KWAZULU-NATAL PROVINCE

EDUCATION
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P2

PREPARATORY EXAMINATIONS

SEPTEMBER 2023

MARKS: 150

TIME: 3 hours

N.B. This question paper consists of 12 pages and 1 information sheet.
This paper has an Answer Booklet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of **11** questions.
2. Answer **ALL** the questions.
3. Number the answers correctly according to the numbering system used in this question paper.
4. Clearly show **ALL** calculations, diagrams, graphs, etc. that you have used in determining your answers.
5. Answers only will **NOT** necessarily be awarded full marks.
6. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
7. If necessary, round off answers to **TWO** decimal places, unless stated otherwise.
8. Diagrams are **NOT** necessarily drawn to scale.
9. An information sheet with formulae is included at the end of the question paper.
10. Write neatly and legibly.

QUESTION 1

Mr Siphokazi supplements his pension by mowing lawns for customers. He measures the areas (x) (in m^2) of 12 of his customers' lawns and the time (y) in minutes, it takes him to mow these lawns. He works 8 hours a day. He recorded the data.

Area (x) (m^2)	360	120	845	602	1 190	530	245	486	350	1 005	320	250
Time (y) (minutes)	50	28	130	75	120	95	55	70	48	110	55	60

1.1 Determine the equation of the least squares regression line. (3)

1.2 Calculate the value of r, the correlation coefficient for the data. (2)

1.3 Given that Mr Siphokazi charges a flat call out fee of R150, as well as R50 per half hour (or part thereof), estimate the charge for mowing a customer's lawn that has an area of 560 m^2.

(For example: 100 minutes would be taken as 2 hours). (3)

1.4 The local high school wants Mr Siphokazi to mow their rugby field which is rectangular, 100 meters long by 70 meters wide.

1.4.1 Use the regression equation found in 1.1 to calculate the time it would take to mow this area. (1)

1.4.2 Is it possible for him to complete this job in a day?
Give a reason for your answer. (1)

[10]

QUESTION 2

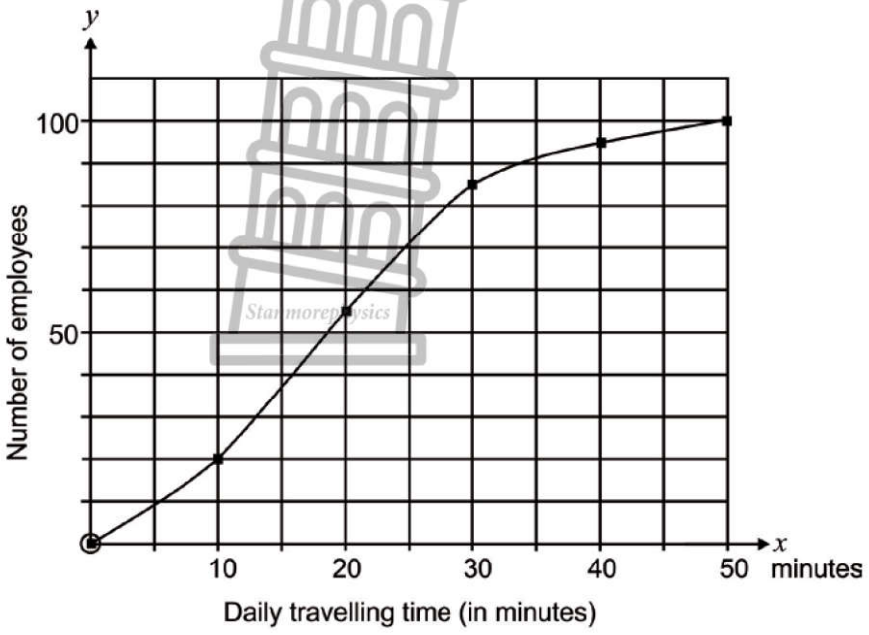
The following table gives the frequency distribution of the daily travelling time (in minutes) from home to work for the employees of a certain company.

Daily travelling time x (in minutes)	**Number of employees (f)**	**Midpoint of Interval**	
$0 \leq x < 10$	20		
$10 \leq x < 20$	35		
$20 \leq x < 30$	30		
$30 \leq x < 40$	10		
$40 \leq x < 50$	5		

2.1 Calculate the estimated mean travelling time. (3)

2.2 Write the modal class of the data. (2)

2.3 An ogive was drawn for the given data.
Construct a box-whisker plot for the data in the ANSWER BOOK. (3)



2.4 State whether the following statements are TRUE or FALSE.

2.4.1 The distribution of these travelling times is positively skewed. (1)

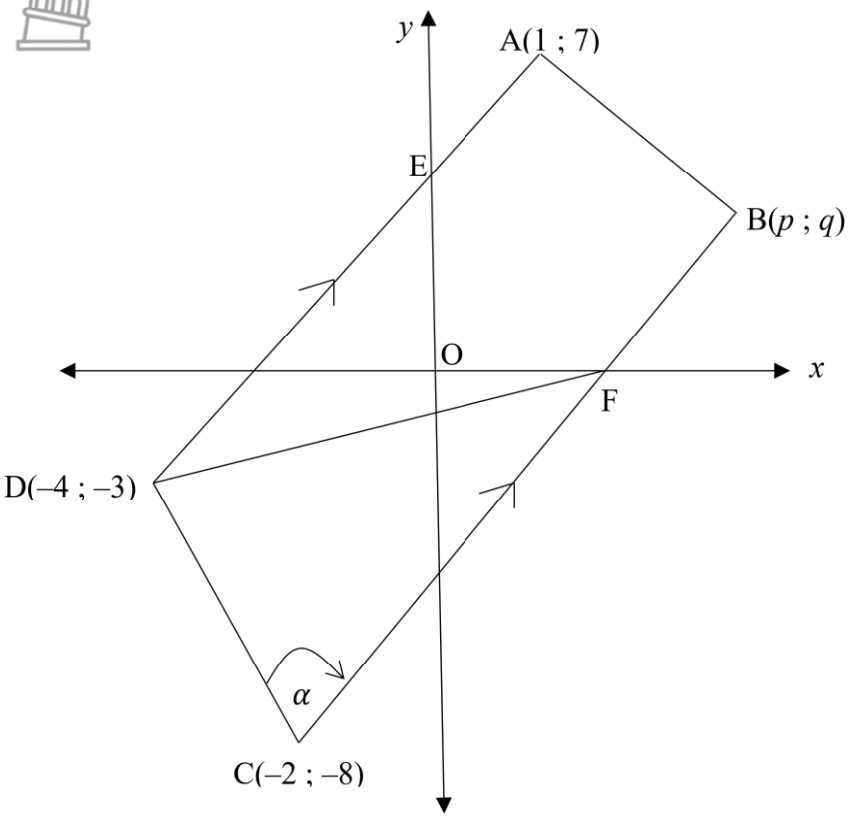
2.4.2 The inter-quartile range for the data is 25. (1)

2.4.3 35 employees take less than 20 minutes. (1)

[11]

QUESTION 3

Trapezium ABCD is drawn below with AD || BC is drawn. The coordinates of the vertices are A(1 ; 7), B(p ; q), C(–2 ; –8) and D(–4 ; –3). BC intersects the x – axis at F. $D\hat{C}B = \alpha$.
AD intersects the y – axis at E.



