education

Department:
Education
North West Provincial Government
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P1

SEPTEMBER 2023
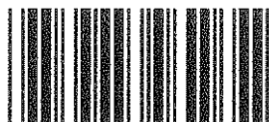
MARKS: 150

TIME: 3 hours

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X10

This question paper consists of 9 pages and 1 information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1.   This question paper consists of 11 questions.

2.   Answer ALL the questions.

3.   Number the answers correctly according to the numbering system used in this question paper.

4.   Clearly show ALL calculations, diagrams, graphs, etc. that you have used in determining your answers.

5.   Answers only will NOT necessarily be awarded full marks.

6.   You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

7.   If necessary, round off answers to TWO decimal places, unless stated otherwise.

8.   Diagrams are NOT necessarily drawn to scale.

9.   An information sheet with formulae is included at the end of the question paper.

10.   Write neatly and legibly.

QUESTION 1

1.1   Solve for x:

1.1.1 $x^2 = 5x$ (3)

1.1.2 $x^2 - 2x - 13 = 0$ (correct to TWO decimal places.)   (3)

1.1.3 $(x-2)(1-x) \leq 0$ (3)

1.1.4 $2\sqrt{2x-1} = x - 11$ (5)

1.2   Solve for x and y simultaneously:

$3x - y = 4$ and $x^2 + xy = 24$ (6)

1.3   Let $S = \left(1 + \frac{1}{7}\right)\left(1 + \frac{1}{8}\right)\left(1 + \frac{1}{9}\right)\left(1 + \frac{1}{10}\right)....\left(1 + \frac{1}{m}\right)$ where $m \varepsilon N$ and $7 < m < 30$.

Calculate all the value(s) of m for which S will be a natural number. (Show
ALL calculations)   (5)
                                                                                           [25]

QUESTION 2

2.1   Given a quadratic number pattern: $-120 ; -99 ; -80 ; -63 ; ...$

2.1.1   Write down the next TWO terms of the pattern.   (2)

2.1.2   Determine the nth term of the number pattern in the form
$T_n = an^2 + bn + c$.   (4)

2.1.3   What value must be added to T_n for the sequence to have only one
value of n for which $T_n = 0$?   (4)

2.2   Given a finite arithmetic series: $9 + 14 + 19 + ... + 124$

2.2.1   Determine the general term of this series in the form $T_n = dn + c$.   (2)

2.2.2   Write the series in sigma notation.   (3)

2.3   Given the arithmetic series: $2^x + 2^{x+1} + 3.2^x + 2^{x+2} + ...$

Calculate the value of x if the sum of the first 30 terms add up to 3 720.   (4)
                                                                                           [19]

QUESTION 3

3.1   Given: 5; 10; 20; ... a geometric sequence.

    3.1.1   Determine the nth term.   (1)

    3.1.2   Calculate the sum of the first 18 terms.   (2)

3.2   The first and second term of a geometric series is given as $(2x-4)$ and
    $(4x^2-16)$ respectively. Determine the value(s) of x for which the
    series will converge.   (4)

3.3   A convergent geometric series has a first term of 2 and $r = \dfrac{1}{\sqrt{2}}$.

    Calculate $\dfrac{S_\infty}{S_2}$.   (3)

                                                                 [10]

QUESTION 4

Given: $f(x) = \dfrac{2}{x-1} - 2$

4.1   Write down the equations of the asymptotes of f.   (2)

4.2   Draw the graph of f. Clearly label ALL the intercepts with the axes and the
    asymptotes on your graph.   (4)

4.3   Determine the equation of the line of symmetry of f for $m < 0$.   (2)

4.4   For which values of x will $f(x) \le -4$?   (2)

                                                                 [10]

QUESTION 5

The graph of $g(x) = a\left(\dfrac{1}{5}\right)^x - 5$ passes through A$(-2 ; -4)$.

