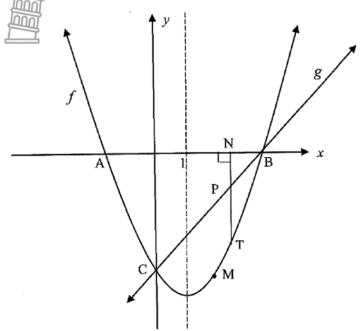
## QUESTION 3

- Given the quadratic sequence: 1; 3; 7; p; ... 3.1
  - (3) 3.1.1 Calculate the value of p.
  - 3.1.2 Determine the n<sup>th</sup> term of the sequence. (4)
  - The first difference between two consecutive terms is 62. (3) Calculate the values of these terms.
- Determine the largest integer value of m if  $\sum_{k=-2}^{m} (2.2^{k+2}) < 2046$ 3.2 (5)

[15]

The following sketch shows the graphs of  $f(x) = x^2 + bx + c$  and g(x) = ax + q.

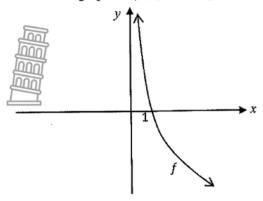
The graph of fintersects the x-axis at A(-2; 0) and at B. C is the y-intercept of both f and g. The axis of symmetry of f is x = 1.



- 4.1 Determine
  - 4.1.1 The coordinates of B (2)
  - 4.1.2 The equation of f in the form  $y = x^2 + bx + c$  (2)
  - 4.1.3 The values of a and q. (4)
  - 4.1.4 The coordinates of the turning point of f. (2)
- 4.2 M is the reflection of C in the axis of symmetry of f. Write down the coordinates of M. (2)
- 4.3 For which values of x is f(x) > g(x)? (2)
- 4.4 Write down the coordinates of the turning point of y = -f(x-2) (2)
- 4.5 N is a point on the x-axis and T is a point on f such that TN is perpendicular to the x-axis. TN intersects g at P. Calculate the maximum length of TP. (4)

[20]

Sketched below is the graph of  $f(x) = -\log_3 x$ 



5.1 Write down the domain of f.

(1)

5.2 Write down the equation of  $f^{-1}$  in the form y = ...

- (2)
- 5.3 Describe the transformation from  $f^{-1}$  to h if  $h(x)=3^{-x}-5$
- (2)
- 5.4 Use the graph of f to solve for x if k(x) is the reflection of f about the x-axis and  $k(x) \ge 3$ .
- [9]

(4)

## **QUESTION 6**

The function f defined by  $f(x) = \frac{a}{x+p} + q$  has the following properties.

- The range of f is  $y \in R$ ,  $y \ne 2$
- The axis of symmetry with a positive gradient is y = x + 1
- The graph of f passes through (0;-4)
- 6.1 Write down the value of q.

(1)

6.2 Calculate the values of a and p.

- (4)
- 6.3 Sketch a neat graph of this function. Your graph must include the intercepts with the axes and asymptotes if any.
- (5)

7.1 Thelma purchased a new car. The bank offered her a loan at an effective interest rate of 10,4% p.a.

Determine the nominal rate compounded monthly that she is required to pay. (4)

7.2 An investor bought shares in a certain company, but found that his money had become half of its original value after a period of 4 years. What was the annual rate of decay, on a reducing balance method, for the shares he had bought?

7.3 Thini borrowed R80 000, exactly five years ago, at 21% interest p.a. She paid back R25 000 exactly three years ago and R55 000 exactly a year ago. She wants to settle her remaining debt today.

7.3.2 How much must Thini pay to settle the debt today? (5)

[15]

(4)

#### **QUESTION 8**

8.1 Given 
$$f(x) = 4x - 2x^2$$
, determine  $f'(x)$  using FIRST PRINCIPLES. (5)

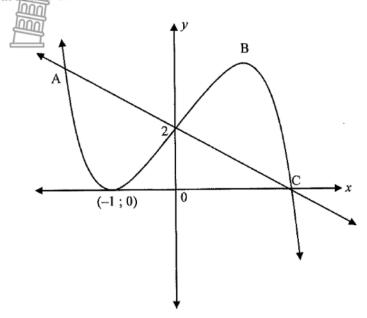
8.2 Determine:

8.2.1 
$$D_x \left[ x^4 - \frac{1}{x^2} \right]$$
 (3)

8.2.2 
$$\frac{dy}{dx} \quad \text{if} \quad x\sqrt{y} = x^2 - 4x \tag{4}$$

Find the equation of the tangent to the graph of  $f(x) = x^3 - 6x^2$  at which the gradient of the tangent to f is equal to 36 and x < 0. (5)

The graph below represents the functions f and g with  $f(x) = ax^3 + cx + 2$  and g(x) = -x + 2. C and (-1; 0) are the x-intercepts of f. The graph of f and g intersect at A and C.