3.1 Calculate the gradient of AD. (2)

3.2 Determine the equation of BC in the form $y = mx + c$. (3)

3.3 Determine the coordinates of F. (2)

3.4 AMCD is a parallelogram, with M on BC. Determine the coordinates of M. (2)

3.5 Show that $\alpha = 48{,}37^{\circ}$. (4)

3.6 Calculate the area of ΔDCF. (4)

[17]

QUESTION 4

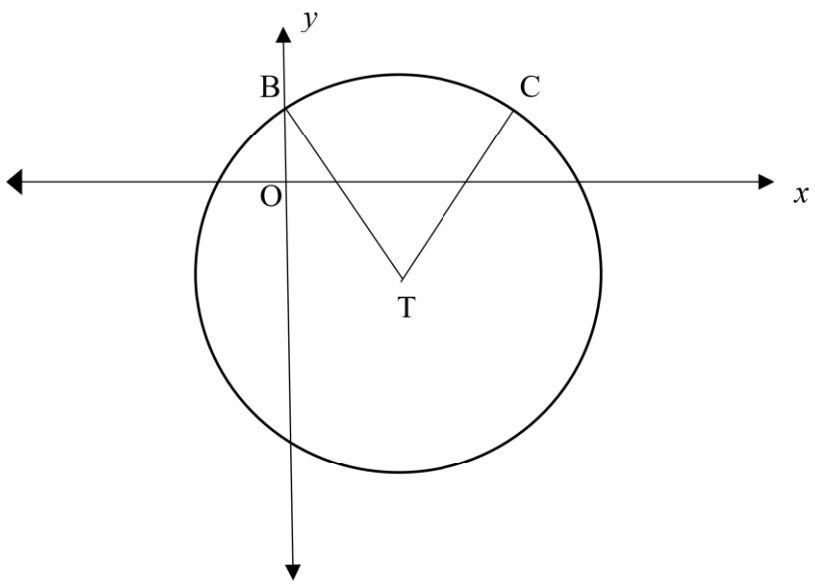
4.1 The equation of a circle is $x^2 + y^2 - 8x + 6y = 15$.

4.1.1 Show that P(2 ; –9) lies on the circle. (2)

4.1.2 Determine the equation of the tangent to the circle at point P(2 ; –9). (6)

4.1.3 A tangent is drawn from Q(–10 ; 12) to the circle. Calculate the length of the tangent. (4)

4.2 The circle, with centre T, and equation $(x - 3)^2 + (y + 2)^2 = 25$ is given below. B is the y – intercept of the circle.



4.2.1 Determine the coordinates of B. (4)

4.2.2 Write down the coordinates of C, if C is the reflection of B in the line $x = 3$. (2)

4.2.3 Another circle with centre M and equation $(x - 12)^2 + (y - 10)^2 = 100$ is given.

(a) Calculate the distance, TM, between the centres. (2)

(b) Do these circles touch or intersect each other? Justify your answer. (2)

[22]

QUESTION 5

5.1 If $\sin 38^\circ = p$, determine the value of the following, **without using a calculator:**

5.1.1 $\cos 218^\circ$ (3)

5.1.2 $\cos 14^\circ$ (3)

5.1.3 $\sin 26^\circ \cos 26^\circ$ (2)

5.2 Evaluate the following trigonometric expression **without using a calculator:**

$$\frac{2\sin 165^\circ \cos 195^\circ}{\cos 45^\circ \sin 15^\circ - \cos 15^\circ \sin 45^\circ}$$ (5)

5.3 Given: $K = \sqrt{3}\cos x + \sin x$.

5.3.1 Write K in the form of $t\sin(x+\theta)$. (3)

5.3.2 Hence, calculate the value of t and θ. (1)

5.3.3 Write down the maximum value of K. (1)

5.4 Prove the identity:

5.4.1 $\frac{2\tan\theta - \sin 2\theta}{2\sin^2\theta} = \tan\theta$ (6)

5.4.2 Hence, determine the values of θ, $\theta \in [180°; 360°]$ which will make the above identity undefined. (2)

[26]

QUESTION 6

6.1 Sketch the graphs of $f(x) = 2\sin x$ and $g(x) = \cos(x - 30^\circ)$ for $x \in [-180^\circ; 180^\circ]$ on the grid in the ANSWER BOOK. Indicate the intercepts with the axes and also the turning points. (6)

6.2 Use your graphs to answer the following questions:

6.2.1 Write down the period of g. (1)

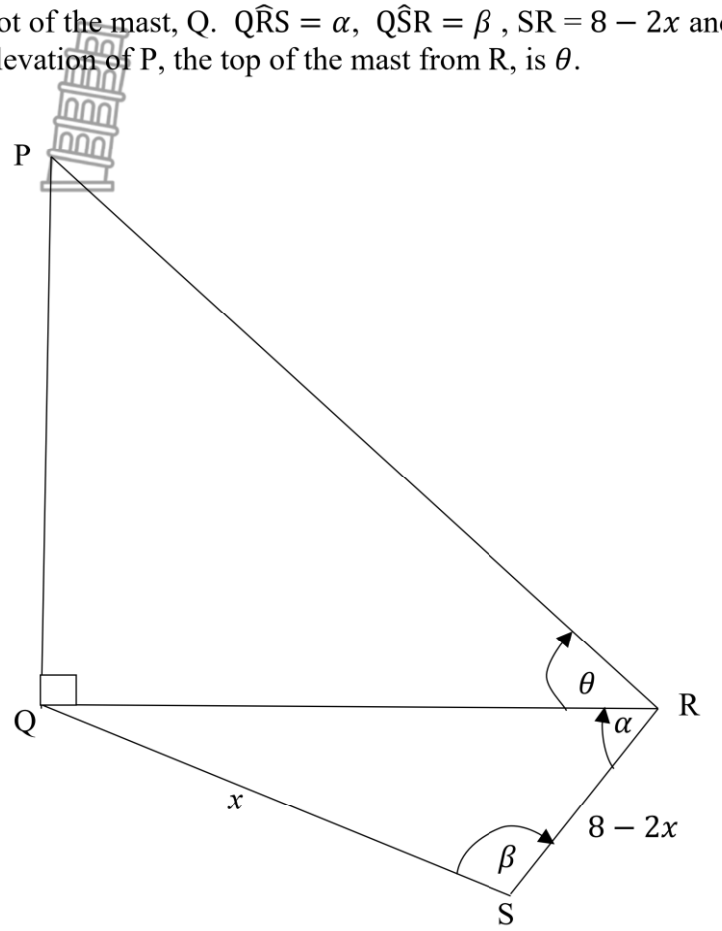
6.2.2 Determine the values of x for which $f(x) > g(x)$. (4)

6.2.3 Write down the values of x for which $f(x) = 1{,}5 + g(x)$. (2)

[13]

QUESTION 7

In the diagram below, PQ is a vertical mast. R and S are two points in the same horizontal plane as the foot of the mast, Q. $\widehat{QRS} = \alpha$, $\widehat{QSR} = \beta$, SR = $8 - 2x$ and SQ = x.
The angle of elevation of P, the top of the mast from R, is θ.