5.1   Show that $a = \dfrac{1}{25}$.   (2)

5.2   Determine the coordinates of the x-intercept of g.   (3)

5.3   Given: $h(x) = \left(\dfrac{1}{5}\right)^x$.

    5.3.1   Determine the equation of $h^{-1}(x)$, the inverse of h, in the form
             $y = ...$ (2)

    5.3.2   Describe the translation from g to h.   (2)
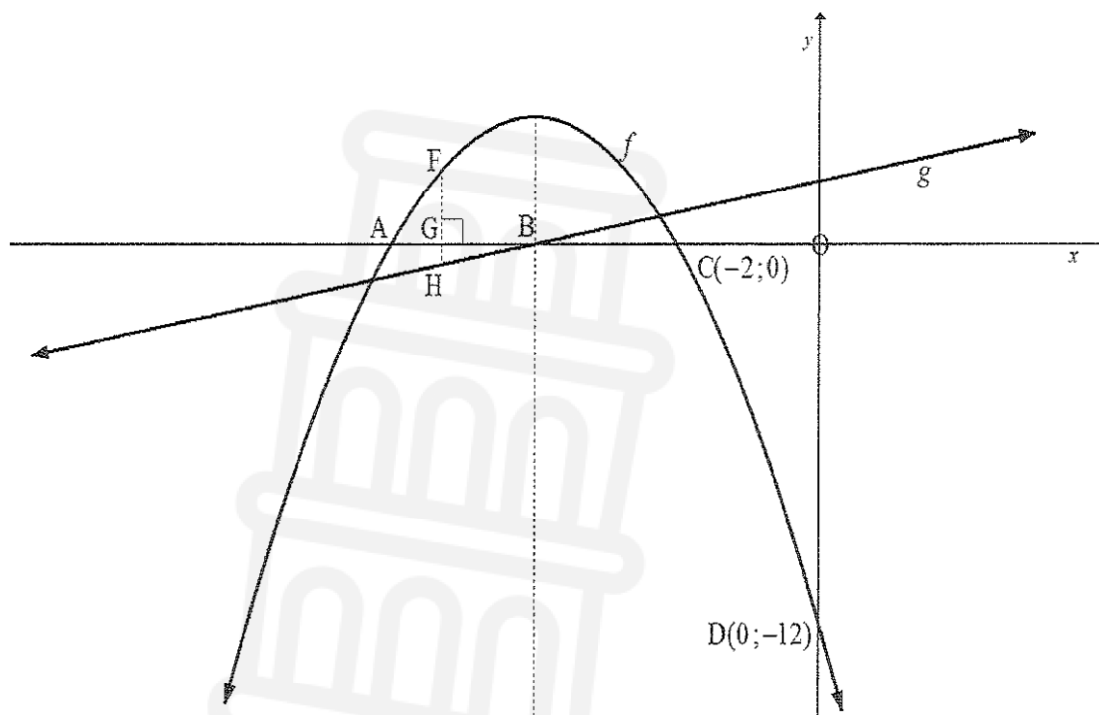
                                                                 [9]

QUESTION 6

The sketch below represents the following graphs: $f(x) = ax^2 + bx + c$ and

$g(x) = \dfrac{1}{2}x + 2$

- The line of symmetry of f cuts the x-axis at B, which is also the x-intercept of g.
- C$(-2\,;\,0)$ is an x-intercept of f.
- D$(0\,;\,-12)$ is the y-intercept of f.

6.1   Calculate the coordinates of B.   (2)

6.2   Determine the equation of f in the form $f(x) = ax^2 + bx + c$.   (4)

6.3   If $f(x) = -x^2 - 8x - 12$, determine the coordinates of G if FH, the vertical length between the points of intersection of f and g, is a maximum.   (4)

6.4   For which values of x will $x.f'(x) > 0$?   (2)

[12]

QUESTION 7

7.1 Convert an effective interest rate of 11,3% p.a. to its equivalent nominal rate per annum, compounded quarterly. (3)

7.2 Lisa opened a savings account and deposited R10 000 immediately into the account. The account paid interest at 5,3% per annum, compounded monthly. She started making additional monthly deposits of R500 into the account three months after the account was opened. Her last monthly deposit of R500 was made 5 years after the account was opened.

How much money was in the account 5 years after the account was opened? (5)

7.3 Sam wants to buy a house and takes out a loan of R860 000. He can only afford to pay R7 200 per month starting one month after the loan is granted. The interest rate is compounded monthly at 9,5% p.a.

7.3.1 Calculate the number of payments that Sam will make to repay the loan. (4)

7.3.2 How much will Sam pay in the last month to settle the loan? (5)

[17]

QUESTION 8

8.1 Determine $f'(x)$ from first principles if $f(x) = 1 - x^2$ (5)

8.2 Determine:

8.2.1 $D_x\left[3x^2 - \dfrac{2}{x}\right]$ (3)

8.2.2 $\dfrac{dy}{dx}$ if $y = \sqrt{x}\left(\sqrt[3]{x} - 5x\right)$ (4)

[12]

QUESTION 9

The graph of $f(x) = ax^3 + bx^2$ has stationary points at $P\left(2 \; ; -\dfrac{1}{3}\right)$ and $O(0 \; ; 0)$
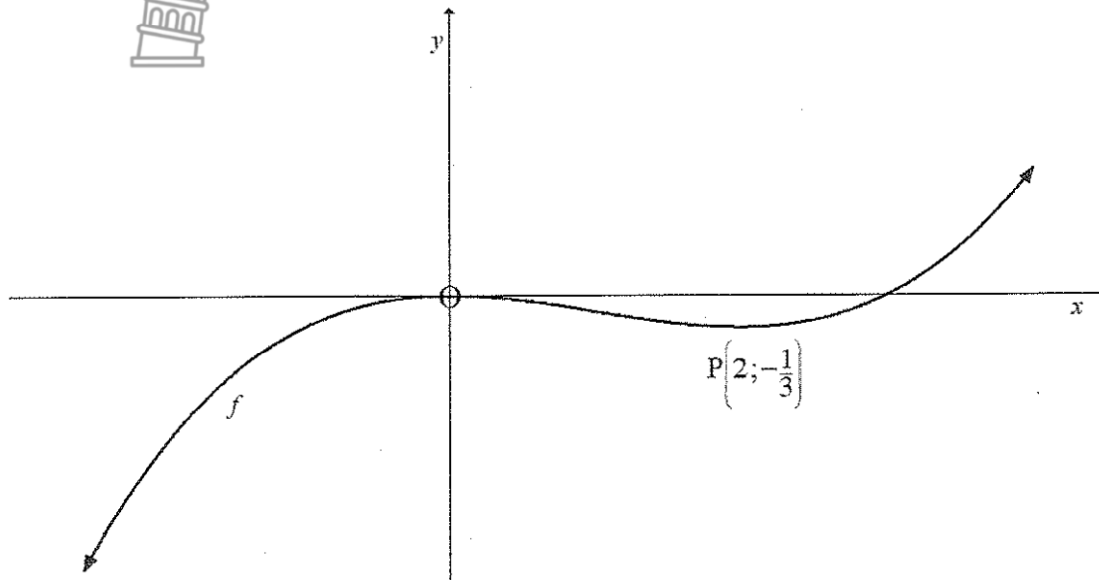
9.1   Prove that $a = \dfrac{1}{12}$ and $b = -\dfrac{1}{4}$.   (4)