- 9.1 Determine the coordinates of C. (1)
- 9.2 Show by calculation that a = -1 and c = 3. (4)
- 9.3 Determine the coordinates of B, the turning point of f. (3)
- 9.4 Find the x-coordinate of the point of inflection of f. (2)
- 9.5 Write down the values of k for which f(x) = k will have only one root. (2)
- 9.6 Write down the values of x for which f'(x) < 0. (2)

[14]



10.1 Given that A and B are independent events, P(B only) = 0.4

P(A and B) = 0.1; P(A only) = x and P(not A or B) = y.

Calculate:

10.1.10 The values of 
$$x$$
 and  $y$ .

(4)

(3)

10.2 In a city 10% of all crimes are violent. 95% of all violent crimes are reported, buy only 45% of non-violent crimes are reported.

10.2.1 Draw a tree diagram showing all the possible outcomes.

(3)

10.2.2 What is the probability that a random crime will be reported?

(3)

[13]

**TOTAL: 150** 



#### INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + in)$$

$$A = P(1 - in)$$

$$A = P(1 - i)^v$$

$$A = P(1 + i)^v$$

$$A = P(1 +$$

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## NATIONAL SENIOR CERTIFICATE

**GRADE 12** 

# MATHEMATICS P1 JUNE 2023 MARKING GUIDELINES

This marking guideline consists of 12 pages.



1.1.1	(x-2)(-x+4) = 0		
	x-2=0 or $-x+4=0$	$\checkmark x = 2$ $\checkmark x = 4$	
A	x = 2 or $x = 4$	$\nabla x = 4$	(2)
1.1.2	$3x^{2} + 2x + 4$	✓ standard form	(2)
1.1.2	0000		
	$3x^2 - 2x - 4 = 0$		
	$-(-2)\pm\sqrt{(-2)^2-4(3)(-4)}$	✓ substitution in correct	
	$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(3)(-4)}}{2(3)}$		
	$x = \frac{2 \pm \sqrt{52}}{6}$	$\checkmark$ values of $x$	
	x = -0.87 or $x = 1.54$		
			(4)
1.1.3	$x - 2\sqrt{x - 1} = 4$		
	$-2\sqrt{x-1} = 4 - x$	✓isolate the surd	
	$(-2\sqrt{x-1})^2 = (4-x)^2$	✓ square both sides	
	,	^	
	$4x - 4 = 16 - 8x + x^2$	Coton to al Comm	
	$x^2 - 12x + 20 = 0$	✓ standard form ✓ factors	
	(x-10)(x-2)=0	$\checkmark$ x-values	
	$x = 10 \text{ or } x \neq 2$	$\checkmark x = 10$ only	(6)
1.1.4	$x^2 - x - 12 > 0$		
	(x-4)(x+3) > 0	✓standard f	]
		✓factors	
		✓ critical values	
	-3 4		ANT FIRST CONTRACTOR C
	$OR$ $\xrightarrow{-3} \qquad 4$ $x < -3 \text{ or } x > 4$	✓ solution	(4)

1.2	$x + 2y = 1$ $x = 1 - 2y$ $(1 - 2y)^{2} - y(1 - 2y) - 5y^{2} = -5$ $1 - 4y + 4y^{2} - y + 2y^{2} - 5y^{2} = -5$ $y^{2} - 5y + 6 = 0$	✓ subject of formulae ✓ substitution ✓ standard form	
	(y-3)(y-2) = 0 y = 3  or  y = 2 x = 1-2(3)  or  x = 1-2(2)	✓ factors  ✓ $y = 3$ or $y = 2$	
	x = -5  or  x = -3	$\checkmark x = -5 \text{ or } x = -3$	(6)
1.3	$5^{x} > 0$ $\therefore 2 - t > 0$ $\therefore t < 2$	✓ condition $\checkmark 2-t>0$ $\checkmark t<2$	(3)
			[25]