7.1 Express PQ in terms QR and a trigonometric ratio of θ. (1)

7.2 Show that: PQ $= \frac{x \sin\beta \tan\theta}{\sin\alpha}$ (4)

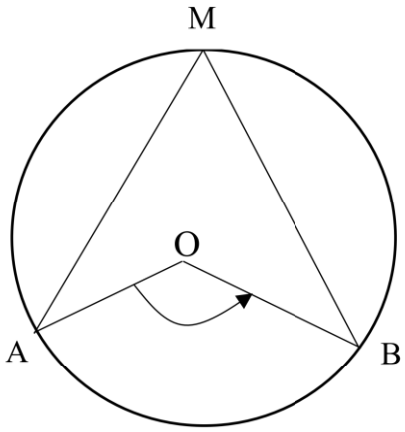
7.3 If $\beta = 60^\circ$, show that the area of $\Delta QSR = 2\sqrt{3}x - \frac{\sqrt{3}}{2}x^2$. (3)

7.4 Determine the value of x for which the area of ΔQSR will be at a maximum. (3)

[11]

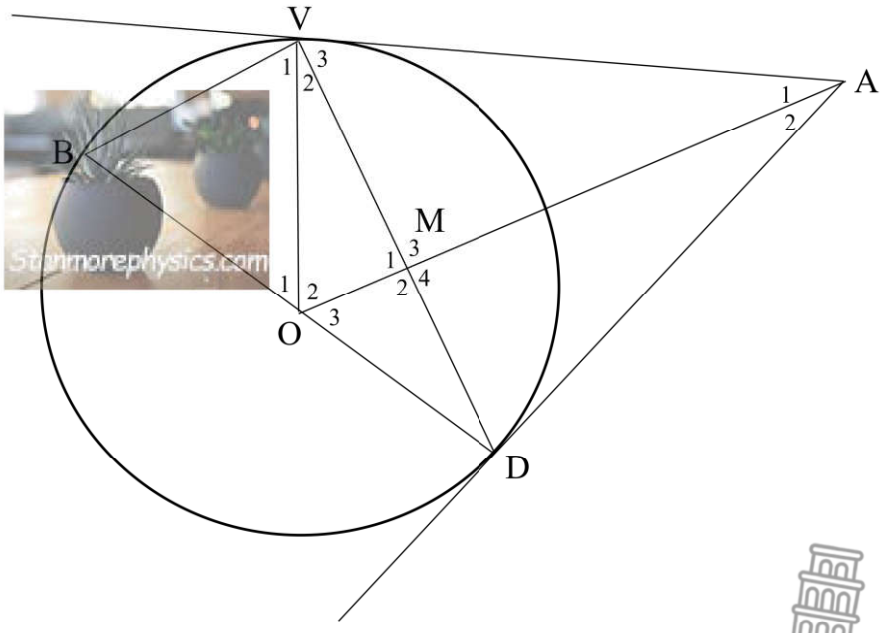
QUESTION 8

8.1 In the diagram O is the centre of the circle and M is a point on the circumference of the circle. Arc AB subtends $A\hat{O}B$ at the centre of the circle and $\hat{M}$ at the circumference of the circle.



Use the diagram to prove the theorem that states that $A\hat{O}B = 2\hat{M}$. (5)

8.2 From a point A outside the circle, center O, two tangents AD and AV are drawn.AO and VD intersect at M. BOD is a diameter of the circle. BV and VO are drawn.
$V\hat{A}D = 40°$



8.2.1 Prove that quadrilateral VODA is cyclic. (2)

8.2.2 Calculate the magnitude of $\hat{O}_1$. (2)

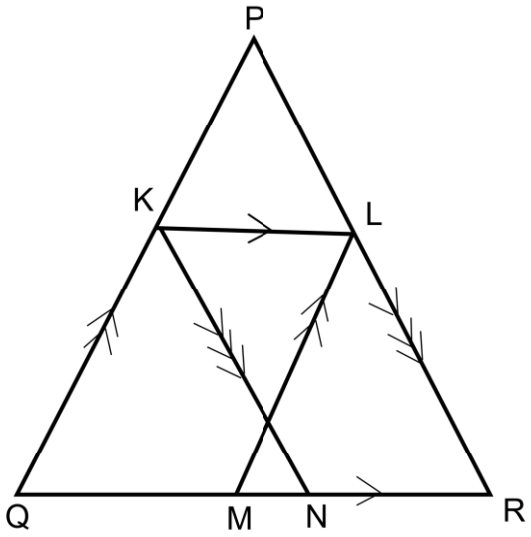
8.2.3 Prove that BV || OA. (5)

[14]

QUESTION 9

9.1 Complete the following statement: A line drawn parallel to one side of a triangle … (2)

9.2 In the figure, KL || QR. M and N are points on QR such that KN || PR and LM || PQ. PK = 3 units, PL = 4 units, LR = 6 units and MN = 1,8 units.



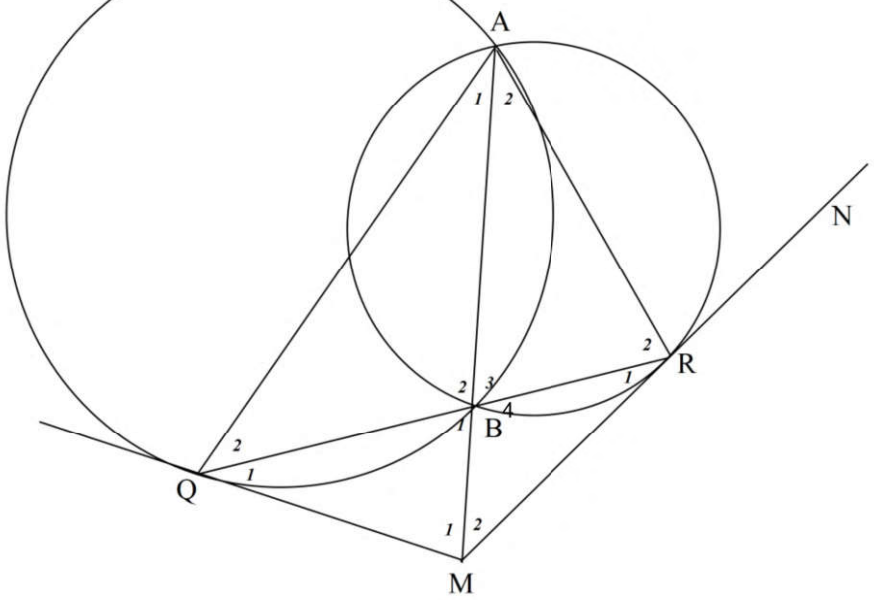
9.2.1 Calculate the length of KQ. (2)

9.2.2 Prove that QM = NR. (2)

[6]

QUESTION 10

In the figure, two circles intersect at A and B. AB produced to M bisects $Q\hat{A}R$. Tangents MQ and MR meet the circles at Q and R respectively. QBR is a straight line. AQ and AR are drawn.



Prove:

10.1 $\Delta MQA \;|||\; \Delta MBQ.$ (3)

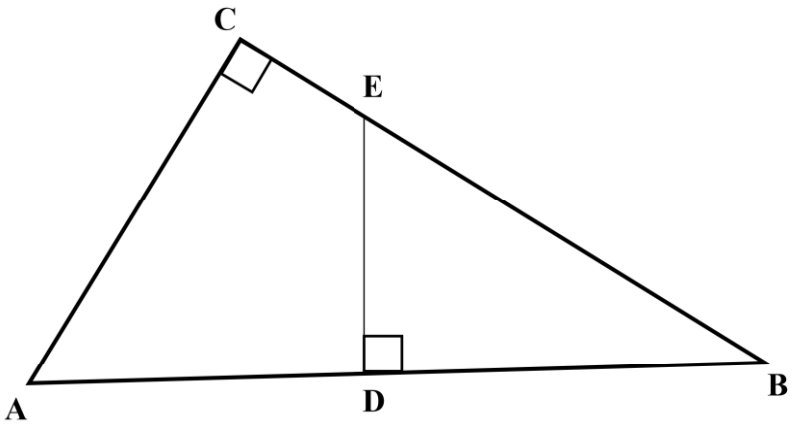
10..2 $MR^2 = AM.MB$ (5)

10.3 $\dfrac{MQ^2}{MR^2} = 1$ (4)

[12]

QUESTION 11

11 ΔABC is right angled at C. ED $\perp$ AB with E on CB and D on AB.
AC = 4,8 cm and AB = 8 cm. AD = DB.