9.2   For which values of x will f be concave down?   (3)

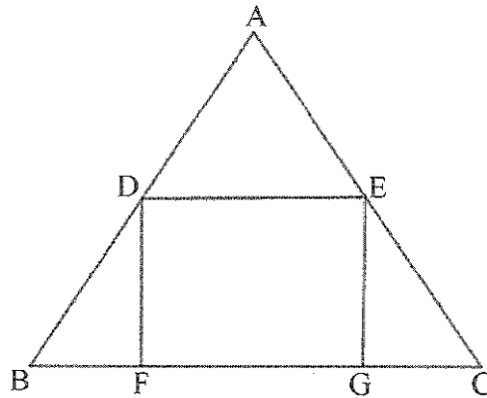9.3   Determine the equation of the tangent to f at $x = -2$ in the form $T_n = mx + n$.   (4)

9.4   Use the graph to determine the values of k for which
$\dfrac{1}{12}x^3 - \dfrac{1}{4}x^2 - k = 0$ will have three real roots.

(2)
[13]

QUESTION 10

In the sketch below, $\triangle ABC$ is an equilateral triangle with side AB = m units.
DEGF is a rectangle with BF = GC = x units.

10.1	Prove that the area of the rectangle DEGF is $\sqrt{3}x(m-2x)$	(3)

10.2	Determine, in terms of m, the maximum area of the rectangle.	(4)
		[7]

QUESTION 11

11.1   Given: $P(A) = 0,3$, $P(B) = 0,43$ and $P(C) = 0,18$.

The events A and B are independent.
The events A and C are mutually exclusive.

11.1.1	What is the probability that A and C will happen simultaneously?	(1)

11.1.2	What is the probability that at least one of A or B will take place?	(3)

11.2   A group of 250 Grade 12 learners participated in a survey. They were asked if they used the social media app TikTok. The results are represented in the table below:

|	Use TikTok	Don't use TikTok	Total
Girls	105	25	130
Boys	68	52	120
Total	173	77	250

11.2.1	A name is drawn randomly out of the group. What is the probability that the name drawn will be a girl and using TikTok?	(1)

11.2.2	Are the events being a girl and using TikTok independent? Motivate your answer by doing appropriate calculations.	(4)

11.3   A group consisting of 5 boys, 6 girls and a teacher goes to the movies. They buy 12 tickets in a row next to one another. One of the seats is next to an aisle.

11.3.1   In how many different ways can they be seated?   (1)

11.3.2   To avoid disruptive behaviour, the teacher considers to place the following restriction on how they may sit:
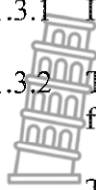
The teacher at the end next to the aisle, and the children in any order as long as Peter and John do not sit together.

Calculate in how many different ways they can be seated.   (3)

11.3.3   Duncan wants to sit next to Suzie. If the whole group of twelve are seated at random, what is the probability that the two of them will sit together?   (3)

[16]

TOTAL: **150**

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni) \qquad A = P(1 - ni) \qquad A = P(1 - i)^n \qquad A = P(1 + i)^n$$

$$T_n = a + (n-1)d \qquad S_n = \frac{n}{2}\left(2a + (n-1)d\right)$$

$$T_n = ar^{n-1} \qquad S_n = \frac{a(r^n - 1)}{r - 1} \; ; \qquad r \neq 1 \qquad S_\infty = \frac{a}{1 - r} \; ; \; -1 < r < 1$$

$$F = \frac{x\left[(1 + i)^n - 1\right]}{i} \qquad P = \frac{x[1 - (1 + i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad M\left(\frac{x_1 + x_2}{2} \; ; \; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

$$In \; \Delta ABC: \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \qquad a^2 = b^2 + c^2 - 2bc.\cos A$$

$$area \; \Delta ABC = \frac{1}{2} ab.\sin C$$

$$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta \qquad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta \qquad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$\bar{x} = \frac{\sum x}{n} \qquad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)} \qquad P(A \; or \; B) = P(A) + P(B) - P(A \; and \; B)$$

$$\hat{y} = a + bx \qquad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$$
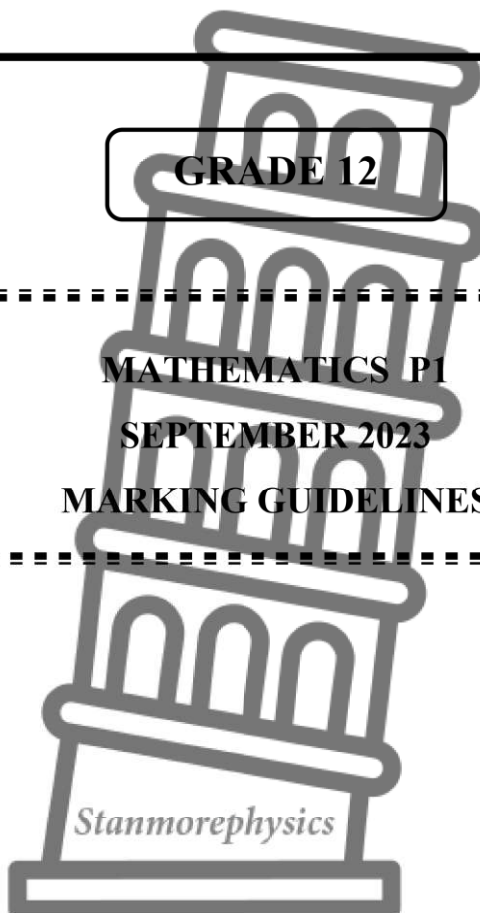
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GRADE 12