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2.1	18	✓answer	(1)
2.2	$T_n = 5n - 2$	✓nth term	
	$T_{253} = 5(253) - 2$	✓substitution	
	T <sub>253</sub> = 1263	✓answer	(3)
2.3	$\sum_{n=2}^{253} (5n-2)$	✓limits	
	1211 - 21 - 21 - 21 - 21 - 21 - 21 - 21	$\sqrt{5n-2}$	(2)
2.4	$S_{253} = \frac{253}{2} [3 + 1263]$	✓substitution	
	_ · ·	✓answer	
	$S_{253} = 160149$		
	OR		
	$S_{253} = \frac{253}{2} [2(3) + (253 - 1)5]$	✓substitution	
	_	✓answer	(2)
	$S_{253} = 160149$		
2.5	8 + 28 + 48 ++ 1248	✓ terms divisible by 4	
	a = 8, d = 20		
	$T_n = 20n - 12$	✓general term	
	20n-12=1248	✓equating	
	n = 63	$\checkmark$ value of $n$ .	(4)
		v value of n.	
			[12]



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3.1.1	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	✓ first differences	
	p-7-4	✓ equating $\checkmark p = 13$	(3)
	p-7-4=2 p=13		
3.1.2	2a = 2 $3a + b = 2$ $a + b + c = 1$		
	a = 1	$\checkmark_{a=1}$	
	b = -1 $c = 1$	$\checkmark_{b=-1}$	
		$\checkmark_{c=1}$	(4)
t	$T_n = n^2 - n + 1$	$\checkmark T_n = n^2 - n + 1$	(4)
3.1.3	2,4,6		
	a=2 $d=2$	✓general term	
	$T_n = 2n$	✓ equating	
	2n = 62	daming	
	n=31		
Į.	Term number 31 and term number 32	✓both 31 and 32	(3)
3.2	$\sum_{k=-2}^{m} 2.2^{k+2} < 2046$		
	$2.2^{0} + 2.2^{1} + 2.2^{2} + 2.2^{3} + + 2.2^{m+2} < 2046$	✓expansion	
	$ \begin{vmatrix} 2+4+8+16++ \\ a=2, & r=2, & n=m-(-2)+1 \end{vmatrix} $	✓ a, r and n	
	$= m+3$ $\therefore \frac{2(2^{m+3}-1)}{2-1} < 2046$	✓ substitution into	
		correct formulae	
	$2^{m+3} - 1 < 1023$ $2^{m+3} < 1024$		
	$2^{m+3} < 2^{10}$		
	m+3<10	√ m < 7	
	m+3<10 $m<7$	$\sqrt{m}=6$	(5)
	$\therefore m = 6$		
	= 0		[15]

4.1.1	-2+x	✓method	T
,,,,,	$\frac{-2+x}{2}=1$	$\checkmark x = 4$	
	x = 4		(2)
	B(4;0) Answer only full marks		
4.1.2	y = 1(x - (-2))(x - 4)	✓method	
	$y = x^2 - 2x - 8$	✓ correct equation	(2)
4.1.3	C(0;-8)	✓ C(0;-8)	
	$a = \frac{-8 - 0}{0 - 4}$	✓substitution	
	0-4	$\checkmark a = 2$	
	$a = \frac{8}{4} = 2$	$\checkmark a = 2$ $\checkmark q = -8$	
	g(x) = 2x - 8		(4)
	a=2		
	q = -8		
4.1.4	$y = (1)^2 - 2(1) - 8$	✓ value of y	
	y = -9	✓ <sub>(1;-9)</sub>	
ļ	(1;-9)	(1, 2)	(2)
4.2	$\frac{0+x}{2}=1$	$\checkmark x = 2$	
	$\begin{vmatrix} 2 \\ x = 2 \end{vmatrix}$		
	$\begin{array}{c} x = 2 \\ M(2; -8) \end{array}$	$\checkmark$ M(2;-8)	(2)
		✓ x < 0	
4.3	x < 0 or $x > 4$	$\sqrt{x} < 0$ $\sqrt{x} > 4$	(2)
			(2)
4.4	(3;9)	$\checkmark x = 3$	
		✓ y = 9	(2)
4.5	$TP = 2x - 8 - (x^2 - 2x - 8)$	✓top function –bottom	
	$TP = -x^2 + 4x$	function	
	TP' = -2x + 4	√derivative	
	-2x+4=0	✓ equating to 0	
	x=2	✓answer	
	$TP = -(2)^2 + 4(2)$		
	TP = 4		