11.1 Calculate BC, correct to 1 decimal digit. (2)

11.2 Complete: $\Delta BAC \,|||\, \ldots$. (1)

11.3 Hence, or otherwise calculate the area of ADEC. (5)

[8]

GRAND TOTAL: 150

INFORMATION SHEET: MATHEMATICS
INLIGTING BLADSY

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1+ni) \qquad A = P(1-ni) \qquad A = P(1-i)^n \qquad A = P(1+i)^n$$

$$T_n = a + (n-1)d \qquad S_n = \frac{n}{2}\left(2a + (n-1)d\right)$$

$$T_n = ar^{n-1} \qquad S_n = \frac{a\left(r^n - 1\right)}{r-1} \;; \quad r \neq 1 \qquad S_\infty = \frac{a}{1-r} \;; \; -1 < r < 1$$

$$F = \frac{x\left[(1+i)^n - 1\right]}{i} \qquad P = \frac{x[1-(1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad \mathrm{M}\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

$$(x-a)^2 + (y-b)^2 = r^2$$

$$In\ \Delta ABC: \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \qquad a^2 = b^2 + c^2 - 2bc.\cos A \qquad area\ \Delta ABC = \frac{1}{2}ab.\sin C$$

$$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta \qquad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta \qquad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$\bar{x} = \frac{\sum f.x}{n} \qquad \sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$$

$$\mathrm{P}(A) = \frac{n(\mathrm{A})}{n(\mathrm{S})} \qquad \mathrm{P(A\ or\ B)} = \mathrm{P(A)} + \mathrm{P(B)} - \mathrm{P(A\ and\ B)}$$

$$\hat{y} = a + bx \qquad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$$

FINAL

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GRADE 12

MATHEMATICS P2

MARKING GUIDELINE

PREPARATORY EXAMINATIONS

SEPTEMBER 2023

MARKS: 150

This marking guideline consists of 14 pages.

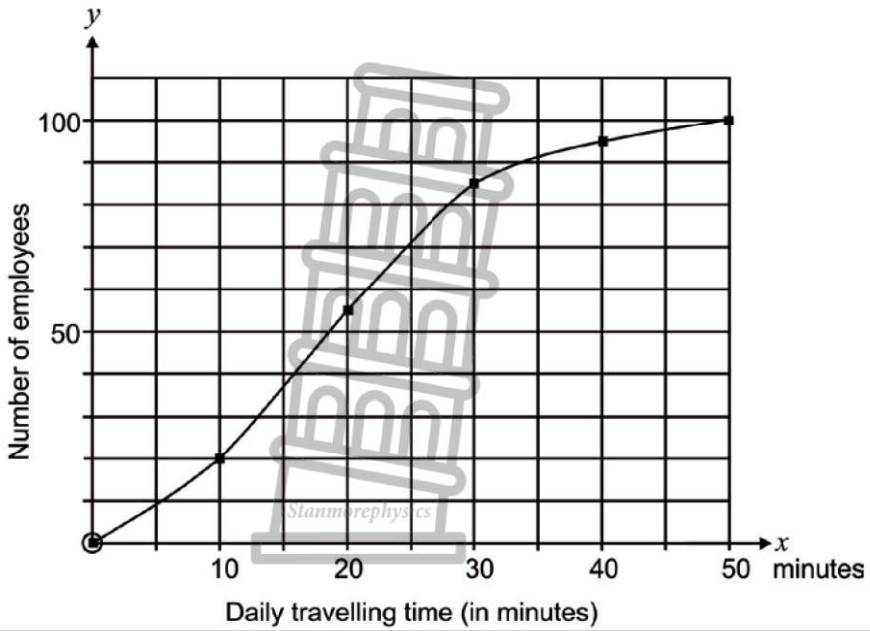
NB: CA APPLIES TO ALL SUB-QUESTIONS IN THIS MARKING GUIDELINE.

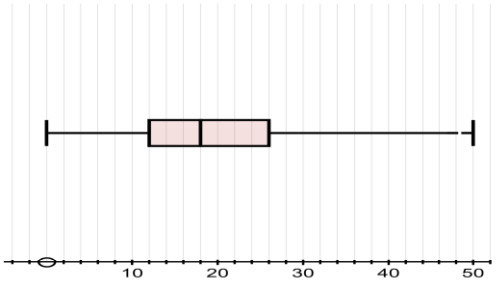
QUESTION 1

1.1	$a = 34{,}90$ $b = 0{,}08$ $y = 34{,}90 + 0{,}08x$ **Answer only: Full Marks**	✓A Value of a ✓A Value of b ✓CA Equation	(3)
1.2	$r = 0{,}88$	✓A✓A Answer	(2)
1.3	$y = 34{,}90 + 0{,}08x$ $y = 34{,}90 + 0{,}08(560)$ $y = 79.7$ minutes = 1.33 hours Total cost = R150 + R150 = R300	✓CA Substitution ✓CA 79,7 minutes ✓CA Answer	(3)
1.4.1	Area = 100 x 70 = 7000 square meters $y = 34{,}90 + 0{,}08(7000)$ $y = 594.9$ minutes = 9.92 hours	✓CA Calculation	(1)
1.4.2	No. The time taken will exceed his daily 8 hour working hours.	✓CA Justification	(1)
			[10]

QUESTION 2

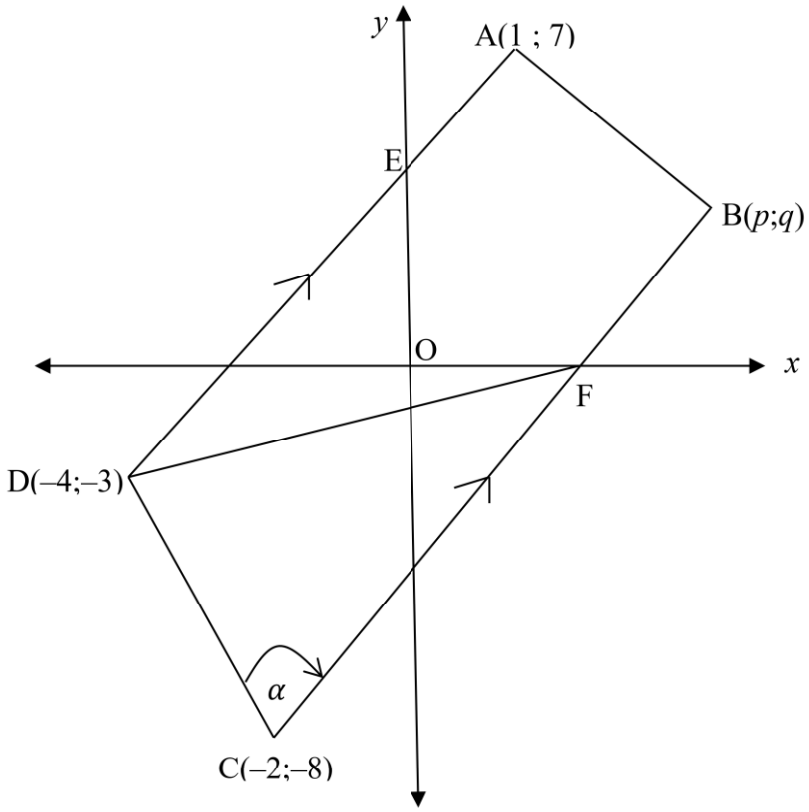
Daily travelling time x(in minutes)	Number of employees (f)	Midpoint of Interval (x)	$f \cdot x$
$0 \leq x < 10$	20	5	100
$10 \leq x < 20$	35	15	525
$20 \leq x < 30$	30	25	750
$30 \leq x < 40$	10	35	350
$40 \leq x < 50$	5	45	225
Total	100		1950