MATHEMATICS P1

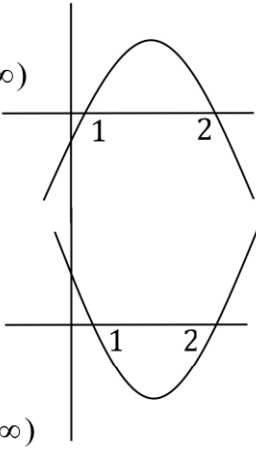SEPTEMBER 2023

MARKING GUIDELINES

MARKS: 150

These marking guidelines consists of 16 pages.
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NOTE:
- If a candidate answers a question TWICE, only mark the FIRST attempt.
- Consistent Accuracy applies in ALL aspects of the marking memorandum.

QUESTION 1

1.1.1	$x^2 = 5x$ $x^2 - 5x = 0$ $x(x-5) = 0$ $x = 0$ or $x = 5$	✓ standard form ✓ factors ✓ x-values (3)
1.1.2	$x^2 - 2x - 13 = 0$ $x = \dfrac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-13)}}{2(1)}$ $x = 4,74$ or $x = -2,74$	✓ substitution into the correct formula ✓ $x = 4,74$ ✓ $x = -2,74$ (3)
1.1.3	$(x-2)(1-x) \le 0$ $x \le 1$ or $x \ge 2$ / $x\varepsilon(-\infty;1]\,\text{or}\,[2;\infty)$ **OR** $(x-2)(x-1) \ge 0$ $x \le 1$ or $x \ge 2$ / $x\varepsilon(-\infty;1]\,\text{or}\,[2;\infty)$	✓ critical values ✓ $x \le 1$ ✓ $x \ge 2$ (3)
1.1.4	$2\sqrt{2x-1} = x - 11$ $4(2x-1) = x^2 - 22x + 121$ $8x - 4 = x^2 - 22x + 121$ $x^2 - 30x + 125 = 0$ $(x-25)(x-5) = 0$ $x = 25$ or $x \ne 5$ **OR** $2\sqrt{2x-1} = x - 11$ $\sqrt{2x-1} = \dfrac{x-11}{2}$ $2x - 1 = \dfrac{x^2 - 22x + 121}{4}$ $8x - 4 = x^2 - 22x + 121$ $x^2 - 30x + 125 = 0$ $(x-25)(x-5) = 0$ $x = 25$ or $x \ne 5$	✓ square on both sides ✓ standard form ✓ factors/formula ✓ both answers ✓ reject $x = 5$ (5) ✓ square on both sides ✓ standard form ✓ factors/formula ✓ both answers ✓ reject $x = 5$ (5)

1.2	$3x - y = 4$	✔ $y = 3x - 4$
	$\qquad y = 3x - 4$	
	$x^2 + xy = 24$	
	$x^2 + x(3x - 4) = 24$	✔ substitution
	$x^2 + 3x^2 - 4x = 24$	✔ standard form
	$4x^2 - 4x - 24 = 0$	
	$x^2 - x - 6 = 0$	
	$(x-3)(x+2) = 0$	✔ factors/formula
	$x = 3$ or $x = -2$	✔ both x-values
	$y = 3(3) - 4$ or $y = 3(-2) - 4$	
	$\quad y = 5 \qquad\qquad y = -10$	✔ both y-values
		(6)
	OR	
	$3x = y + 4$	
	$x = \dfrac{y + 4}{3}$	✔ $x = \dfrac{y+4}{3}$
	$x^2 + xy = 24$	
	$\left(\dfrac{y+4}{3}\right)^2 + y\left(\dfrac{y+4}{3}\right) = 24$	✔ substitution
	$\dfrac{y^2 + 8y + 16}{9} + \dfrac{y^2 + 4y}{3} = 24$	
	$y^2 + 8y + 16 + 3y^2 + 12y = 216$	
	$4y^2 + 20y - 200 = 0$	✔ standard form
	$y^2 + 5y - 50 = 0$	
	$(y+10)(y-5) = 0$	✔ factors/formula
	$y = -10$ or $y = 5$	✔ both y-values
	$x = \dfrac{(-10)+4}{3}$ or $x = \dfrac{(5)+4}{3}$	
	$\quad x = -2 \qquad\qquad x = 3$	✔ both x-values (6)
1.3	$S = \left(1 + \dfrac{1}{7}\right)\left(1 + \dfrac{1}{8}\right)\left(1 + \dfrac{1}{9}\right)\ldots\left(1 + \dfrac{1}{m}\right)$	✔ $\left(\dfrac{8}{7}\right)\left(\dfrac{9}{8}\right)\left(\dfrac{10}{9}\right)\ldots$
	$S = \left(\dfrac{8}{7}\right)\left(\dfrac{9}{8}\right)\left(\dfrac{10}{9}\right)\ldots\left(\dfrac{m+1}{m}\right)$	✔ $\dfrac{m+1}{m}$
	$S = \dfrac{m+1}{7}$	✔ $S = \dfrac{m+1}{7}$
	For S to be a natural number, $m+1$ must be a multiple of 7	
	$m + 1 = 14$ or $m + 1 = 21$ or $m + 1 = 28$	✔ multiples 14,21,28
	$m = 13 \qquad\qquad m = 20 \qquad\qquad m = 27$	✔ answer (5)
		[25]