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OR Completing the square $-(x^{2} - 4x)$ $= -[(x+2)^{2} - 4]$	✓ factoring a negative ✓ completing the square ✓ ✓ maximum 4 when	
$= -[(x+2)^2 - 4]$ $= -(x-2)^2 + 4$	x = 2	(4
There is a maximum of 4 when $x = 2$		
		] [

5.1	$x \ge 0$ <b>OR</b> $x \in (0, \infty)$	√domain	(1)
5.2	$y = -\log_3 x$		
	$y = \log_3 x^{-1}$	✓ interchanging $x$ and $y$	
	$y = -\log_3 x$ $y = \log_3 x^{-1}$ $x = \log_3 y^{-1}$	• Interestanging x and y	
	$3^x = y^{-1}$		
	$y = \left(\frac{1}{3}\right)^x \text{ or } y = 3^{-x}$		
	$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	✓ correct equation	(2)
			(2)
5.3	f is shifted 5 units vertically down	✓5 units	
		√down	(2)
5.4	$\log_3 x = 3$	$\checkmark x = 27$	
	$x=3^3$	$\checkmark -\log_3 x \ge -3$	
	x = 27	$\checkmark$ $\checkmark$ $0 < x \le 27$	
	$\therefore -\log_3 x \ge -3$		(4)
	$0 < x \le 27$		
			[9]
			[

6.1	q = 2	√answer	(1)
6.2	$y = x + p + 2$ $\therefore p + 2 - 1$ $p = 1$ $p = 1$ $y = x + 1$ $2 = x + 1$ $x = 1$ $x = 1$ $-4 = \frac{a}{0 - 1} + 2$ $a = 6$ $y = \frac{6}{x - 1} + 2$ $0 = \frac{6}{x - 1} + 2$ $x = -2$ $(-2; 0)$	$\sqrt{p+2}$ $\sqrt{p} = -1$ $\sqrt{\text{sub } (0;-4)}$ $\sqrt{a} = 6$	(4)
	y = 1 $-2$ $-4$ $x$	✓x intercept ✓y intercept ✓asymptote ✓shape of graph	(5) [10]

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7.1 $1 + i_{eff} = \left(1 + \frac{0.104}{12}\right)^{12}$ $i_{eff} = i_{o}   09103376 - 1$ $i = \frac{0,104}{12}$	
$i_{eff} = 1.509103376 - 1$ $i = \frac{1}{12}$	
√ 12	
: 4400104	İ
i <sub>eff</sub> 10.91%	
✓ substitution into the	
correct formula	
<b>√10,91%</b>	
V 10,9170	(4)
7.2 0.5 B = B(1 .04) ✓ subs in correct formula	
0.5P = P(1-i)	
$0.5 = (1-i)^4$ \text{divide by } P	
$\sqrt{40,5} = 1 - i$	
$i = 1 - \sqrt[4]{0.5}$	
$i = 0.1591$ $\checkmark i = 15.91\%$	
i = 15,91%	(4)
7.3.1 Loan with interest to date	
Coloring control formula	
= 80000(1 + 0,21)	
$= R207499,40$ $\checkmark R207499,40$	
	(2)
7.3.2 Repayments with interest to date $\checkmark 25000(1+0.21)^3$	
$= 25000(1+0.21)^3 + 55000(1+0.21)^1$ $\checkmark 55000(1+0.21)^1$	
$= R101673,20$ $\checkmark R101673,20$	
Outstanding balance =Loan with interest to date-	
Repayments with interest to date   √105826,20	
=R207499,40 - R10 673,20	
=R105826,20	(5)
	[15]