2.1	Estimated Mean $= \frac{1950}{100} = 19{,}5$ **Answer only: Full Marks**	✓A 1950 ✓A 100 ✓CA Answer	(3)
2.2	$10 \leq x < 20$	✓A✓A Answer	(2)
2.3	See Diagram	✓A Minimum and Maximum value ✓A 1st and 3rd Quartiles ✓A 2nd Quartile	(3)
	 Five number summary: 0 ; 12 ; 18 ; 26 ; 50 **Accept:** ± 1 deviation on quartiles		

2.4.1	True	✓A Answer	(1)
2.4.2	False	✓A Answer	(1)
2.4.3	True	✓A Answer	(1)
			[11]

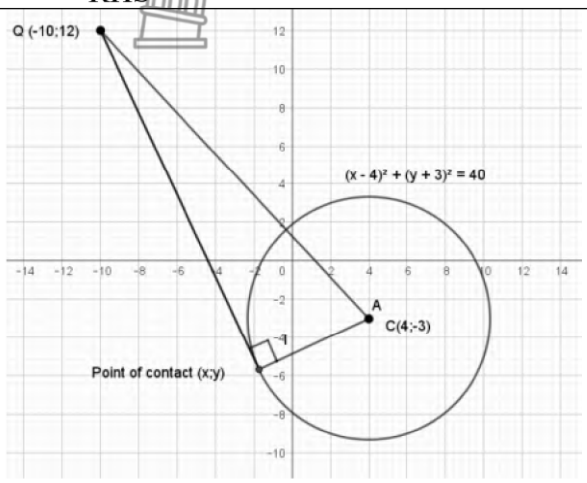
QUESTION 3



3.1	$m_{AD} = \frac{7+3}{1+4} = \frac{10}{5} = 2$	✓A Substitution of points A and D ✓CA Answer	(2)
3.2	$m_{AD} = m_{BC} = 2$ …..(AD || BC) $y = mx + c$ $-8 = 2(-2) + c$ $-4 = c$ $y = 2x - 4$	✓CA Gradient of BC ✓CA Substitution of point C and gradient ✓CA Answer	(3)

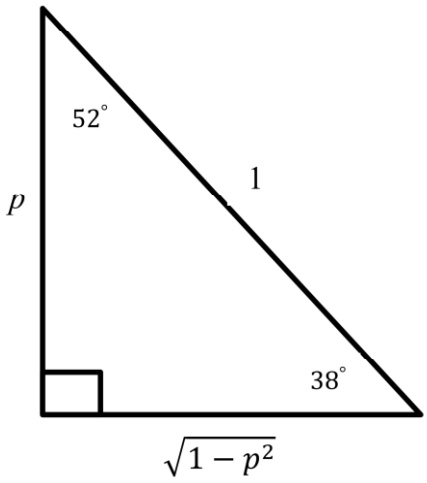
3.3	Let $y = 0$: $0 = 2x - 4$ $x = 2$ $F(2\,;0)$	 ✓CA $0 = 2x - 4$ ✓CA x – value	 (2)
3.4	M(3 ; 2)	✓A x – value ✓A y – value	(2)
3.5	$m_{CD} = \frac{-3+8}{-4+2} = \frac{5}{-2}$ Inclination of CD: $\tan\theta_1 = 180^\circ - 68{,}2^\circ = 111{,}8^\circ$ Inclination of CF: $\tan\theta_2 = 2$ $\theta_2 = 63{,}43^\circ$ Therefore $\alpha = 111{,}8^\circ - 63{,}43^\circ$ $= 48{,}37^\circ$ **OR** DF = $\sqrt{(2+4)2+(0+3)2}$ = $\sqrt{45}$ DC = $\sqrt{29}$ CF = $\sqrt{80}$ $(\sqrt{45})^2 = (\sqrt{29})^2 + (\sqrt{80})^2 - 2\sqrt{29}\sqrt{80}\cos\alpha$ $\alpha = 48{,}37^\circ$	✓A Gradient of CD ✓A Inclination of CD ✓A Inclination of CF ✓A subtraction of the angles ✓A Distance of DF ✓A Distance of DC ✓A Distance of CF ✓A subst. into cosine rule	(4)
3.6	DC = $\sqrt{(-3+8)^2+(-4+2)^2} = \sqrt{29}$ FC = $\sqrt{(0+8)^2+(2+2)^2} = \sqrt{80}$ Area of ΔDCF = $\frac{1}{2}(\sqrt{29})(\sqrt{80})\sin 48{,}37^\circ$ = 18 square units.	✓A Length of CD ✓CA Length of FC ✓CA Substitution into area formula ✓CA Answer	(4)
			[17]

QUESTION 4

4.1.1	$x^2 + y^2 - 8x + 6y = 15$ LHS = $(2)^2 + (-9)^2 - 8(2) + 6(-9)$ $= 4 + 81 - 16 - 54$ = 15 = RHS	✓A Subst. of point ✓A Simplification	(2)
4.1.2	 $(x-4)^2 + (y+3)^2 = 40$ Centre: C(4 ; –3) P(2 ; –9) $m_{Radius} = \frac{-3+9}{4-2} = \frac{6}{2} = 3$ $m_{Tangent} = -\frac{1}{3}$ Equation of Tangent: $y = mx + c$ $-9 = -\frac{1}{3}(2) + c$ $-\frac{25}{3} = c$ $y = -\frac{1}{3}x - \frac{25}{3}$	✓A writing the equation as $(x-4)^2 + (y+3)^2 = 40$ ✓CA Centre of circle ✓CA Gradient of radius ✓CA Gradient of tangent ✓CA Substitution ✓CA Answer	(6)
4.1.3	r^2: $(x-4)^2 + (y+3)^2 = 40$ $r^2 = 40$ $(distance\ Q\ to\ the\ centre)^2 = (-10-4)^2 + (12+3)^2$ $= 421$ (Length of tangent)$^2 = 421 - 40 = 381$ Length of tangent = $\sqrt{381}$	✓CA Calculation of r^2 ✓CA distance calculation ✓CA Tangent calculation ✓CA Answer	(4)

4.2.1	$(x-3)^2+(y+2)^2=25$ Let $x=0$: $(0-3)^2+(y+2)^2=25$ $(y+2)^2=16$ $y+2=\pm 4$ $y=-6$ or $y=2$ B(0 ; 2)	✓A Letting $x=0$ ✓A Simplification ✓CA y – values ✓CA Answer	(4)
4.2.2	C(6 ; 2)	✓CA x – value ✓CA y – value	(2)
4.2.3 (a)	T(3 ; –2) and M(12 ; 10) $TM^2 = (12-3)^2 + (10+2)^2 = 225$ TM = 15 units	✓A Coordinates of M ✓CA Answer	(2)
(b)	Radius, center T = 5 units and Radius, center M = 10 units Sum of radii = 15 units Circles **touch** . TM= Sum of radii	✓CA Sum of radii ✓CA Justification	(2)
			[22]