QUESTION 2

2.1.1	$-120 \quad -99 \quad -80 \quad -63$ $\quad\quad 21 \quad\quad 19 \quad\quad 17$ $\quad\quad\quad -2 \quad -2 \quad -2$ The next TWO terms: $\quad -48 \ ; \ -35$	✓ -48 ✓ -35 (2)
2.1.2	$2a = -2 \quad\quad 3a+b = 21 \quad a+b+c = -120$ $a = -1 \quad\quad 3(-1)+b = 21 \quad (-1)+(24)+c = -120$ $\quad\quad\quad\quad\quad\quad\quad b = 24 \quad\quad\quad c = -143$ $T_n = -n^2 + 24n - 143$	✓ 2nd diff $= -2$ ✓ $a = -1$ ✓ $b = 24$ ✓ $c = -143$ (4)
2.1.3	$T'n = -2n + 24 = 0$ $\quad\quad\quad\quad\quad n = 12$ $T_n = -(12)^2 + 24(12) - 143$ $T_n = 1$ A maximum of 1 Add -1 to Tn **OR** $n = \dfrac{-(24)}{2(-1)} = 12$ $T_n = -(12)^2 + 24(12) - 143$ $T_n = 1$ A maximum of 1 Add -1 to Tn **OR** $T_n = -n^2 + 24n - 143 + k$ $\Delta = (24)^2 - 4(-1)(k - 143)$ $= 576 + 4k - 572$ $= 4k + 4$ but $\Delta = 0$ $4k + 4 = 0$ $k = -1$	✓ method ✓ $n = 12$ ✓ maximum 1 ✓ -1 (4) ✓ method ✓ $n = 12$ ✓ maximum 1 ✓ -1 (4) ✓ method ✓ $\Delta = 4k + 4$ ✓ $\Delta = 0$ ✓ -1 (4)
2.2.1	$9 + 14 + 19 + ... + 124$ $T_n = (9) + (n-1)(5)$ Answer only: full marks $T_n = 5n + 4$	✓ substitution into the correct formula ✓ $T_n = 5n + 4$ (2)
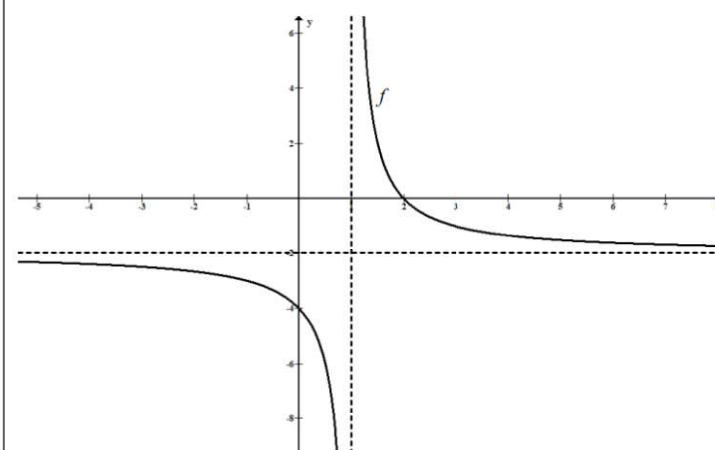
2.2.2	$T_n = 5n + 4 = 124$ $5n = 120$ $n = 24$ $\displaystyle\sum_{n=1}^{24} (5n + 4)$	✓ $= 124$ ✓ $n = 24$ ✓ answer (3)
2.3	$2^x + 2.2^x + 3.2^x + 4.2^x ....$ $a = 2^x$ $d = 2^x$ $S_{30} = \dfrac{30}{2}\left[2(2^x) + 29(2^x)\right]$ $3720 = 15(31.2^x)$ $248 = 31.2^x$ $8 = 2^x$ $2^3 = 2^x$ $x = 3$	✓ $a = 2^x$ and $d = 2^x$ ✓ substitution into the correct formula ✓ $2^x = 8$ ✓ $x = 3$ (4)
		[19]

QUESTION 3

3.1.1	$5; 10; 20; ....$ $T_n = a.r^{n-1}$ $T_n = (5)(2)^{n-1}$	✓ answer (1)
3.1.2	$S_n = \dfrac{a\left(r^n - 1\right)}{r - 1}$ $S_{18} = \dfrac{5\left[(2)^{18} - 1\right]}{2 - 1}$ $S_{18} = 1310\,715$	✓ substitution into the correct formula ✓ answer (2)
3.2	$r = \dfrac{(2x+4)(2x-4)}{2x-4} = 2x + 4$ Converge : $-1 < r < 1$ $-1 < 2x + 4 < 1$ $-5 < 2x < -3$ $-\dfrac{5}{2} < x < -\dfrac{3}{2}$	✓ $r = 2x + 4$ ✓ $-1 < r < 1$ ✓ substitution ✓ answer (4)