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0.1			
8.1	$f(x+h) = 4(x+h) - 2(x+h)^2$		
	$= 4x + 4h - 2x^2 - 4xh - 2h^2$	$\checkmark f(x+h)$	
	$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ $f'(x) = \lim_{h \to 0} \frac{4x + 4h - 2x^2 - 4xh - 2h^2 - (4x - 2x^2)}{h}$		
	$h \to 0$	✓ subt. into correct	
ž	$f'(x) = \lim_{h \to 0} \frac{4h - 4xh - 2h^2}{h}$	formula	
	$f'(x) = \lim_{h \to 0} \frac{h(4 - 4x - 2h)}{h}$	$\checkmark 4h-4xh-2h^2$	
	$ \begin{array}{c} h \to 0 \\ f'(x) = 4 - 4x \end{array} $	✓ factoring h	
		✓answer	(5)
8.2.1	$D_{x}[x^{4}-x^{-2}]$	✓ x <sup>-2</sup>	
4444	$=4x^3+2x^{-1}$	$\checkmark 4x^3 \qquad \checkmark 2x^{-1}$	(3)
8.2.2	$\sqrt{y} = \frac{x^2}{x} - \frac{4x}{x}$ $\sqrt{y} = x - 4$	✓ dividing by x	
		✓squaring	
	$y = (x-4)^{2}$ $y = x^{2} - 8x + 16$		
- June 1	$\frac{dy}{dx} = 2x - 8$	√√answer	(4)
8.3	m = 36	$\checkmark f'(x) = 36$	
	$f'(x) = 3x^2 - 12x$		
	$3x^2 - 12x = 36$		
	$3x^2 - 12x - 36 = 0$	✓factors	
	(x-6)(x+2) = 0	$\checkmark x = -2$	
	$x \neq 6 \text{ or } x = -2$		
	$f(-2) = (-2)^3 - 6(-2)^2$	$\checkmark y = -32$	
	f(-2) = -32		
	y - (-32) = 36(x - (-2))	√equation	(5)
	y = 36x + 40	1000	[17]
			[[-,]

9.1	C(2;0)	✓ answer	(1)
9.2	$0 = a(-1)^3 + c(-1) + 2$	✓ substitute (-1; 0)	
	a+c $=2$	✓ subject of formula	
	c=2-a		
	$0 = a(2)^3 + c(2) + 2$	✓substitute (2;0)	
	c + 4a = -1		
	2-a+4a=-1	✓ sub in second	
	3a = -3	equation	
	a = -1		
	c = 2 - (-1)		(4)
	c = 3		(4)
9.3	$f'(x) = -3x^2 + 3$	✓ equating derivative	
	$0 = -3x^2 + 3$	to zero	
	0 = -3(x-1)(x+1)	✓x-value	
	x=1		
	$f(1) = -(1)^3 + 3(1) + 2$		
	= 4	✓y-value	
	B(1:4)		(3)
9.4	1.1	✓method	
9.4	$\frac{-1+1}{2}=0$	Method	
	OR		
	$f'(x) = -3x^2 + 3$ $f''(x) = -6x$		
	-6x = 0	✓answer	(2)
	x = 0	(h < 0	
9.5	k < 0  or  k > 4	$\checkmark k < 0$	
		√k > 4	(2)
9.6	x < -1 or $x > 1$	✓ both critical values	
		$\checkmark x < -1$ or $x > 1$	(2)
			[14]
L		<u> </u>	

1011	D/D) 01 · 0.4	(D(D) 0 5	
10.1.1	P(B) = 0.1 + 0.4	$\checkmark$ P(B) = 0,5	
	looo	$A \cap B(A) \cup B(B) = 0.1$	
	$P(A) \times P(B) = 0,1$	$\checkmark P(A) \times P(B) = 0,1$	
	(x+0,1)(0,5)=0,1		
	x = 0,1	$\checkmark x = 0,1$	
	y = 1 - 0.1	( -00	(4)
	y = 0.9	$\checkmark y = 0.9$	(,)
10.1.2	P(A  or  B) = P(A) + P(B) - P(A  and  B)	√formulae	
	= 0,2+0,5-0,1	$\checkmark$ P(A) = 0,2	
	= 0,6	✓answer	(3)
10.2.1			
	oos / R VR		
	0,95 K VR	✓ 0,1×0,95	
	V		
	0,1 0,05 NR VNR		
		✓ 0,1×0,05	
	0,9 R NVR		
	NV<	✓ non violent	
	0,55 NR NVNR		
			(3)
10.2.2	POT 1 ( ) POT 1 ( )		
10.2.2	P(Violent crime rep) or P(Non -violent crime rep)	✓ 0,1×0,95	
	$= 0.1 \times 0.95 + 0.9 \times 0.45$	✓ 0,9×0,45	
	= 0,5	✓answer	(2)
			(3)
			[13]

**TOTAL MARKS: 150**