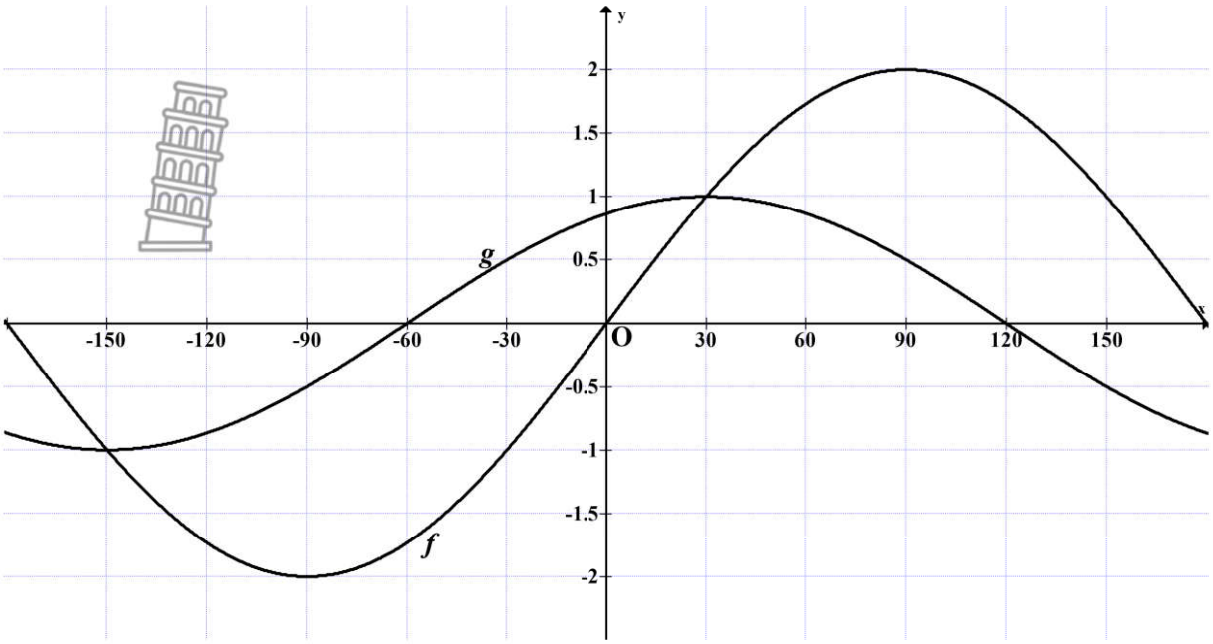
QUESTION 5

5.1			
5.1.1	$\cos 218^\circ$ $= -\cos 38^\circ$ $= -\frac{\sqrt{1-p^2}}{1}$	✓A Calculation of $\sqrt{1-p^2}$ ✓A Reduction ✓CA Answer	(3)

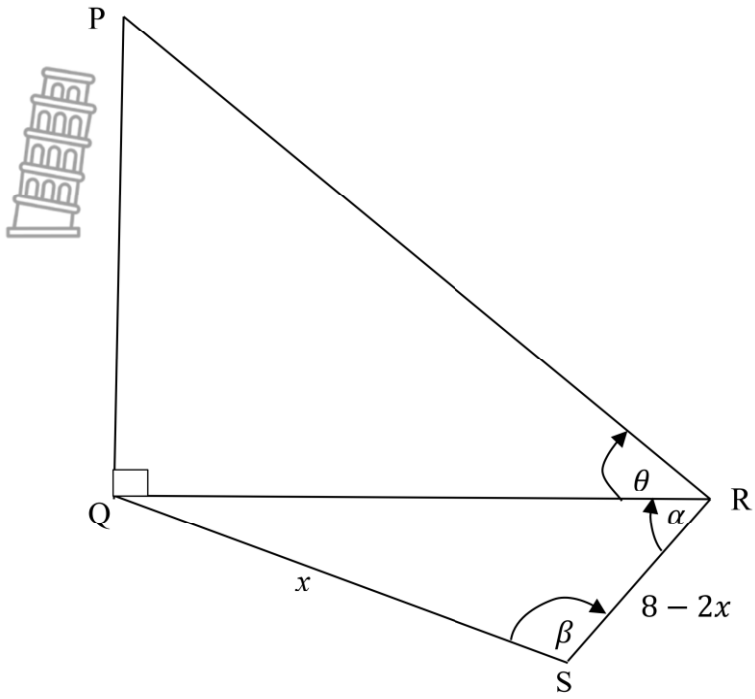
5.1.2	$\cos 14^\circ$ $= \cos(52^\circ - 38^\circ)$ $= \cos 52^\circ \cos 38^\circ + \sin 52^\circ \sin 38^\circ$ $= \left(\frac{p}{1}\right)\left(\frac{\sqrt{1-p^2}}{1}\right) + \left(\frac{\sqrt{1-p^2}}{1}\right)\left(\frac{p}{1}\right)$ $= 2p\sqrt{1-p^2}$ **OR** $\cos 24° = \sin 76°$ $= \sin 2(38°)$ $= 2\sin 38° \cos 38°$ $= 2p\sqrt{1-p^2}$	✓A Writing as difference ✓A Expansion ✓CA Answer ✓ $\sin 76°$ ✓double angle ✓answer	(3)
5.1.3	$\sin 26^\circ \cos 26^\circ$ $= \frac{1}{2}\sin 52^\circ$ $= \frac{1}{2}\sqrt{1-p^2}$	✓A Double angle ✓CA Answer	(2)
5.2	$\frac{2\sin 165^\circ \cos 195^\circ}{\cos 45^\circ \sin 15^\circ - \cos 15^\circ \sin 45^\circ}$ $= \frac{2\sin 15^\circ.(-\cos 15^\circ)}{\cos 45^\circ \sin 15^\circ - \cos 15^\circ \sin 45^\circ}$ $= \frac{-2\sin 30^\circ}{\sin(15^\circ - 45^\circ)}$ $= \frac{-2\sin 30^\circ}{\sin(-30^\circ)}$ $= \frac{-2\sin 30^\circ}{-\sin 30^\circ}$ $= 2$	✓A $-\cos 15^\circ$ ✓A $-2\sin 30^\circ$ ✓A $\sin(15^\circ - 45^\circ)$ ✓A $-\sin 30^\circ$ ✓CA Answer	(5)
5.3.1	$K = \sqrt{3}\cos x + \sin x$ $K = 2\left(\frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x\right)$ $K = 2(\sin 60^\circ \cos x + \cos 60^\circ \sin x)$ K = $2\sin(60^\circ + x)$	✓A $2\left(\frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x\right)$ ✓A $\sin 60^\circ$ and $\cos 60^\circ$ ✓A $2\sin(60^\circ + x)$	(3)