3.3	$\dfrac{S_\infty}{S_2} = \dfrac{\dfrac{2}{1-\dfrac{1}{\sqrt{2}}}}{\dfrac{2\left(1-\left(\dfrac{1}{\sqrt{2}}\right)^2\right)}{1-\dfrac{1}{\sqrt{2}}}}$ $=\dfrac{1}{1-\dfrac{1}{2}}$ $=2$	✓ S_∞ ✓ S_2 ✓ dividing ✓ answer (3)
		[10]

QUESTION 4

4.1	$x=1$ $y=-2$	✓ $x=1$ ✓ $y=-2$ (2)
4.2		✓ x-asymptote and y-asymptote ✓ x-intercept ✓ y-intercept ✓ form-decreasing (4)
4.3	$y=-x+c$ OR $y-y_1=-1(x-x_1)$ $-2=-(1)+c$ $y=-x+1-2$ $-1=c$ $y=-x-1$ $\quad y=-x-1$ Answer only: full marks	✓ method ✓ answer (2)
4.4	$0\le x<1$ **OR** $x\,\varepsilon\,[\,0\,;\,1)$	✓ 0 and 1 ✓ inequalities (2) ✓ 0 and 1 ✓ brackets (2)
		[10]

QUESTION 5

5.1	$y = a\left(\dfrac{1}{5}\right)^{x} - 5$ $-4 = a\left(\dfrac{1}{5}\right)^{-2} - 5$ $1 = a(25)$ $\dfrac{1}{25} = a$	✓ substitution ✓ simplification (2)
5.2	$0 = \dfrac{1}{25}\left(\dfrac{1}{5}\right)^{x} - 5$ $5 = \dfrac{1}{25}\left(\dfrac{1}{5}\right)^{x}$ $125 = \left(\dfrac{1}{5}\right)^{x}$ $5^{3} = 5^{-x}$ $x = -3 \quad (-3;0)$	✓ $y = 0$ ✓ simplifying ✓ answer (3)
5.3.1	$h:\ y = \left(\dfrac{1}{5}\right)^{x}$ $x = \left(\dfrac{1}{5}\right)^{y}$ Answer only: full marks $y = \log_{\frac{1}{5}} x \ \text{ or }\ y = -\log_{5} x$	✓ **s**wop x and y ✓ answer (2)
5.3.2	$g:\ y = \left(\dfrac{1}{5}\right)^{2}\cdot\left(\dfrac{1}{5}\right)^{x} - 5$ $g(x) = \left(\dfrac{1}{5}\right)^{x+2} - 5$ 2 units right and 5 units up	✓ rewriting g ✓ 2 units right and 5 units up (2)
		[9]

QUESTION 6

6.1	$y = \dfrac{1}{2}x + 2$	
	$0 = \dfrac{1}{2}x + 2$	✓ $y = 0$
	$x = -4$	
	$B(-4; 0)$	✓ answer (2)
6.2	$y = a(x - x_1)(x - x_2)$	✓ coordinates of $A(-6; 0)$
	$y = a(x + 2)(x + 6)$	✓ substitution
	$-12 = a(0 + 2)(0 + 6)$	
	$-12 = 12a$	
	$a = -1$	✓ $a = -1$
	$y = -(x^2 + 8x + 12)$	
	$y = -x^2 - 8x - 12$	✓ answer (4)
	OR	
	$(-2; 0):\ 0 = a(-2)^2 + b(-2) - 12$	✓ substitution
	$\qquad\qquad 12 = 4a - 2b$	
	$\qquad\qquad 6 = 2a - b........(1)$	
	and $\quad \dfrac{-b}{2a} = -4$	✓ substitution
	$\qquad\quad -b = -8a$	
	$\qquad\quad\ b = 8a.....(2)$	
	$6 = 2a - 8a$	
	$6 = -6a$	
	$a = -1$	✓ $a = -1$
	$b = 8(-1) = -8$	
	$y = -x^2 - 8x - 12$	✓ answer (4)

6.3	$FH = -x^2 - 8x - 12 - \left(\dfrac{1}{2}x + 2\right)$	✓ $f(x) - g(x)$
	$= -x^2 - 8x - 12 - \dfrac{1}{2}x - 2$	
	$= -x^2 - \dfrac{17}{2}x - 14$	✓ FH ito x
	$\dfrac{dFH}{dx} = -2x - \dfrac{17}{2} = 0 \quad \text{or} \quad x = -\dfrac{b}{2a}$	✓ method
	$\qquad\qquad -2x = \dfrac{17}{2} \qquad x = -\dfrac{(-8,5)}{2(-1)}$	
	$\qquad\qquad x = -\dfrac{17}{4} \qquad x = -\dfrac{17}{4}$	
	$G\left(-\dfrac{17}{4}; 0\right)$	✓ answer (4)
6.4	$-4 < x < 0$	✓ answer ✓ inequility (2)
	OR	
	$x\varepsilon(-4; 0)$	✓ -4 and 0 ✓ inequility (2)
		[12]

QUESTION 7

7.1	$1 + i_{eff} = \left(1 + \dfrac{i^m}{m}\right)^m$	
	$1 + 0,113 = \left(1 + \dfrac{i^4}{4}\right)^4$	✓ substitution
	$1,027... = 1 + \dfrac{i^4}{4}$	✓ 4th root
	$0,027... = \dfrac{i^4}{4}$	
	$0,1085.. = i^4$	
	$r = 10,85\%$	✓ answer (3)