5.3.2	$t = 2$ and $\theta = 60^\circ$	✓CA t – value and $\theta = 60^\circ$	(1)
5.3.3	2	✓CA Answer	(1)
5.4.1	LHS $= \dfrac{2\tan\theta - \sin 2\theta}{2\sin^2\theta}$ $= \dfrac{2\left(\dfrac{\sin\theta}{\cos\theta}\right) - 2\sin\theta\cos\theta}{2\sin^2\theta} \times \dfrac{\cos\theta}{\cos\theta}$ $= \dfrac{2\sin\theta - 2\sin\theta\cos^2\theta}{2\sin^2\theta\cos\theta}$ $= \dfrac{2\sin\theta\left(1-\cos^2\theta\right)}{2\sin^2\theta\cos\theta}$ $= \dfrac{2\sin\theta \,.\, \sin^2\theta}{2\sin^2\theta \,.\, \cos\theta}$ $= \dfrac{\sin\theta}{\cos\theta}$ $= \tan\theta$ = LHS	✓A $\dfrac{\sin\theta}{\cos\theta}$ ✓A $2\sin\theta\cos\theta$ ✓A simplification ✓A factorizing ✓A $1-\cos^2\theta = \sin^2\theta$ ✓A simplified to $\dfrac{\sin\theta}{\cos\theta}$	(6)
5.4.2	$2\sin^2\theta = 0$ $\sin\theta = 0$ $\therefore \theta = 180^\circ$ and 360° $\theta = 270^\circ$	✓ 180° and 360° ✓ 270°	(2)
			[26]

QUESTION 6

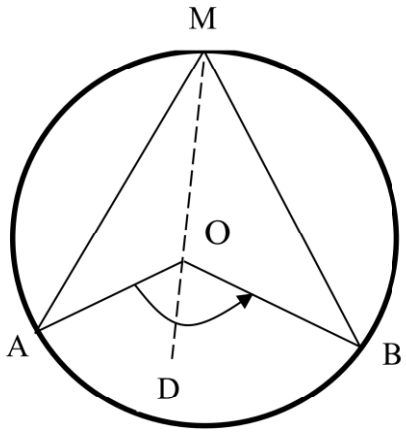
6.1			
	Graph of f: 1A mark for x – intercepts 1A marks for minimum and maximum points 1A mark for shape Graph of g: 1A mark for end points 1A mark for x – intercepts 1A mark for y – intercept		(6)
6.2.1	360°	✓A Answer	(1)
6.2.2	$x \in [-180^\circ ; -150^\circ) \cup (30^\circ; 180^\circ]$	$[-180^\circ ; -150^\circ)$ ✓A : Notation ✓CA : values $(30^\circ; 180^\circ]$ ✓A : Notation ✓CA : values	(4)
6.2.3	$f(x) = 1.5 + g(x)$ $f(x) - g(x) = 1.5$ $x = 90^\circ$ or $x = 150^\circ$	✓A $f(x) - g(x) = 1.5$ ✓A Answer	(2)
			[13]

QUESTION 7

			
7.1	$\frac{PQ}{QR} = \tan\theta$ $PQ = QR\tan\theta$	✓A Answer	(1)
7.2	$\frac{QR}{\sin\hat{S}} = \frac{QS}{\sin QRS}$ $\frac{QR}{\sin\beta} = \frac{x}{\sin\alpha}$ $QR = \frac{x\sin\beta}{\sin\alpha}$ $PQ = \frac{x\sin\beta\tan\theta}{\sin\alpha}$	✓A Sine rule formula ✓A Subs. Sine rule ✓A Making QR a subj. of the formula ✓A Subst. of QR	(4)
7.3	Area of $\Delta QSR = \frac{1}{2}(x)(8-2x)\sin 60^\circ$ $= \frac{1}{2}(x)(8-2x)\left(\frac{\sqrt{3}}{2}\right)$ $= \frac{\sqrt{3}}{4}(x)(8-2x)$ $= \sqrt{3}(x)\left(2-\frac{1}{2}x\right)$ $= 2\sqrt{3}x - \frac{\sqrt{3}}{2}x^2$	✓A Subst. into Area rule ✓A $\frac{\sqrt{3}}{2}$ ✓ A Simplifying	(3)

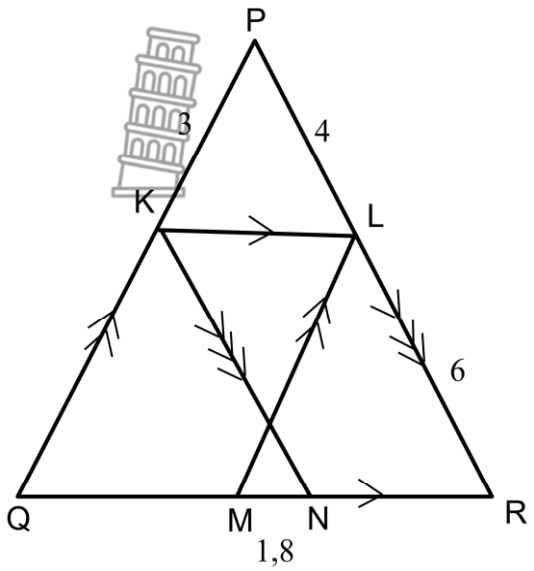
7.4	For Max Area: $x = -\frac{b}{2a}$ $x = -\frac{(2\sqrt{3})}{2\left(-\frac{\sqrt{3}}{2}\right)}$ $x = 2$	✓A Formula ✓ A Substitution into formula ✓ CA Answer	(3)
			[11]

QUESTION 8

8.1	 Constr: Join MO and produce to D. $A\hat{O}D = O\hat{A}M + A\hat{M}O$ … (Ext. o∠f Δ) $B\hat{O}D = O\hat{B}M + B\hat{M}O$ … (Ext. ∠ of Δ) But $O\hat{A}M = A\hat{M}O$ and $O\hat{B}M = B\hat{M}O$ …(Radii =) $\therefore A\hat{O}D + B\hat{O}D = 2A\hat{M}O + 2B\hat{M}O$ $A\hat{O}B = 2(A\hat{M}O + B\hat{M}O)$ $A\hat{O}B = 2\hat{M}$ **NOTE No construction : No marks**	✓A Construction ✓A S/R ✓A S ✓A S ✓A S	(5)
8.2.1	$O\hat{V}A = O\hat{D}A = 90^\circ$ …. (Radius ⊥ Tangent) VODA is a cyclic quad. (Converse of opposite angles of quad. Supplementary)	✓A S/R ✓A R	(2)
8.2.2	$\hat{O}_1 = 40^\circ$ …(Exterior angle of cyclic quad = int. opp. Angle)	✓S ✓R	(2)