7.2	$A = 10\,000\left(1 + \dfrac{0,053}{12}\right)^{60}$	✓ substitution into the correct formula
	$A = 13\,026,71$	✓ $n = 60$ and $i = \dfrac{0,053}{12}$
	$F = \dfrac{500\left[\left(1 + \dfrac{0,053}{12}\right)^{58} - 1\right]}{\dfrac{0,053}{12}}$	✓ substitution into the F formula
		✓ $n = 58$
	$F = R32\,970,51$	
	$\text{Total} = 13\,026,71 + 32\,970,51$	
	$\qquad = R45\,997,22$	✓ answer (5)
	OR	
	$= 10\,000\left(1 + \dfrac{0,053}{12}\right)^{60} + \dfrac{500\left[\left(1 + \dfrac{0,053}{12}\right)^{58} - 1\right]}{\dfrac{0,053}{12}}$	✓ substitution into the correct formula
		✓ $n = 60$ and $i = \dfrac{0,053}{12}$
		✓ substitution into the F formula
		✓ $n = 58$
	$= R45\,997,22$	✓ answer (5)
7.3.1	$860\,000 = \dfrac{7\,200\left[1 - \left(1 + \dfrac{0,095}{12}\right)^{-n}\right]}{\dfrac{0,095}{12}}$	✓ substitution into the correct formula
		✓ $i = \dfrac{0,095}{12}$
	$0,945... = 1 - \left(1 + \dfrac{0,095}{12}\right)^{-n}$	
	$-0,054... = -\left(1 + \dfrac{0,095}{12}\right)^{-n}$	
	$0,054... = \left(1 + \dfrac{0,095}{12}\right)^{-n}$	
	$-n = \log_{(1,007..)} 0,054...$	✓ correct use of log
	$-n = -369,212...$	
	Sam will have 370 installments	✓ answer (4)

7.3.2	$A = 860\,000\left(1+\dfrac{0,095}{12}\right)^{369}$ $A = R15\,782\,859,31$ $F = \dfrac{7\,200\left[\left(1+\dfrac{0,095}{12}\right)^{369} - 1\right]}{\dfrac{0,095}{12}}$ $F = R15\,781\,334,69$ Balance $= R1\,524,62$ (after 369 installments) Last installment $= 1\,524,62\left(1+\dfrac{0,095}{12}\right)^{1}$ $\qquad\qquad\quad = R1\,536,69$	✓ loan and interest ✓ $n = 369$ ✓ payment and interest ✓ method ✓ answer (5)
	OR $P = \dfrac{7\,200\left(1-\left(1+\dfrac{0,095}{12}\right)^{-0,2127679...}\right)}{\dfrac{0,095}{12}}$ $P = R1\,524,62$ (after 369 installments) Last installment $= 1\,524,62\left(1+\dfrac{0,095}{12}\right)^{1}$ $\qquad\qquad\quad = R1\,536,69$	✓ method ✓ $n = -0,2127679...$ ✓ balance ✓ method ✓ answer (5)
		[17]

Mathematics P1 NW/September 2023
NSC-Marking guidelines

QUESTION 8

8.1	$f(x) = 1 - x^2$ $f(x+h) = 1-(x+h)^2$ $= 1-(x^2+2xh+h^2)$ $= 1-x^2-2xh-h^2$ $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $= \lim_{h\to 0}\frac{1-x^2-2xh-h^2-(1-x^2)}{h}$ $= \lim_{h\to 0}\frac{-2xh-h^2}{h}$ $= \lim_{h\to 0}\frac{h(-2x-h)}{h}$ $= \lim_{h\to 0}(-2x-h)$ $f'(x) = -2x$	✓ $=1-x^2-2xh-h^2$ ✓ substitution ✓ $\frac{-2xh-h^2}{h}$ ✓ $-2x-h$ ✓ $-2x$ (5)
8.2.1	$D_x\left[3x^2-\frac{2}{x}\right]$ $D_x\left[3x^2-2x^{-1}\right]$ $= 6x+2x^{-2}$	✓ $-2x^{-1}$ ✓ $6x$ ✓ $2x^{-2}$ (3)
8.2.2	$y = \sqrt{x}\left(\sqrt[3]{x}-5x\right)$ $y = x^{\frac{1}{2}}\left(x^{\frac{1}{3}}-5x\right)$ $y = x^{\frac{5}{6}}-5x^{\frac{3}{2}}$ $\frac{dy}{dx} = \frac{5}{6}x^{-\frac{1}{6}}-\frac{15}{2}x^{\frac{1}{2}}$	✓ change from surd to exponential form ✓ $x^{\frac{5}{6}}$ and $-5x^{\frac{3}{2}}$ ✓ $\frac{5}{6}x^{-\frac{1}{6}}$ ✓ $-\frac{15}{2}x^{\frac{1}{2}}$ (4)
		[12]

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QUESTION 9

9.1	$f(x) = ax^3 + bx^2$ $-\dfrac{1}{3} = a(2)^3 + b(2)^2$ $-\dfrac{1}{3} = 8a + 4b.....(1)$ $f'(x) = 3ax^2 + 2bx$ $f'(2) = 3a(2)^2 + 2b(2) = 0$ $\qquad 12a + 4b = 0$ $\qquad\qquad 4b = -12a$ $\qquad\qquad b = -3a.....(2)$ $-\dfrac{1}{3} = 8a + 4(-3a)$ $-\dfrac{1}{3} = 8a - 12a$ $-\dfrac{1}{3} = -4a$ $a = \dfrac{1}{12}$ $b = -\dfrac{1}{4}$	✓ substitution $\left(2 \ ; -\dfrac{1}{3}\right)$ ✓ $f'(x) = 0$ ✓ substitution $x = 2$ ✓ solve simultaneously (4)
9.2	$f''(x) = \dfrac{1}{2}x - \dfrac{1}{2}$ $\qquad \dfrac{1}{2}x - \dfrac{1}{2} < 0$ $\qquad\qquad x < 1 \quad / \quad x\varepsilon(-\infty \ ; 1)$	✓ $f''(x)$ ✓ $f''(x) < 0$ ✓ answer (3)