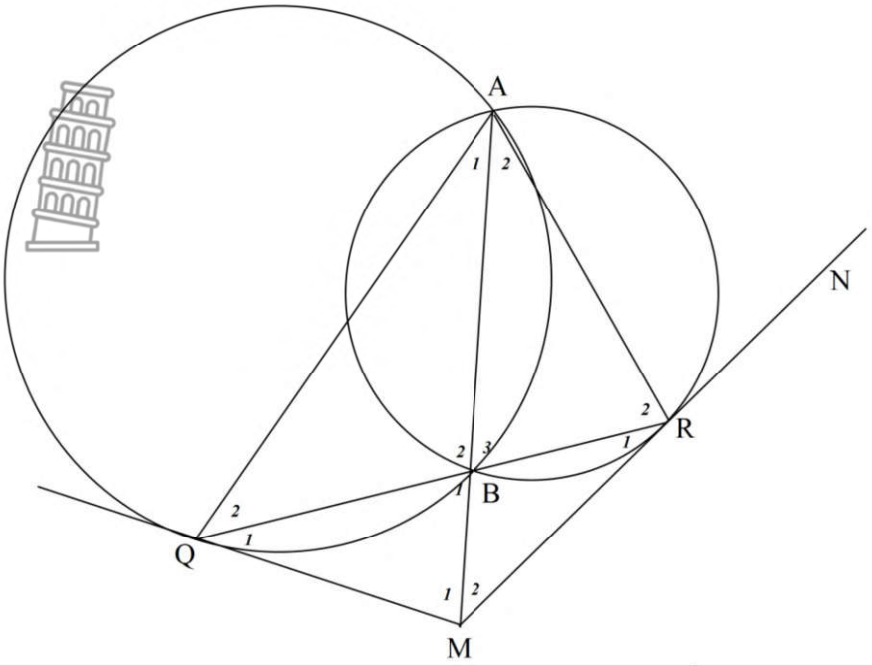
8.2.3	$\hat{V}_1 = \frac{180^\circ - 40^\circ}{2} = 70^\circ$ …..(sum of $\angle s$ of Δ; radii)	✓ S/R	
	$A\hat{D}M = \frac{180^\circ - 40^\circ}{2} = 70^\circ$ …..(sum of $\angle s$ of Δ; Tangents drawn from a common point A)	✓ S/R	
	$A\hat{D}M = \hat{O}_2 = 70^\circ$ … (Angles subtended by common chord AV)	✓S ✓R	
	BV ‖ OA ……(Converse of Alt $\angle s$ or Alt $\angle s$ are =)	✓ R	(5)
	OR		
	In ΔOVA and ΔODA OV=OD.....(radii) OA=OA.....(common) AV=AD.....(tans from the same point) ΔOVA ≡ ΔODA.....(SSS) $A_1=A_2$.....(≡ Δs) =20° $A_2=V_2$.....(∠s in the same segment) =20° $V_1=90^\circ-20^\circ=70^\circ$ $O_2=180^\circ-[OVA+A_1]$ $=180^\circ-[90^\circ+20^\circ]$ $=70^\circ$ $\therefore V_1 = O_2$.....(both = 70°) ∴ BV‖OA..... (alt∠s are = or conv.alt∠s)	✓A ΔOVA ≡ ΔODA.....(SSS) ✓A S/R ✓A $V_1=70^\circ$ ✓A $O_2=70^\circ$ ✓A R	
			[14]

QUESTION 9

9.1	Divides the other two sides, proportionally.	✓S <u>divides</u> the other two sides ✓S proportionally	(2)
9.2			

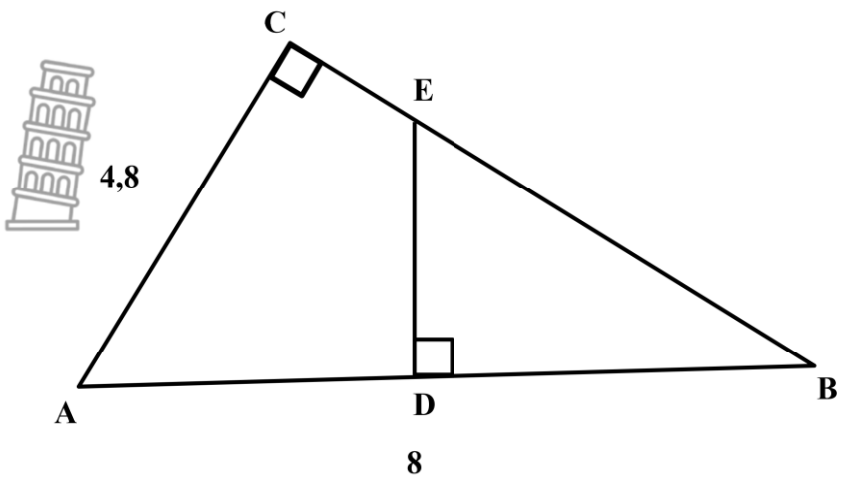
9.2.1	$\frac{KQ}{3}=\frac{6}{4}$ ……..(Prop. Thm; KL || QR) KQ = 4,5 units	✓ S/R ✓ Answer	(2)
9.2.2	KL = QM ….(Opposite sides of $\|\|^m$ QKLM are equal) KL = NR …. (Opposite sides of $\|\|^m$ NKLR are equal) Therefore QM = NR (both = KL) **OR** $\frac{QN}{QR}=\frac{QK}{KP}$ Prop Thm, KN||PR $\frac{4,5}{3}=\frac{3}{2}$ $\frac{MR}{QM}=\frac{RL}{PL}$ Prop Thm, ML||QP $=\frac{6}{4}=\frac{3}{2}$ $\therefore \frac{QN}{NR}=\frac{MR}{QM}$ $\frac{QM+1,8}{NR}=\frac{NR+1,8}{QM}$ $\therefore QM(QM+1,8)=MR(NR+1,8)$ $\therefore QM=NR$	✓ S/R ✓ S/R ✓ $\frac{QN}{QR}=\frac{QK}{KP}$ Prop Thm, KN||PR OR $\frac{MR}{QM}=\frac{RL}{PL}$ Prop Thm, ML||QP ✓ $\frac{QM+1,8}{NR}=\frac{NR+1,8}{QM}$	(2) **[16]**

QUESTION 10



10.1	In Δ's MQA and MBQ $\hat{M}_1 = \hat{M}_1$ …….(Common) $\hat{A}_1 = \hat{Q}_1$ …….(Tan-Chord Theorem) $A\hat{Q}M = \hat{B}_1$ …….(Remaining $\angle s$ of Δ's) ΔMQA ||| ΔMBQ ……(AAA)	✓ S/R ✓ S/R ✓ R	(3)
10.2	In Δ's MAR and MRB $\hat{M}_2 = \hat{M}_2$ …….(Common) $\hat{A}_2 = \hat{R}_1$ …….(Tan-Chord Thm) $A\hat{R}M = R\hat{B}M$ …….(Remaining $\angle s$ of Δ's) ΔMAR ||| ΔMRB ……(AAA) $\frac{MA}{MR} = \frac{MR}{MB}$ ……(Δ's similar) $MR^2 = AM.MB$	✓ S ✓ S/R ✓ R ✓S A✓R	(5)
10.3	$\frac{MQ}{MB} = \frac{MA}{MQ}$ ……(from 10.1) $MQ^2 = MB.MA = \ldots\ldots(1)$ Also $MR^2 = AM.MB$ Now $\frac{MQ^2}{MR^2} = \frac{MB.MA}{AM.MB}$ $\therefore \frac{MQ^2}{MR^2} = 1$	✓ $\frac{MQ}{MB} = \frac{MA}{MQ}$ ✓ $MQ^2 = MB.MA$ ✓✓ $\frac{MQ^2}{MR^2} = \frac{MB.MA}{AM.MB}$	(4)
			[12]

QUESTION 11

			
11.1	$BC^2 = 8^2 - 4{,}8^2$ ……..(TOP) $BC = 6{,}4$ cm	✓ S ✓ S	(2)
11.2	ΔBED	✓ S	(1)
11.3	$\frac{BA}{BE} = \frac{AC}{ED} = \frac{BC}{BD}$ ΔBAC ||| ΔBED $\frac{8}{BE} = \frac{4{,}8}{ED} = \frac{6{,}4}{4}$ $ED = \frac{4{,}8 \times 4}{6{,}4} = 3\ cm$ Area of ΔEDB $= \frac{1}{2}(4)(3) = 6\ cm^2$ Area of ΔABC $= \frac{1}{2}(4{,}8)(6{,}4) = 15{,}6\ cm^2$ Therefore, Area of ADEC $= 15{,}6 - 6 = 9{,}6\ cm^2$	✓A S ✓CA Value of ED ✓CA Area of ΔEDB ✓CA Area of ΔABC ✓CA Area of ADEC	(5)
			[8]

GRAND TOTAL: 150