9.3	$f(x) = \dfrac{1}{12}x^3 - \dfrac{1}{4}x^2$ $f(-2) = \dfrac{1}{12}(-2)^3 - \dfrac{1}{4}(-2)^2 = -\dfrac{5}{3}$ $\left(-2; -\dfrac{5}{3}\right)$ $m = f'(x) = \dfrac{1}{4}x^2 - \dfrac{1}{2}x$ $f'(-2) = \dfrac{1}{4}(-2)^2 - \dfrac{1}{2}(-2)$ $m = 2$ $y = 2x + c$ $-\dfrac{5}{3} = 2(-2) + c$ $c = \dfrac{7}{3}$ $y = 2x + \dfrac{7}{3}$	✓ $y = -\dfrac{5}{3}$ ✓ gradient ✓ substitution of x, y and m ✓ answer (4)
9.4	$-\dfrac{1}{3} < k < 0$ / $k\,\varepsilon\left(-\dfrac{1}{3}; 0\right)$	✓✓ answer (2)
		[13]

QUESTION 10

| 10.1 | $\tan 60° = \dfrac{DF}{x} = \sqrt{3}$

 $DF = \sqrt{3}x$
 $\therefore$ Area rectangle $= DF \times DE$
 $\qquad\qquad\qquad = \sqrt{3}x(m - 2x)$ | ✓ $\tan 60° = \sqrt{3}$
 ✓ $DF = \sqrt{3}x$
 ✓ $DE = (m - 2x)$

 (3) |

10.2	$\text{Area} = \sqrt{3}mx - 2\sqrt{3}x^2$	
	$\dfrac{d\text{A}}{dx} = \sqrt{3}m - 4\sqrt{3}x = 0$	✓ $f'(x) = 0$
	$x = \dfrac{m}{4}$	✓ $x = \dfrac{m}{4}$
	$\text{Max Area} = \sqrt{3}x(m - 2x)$	
	$= \sqrt{3}\left(\dfrac{m}{4}\right)\left(m - 2\left(\dfrac{m}{4}\right)\right)$	✓ substitution
	$= \dfrac{\sqrt{3}}{8}m^2$	✓ $\dfrac{\sqrt{3}}{8}m^2$ (4)
	OR	
	$\dfrac{d\text{A}}{dx} = \sqrt{3}m - 4\sqrt{3}x = 0$	✓ $f'(x) = 0$
	$x = \dfrac{m}{4}$	✓ $x = \dfrac{m}{4}$
	$\sqrt{3}mx - 2\sqrt{3}(x^2)$	
	$= \sqrt{3}m\left(\dfrac{m}{4}\right) - 2\sqrt{3}\left(\dfrac{m}{4}\right)^2$	✓ substitution
	$= \dfrac{\sqrt{3}m^2}{4} - \dfrac{\sqrt{3}m^2}{8}$	
	$= \dfrac{2\sqrt{3}m^2 - 8\sqrt{3}m^2}{8}$	
	$= \dfrac{\sqrt{3}m^2}{8}$	✓ $\dfrac{\sqrt{3}}{8}m^2$ (4)
		[7]

QUESTION 11

11.1.1	$P(A \text{ and } C) = 0$	✓ answer	(1)
11.1.2	$P(A \text{ and } B) = P(A) \times P(B)$ $P(A \text{ and } B) = 0,3 \times 0,43$ $P(A \text{ and } B) = 0,129$ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ $P(A \text{ or } B) = 0,3 + 0,43 - 0,129$ $P(A \text{ or } B) = 0,6$	✓ $P(A \text{ and } B) = 0,129$ ✓ substitution ✓ answer	(3)
11.2.1	$P(G \text{ and } T) = \dfrac{105}{250} = \dfrac{21}{50} = 42\%$	✓ answer	(1)
11.2.2	Independent: $P(T \text{ and } G) = P(T) \times P(G)$ $\begin{array}{ll} P(T) \times P(G) & P(G \text{ and } T) \\ = \dfrac{173}{250} \times \dfrac{130}{250} & = \dfrac{105}{250} \\ = 0.36 & = 0.42 \end{array}$ $P(G \text{ and } T) \neq P(G) \times P(T)$ Events are not independent.	✓ $P(T) = \dfrac{173}{250}$ ✓ $P(G) = \dfrac{130}{250}$ ✓ $P(T) \times P(G) = 0,36$ ✓ answer	(4)
11.3.1	$12!$	✓ answer	(1)
11.3.2	Pieter and John next to one another $= 10!2!$ $11! - 10!2!$ $= 32\,659\,200$	✓ $10!2!$ ✓ $11! - 10!2!$ ✓ answer	(3)
11.3.3	$\dfrac{11!2!}{12!}$ $= \dfrac{1}{6}$	✓ $11!\,2!$ ✓ $12!$ ✓ answer	(3)
			[16]
		TOTAL:	**150**